1. [4 *points*]

(a) Write $\sin x + \cos x$ in the form $R\sin(x+\alpha)$, where R and α are to be found with $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(b) Hence solve the equation:

$$\sin x + \cos x = \frac{\sqrt{2}}{2}$$

for $0 \leqslant x \leqslant 2\pi$.

2. [5 *points*]

(a) Show that:

$$4\cos^3 x = \cos(3x) + 3\cos(x)$$

(b) Hence solve the equation:

$$\cos(3x) + 3\cos(x) = -\frac{3\sqrt{3}}{2}$$

3. [7 *points*]

Let $f(x) = \arcsin(x) + \arccos(x)$.

(a) State the domain of $f(x)$.

(b) Calculate $f(\frac{1}{2})$.

(c) With reference to the graphs of $\arcsin(x)$ and $\arccos(x)$, or otherwise, explain why $0 < f(x) < \pi$.

(d) By simplifying the expression for $\sin(f(x))$, or otherwise, sketch the graph of $f(x)$.

4. [4 *points*]

Given that $\sec\alpha = -\frac{3}{2}$ and $\cot\beta = \frac{1}{2}$ with $0 < \alpha < \pi < \beta < 2\pi$, find the exact value of $\cos(\alpha + \beta)$.