Composition and inverse [74 marks]

1. [Maximum mark: 5] SPM.1.SL.TZ0.5

The functions f and g are defined such that $f(x) = \frac{x+3}{4}$ and $g(x) = 8x + 5$.

(a) Show that $(g \circ f)(x) = 2x + 11$. [2]

(b) Given that $(g \circ f)^{-1}(a) = 4$, find the value of a. [3]

2. [Maximum mark: 5] 23N.1.AHL.TZ1.1

Consider the functions $f(x) = x - 3$ and $g(x) = x^2 + k^2$, where k is a real constant.

(a) Write down an expression for $(g \circ f)(x)$. [2]

(b) Given that $(g \circ f)(2) = 10$, find the possible values of k. [3]

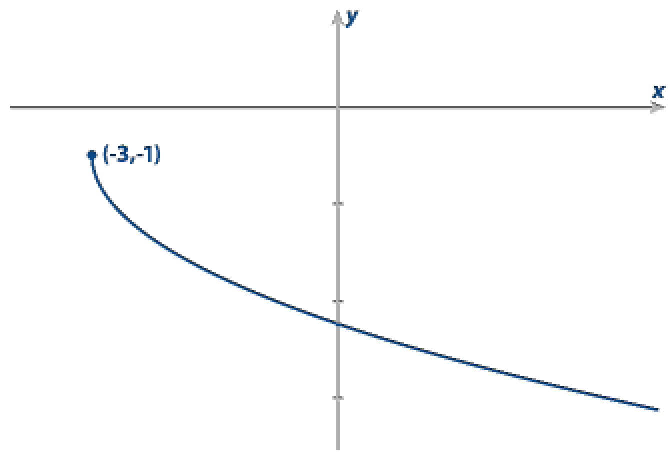
3. [Maximum mark: 6] EXN.1.SL.TZ0.5

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = x - 2$ and $g(x) = ax + b$, where $a,\ b \in \mathbb{R}$.

Given that $(f \circ g)(2) = -3$ and $(g \circ f)(1) = 5$, find the value of a and the value of b. [6]

4. [Maximum mark: 14] EXN.1.SL.TZ0.8

The following diagram shows the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$.



(a) Describe a sequence of transformations that transforms the graph of $y = \sqrt{x}$ for $x \geq 0$ to the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$. [3]

A function f is defined by $f(x) = -1 - \sqrt{x+3}$ for $x \geq -3$.

(b) State the range of f. [1]

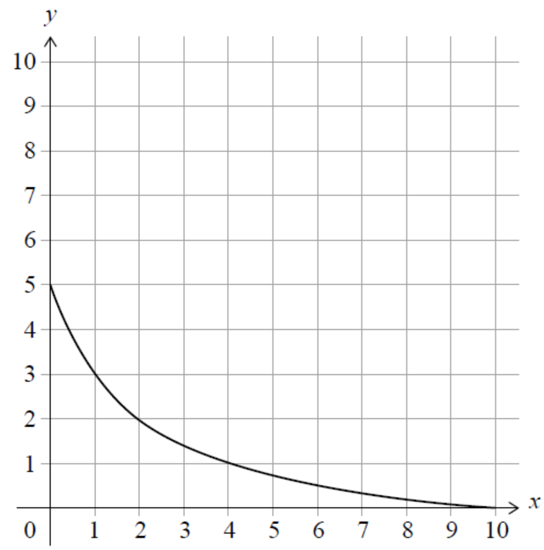
(c) Find an expression for $f^{-1}(x)$, stating its domain. [5]

(d) Find the coordinates of the point(s) where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect. [5]

5. [Maximum mark: 5] 24M.1.SL.TZ2.1

The graph of $y = f(x)$ for $0 \leq x \leq 10$ is shown in the following diagram.

The graph intercepts the axes at $(10,\ 0)$ and $(0,\ 5)$.



(a) Write down the value of

(a.i) $f(4)$; [1]

(a.ii) $f \circ f(4)$; [1]

(a.iii) $f^{-1}(3)$. [1]

(b) On the axes above, sketch the graph of $y = f^{-1}(x)$. Show clearly where the graph intercepts the axes. [2]

6. [Maximum mark: 7] 23M.1.AHL.TZ1.1

The function f is defined by $f(x) = \frac{7x+7}{2x-4}$ for $x \in \mathbb{R}, x \neq 2$.

(a) Find the zero of $f(x)$. [2]

(b) For the graph of $y = f(x)$, write down the equation of

(b.i) the vertical asymptote; [1]

(b.ii) the horizontal asymptote. [1]

(c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

7. [Maximum mark: 7] 23M.1.AHL.TZ2.5

The functions f and g are defined for $x \in \mathbb{R}$ by

$f(x) = ax + b$, where $a, b \in \mathbb{Z}$

$g(x) = x^2 + x + 3$.

Find the two possible functions f such that
$(g \circ f)(x) = 4x^2 - 14x + 15$.

[7]

8. [Maximum mark: 5] 22M.1.SL.TZ2.1

The following table shows values of $f(x)$ and $g(x)$ for different values of x.

Both f and g are one-to-one functions.

x	–2	0	3	4
$f(x)$	8	4	0	–3
$g(x)$	–5	–2	4	0

(a) Find $g(0)$. [1]

(b) Find $(f \circ g)(0)$. [2]

(c) Find the value of x such that $f(x) = 0$. [2]

9. [Maximum mark: 20] 22M.1.AHL.TZ2.11

A function f is defined by $f(x) = \frac{1}{x^2-2x-3}$, where $x \in \mathbb{R},\ x \neq -1,\ x \neq 3$.

(a) Sketch the curve $y = f(x)$, clearly indicating any asymptotes with their equations. State the coordinates of any local maximum or minimum points and any points of intersection with the coordinate axes. [6]

A function g is defined by $g(x) = \frac{1}{x^2-2x-3}$, where $x \in \mathbb{R},\ x > 3$.

The inverse of g is g^{-1}.

(b.i) Show that $g^{-1}(x) = 1 + \frac{\sqrt{4x^2+x}}{x}$. [6]

(b.ii) State the domain of g^{-1}. [1]

A function h is defined by $h(x) = \arctan\frac{x}{2}$, where $x \in \mathbb{R}$.

(c) Given that $(h \circ g)(a) = \frac{\pi}{4}$, find the value of a.

Give your answer in the form $p + \frac{q}{2}\sqrt{r}$, where $p,\ q,\ r \in \mathbb{Z}^+$. [7]