Sequences [119 marks]

1. [Maximum mark: 5] 24M.1.AHL.TZ1.5

Consider a geometric sequence with first term 1 and common ratio 10.

S_n is the sum of the first n terms of the sequence.

(a) Find an expression for S_n in the form $\frac{a^n-1}{b}$, where $a,\ b \in \mathbb{Z}^+$. [1]

(b) Hence, show that $S_1 + S_2 + S_3 + \ldots + S_n = \frac{10(10^n-1)-9n}{81}$. [4]

2. [Maximum mark: 16] 24M.1.AHL.TZ2.10

Consider the arithmetic sequence $a,\ p,\ q \ldots$, where $a,\ p,\ q \neq 0$.

(a) Show that $2p - q = a$. [2]

Consider the geometric sequence $a,\ s,\ t \ldots$, where $a,\ s,\ t \neq 0$.

(b) Show that $s^2 = at$. [2]

The first term of both sequences is a.

It is given that $q = t = 1$.

(c) Show that $p > \frac{1}{2}$. [2]

Consider the case where $a = 9,\ s > 0$ and $q = t = 1$.

(d) Write down the first four terms of the

(d.i) arithmetic sequence; [2]

(d.ii) geometric sequence. [2]

The arithmetic and the geometric sequence are used to form a new arithmetic sequence u_n.

The first three terms of u_n are $u_1 = 9 + \ln 9,\ u_2 = 5 + \ln 3$, and $u_3 = 1 + \ln 1$.

(e.i) Find the common difference of the new sequence in terms of $\ln 3$. [3]

(e.ii) Show that $\sum_{i=1}^{10} = -90 - 25 \ln 3$. [3]

3. [Maximum mark: 7] 23N.1.SL.TZ1.4

The sum of the first n terms of an arithmetic sequence is given by $S_n = pn^2 - qn$, where p and q are positive constants.

It is given that $\mathrm{S}_5 = 65$ and $S_6 = 96$.

(a) Find the value of p and the value of q. [5]

(b) Find the value of u_6. [2]

4. [Maximum mark: 14] 23M.1.AHL.TZ1.10

Consider the arithmetic sequence $u_1, u_2, u_3, \ldots$.

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

(a.i) Find the sum of the first five terms. [2]

(a.ii) Given that $S_6 = 60$, find u_6. [2]

(b) Find u_1. [2]

(c) Hence or otherwise, write an expression for u_n in terms of n. [3]

Consider a geometric sequence, v_n, where $v_2 = u_1$ and $v_4 = u_6$.

(d) Find the possible values of the common ratio, r. [3]

(e) Given that $v_{99} < 0$, find v_5. [2]

5. [Maximum mark: 18] 22M.1.AHL.TZ1.10

Consider the series $\ln x + p \ln x + \frac{1}{3} \ln x + \ldots$, where $x \in \mathbb{R},\ x > 1$ and $p \in \mathbb{R},\ p \neq 0$.

Consider the case where the series is geometric.

(a.i) Show that $p = \pm \frac{1}{\sqrt{3}}$. [2]

(a.ii) Hence or otherwise, show that the series is convergent. [1]

(a.iii) Given that $p > 0$ and $S_\infty = 3 + \sqrt{3}$, find the value of x. [3]

Now consider the case where the series is arithmetic with common difference d.

(b.i) Show that $p = \frac{2}{3}$. [3]

(b.ii) Write down d in the form $k \ln x$, where $k \in \mathbb{Q}$. [1]

(b.iii) The sum of the first n terms of the series is $\ln\left(\frac{1}{x^3}\right)$.

Find the value of n. [8]

6. [Maximum mark: 15] 21N.1.SL.TZ0.8

Consider the function $f(x) = a^x$ where $x,\ a \in \mathbb{R}$ and $x > 0,\ a > 1$.

The graph of f contains the point $\left(\frac{2}{3}, 4\right)$.

(a) Show that $a = 8$. [2]

(b) Write down an expression for $f^{-1}(x)$. [1]

(c) Find the value of $f^{-1}\left(\sqrt{32}\right)$. [3]

Consider the arithmetic sequence
$\log_8 27\ ,\ \log_8 p\ ,\ \log_8 q\ ,\ \log_8 125$, where $p > 1$ and $q > 1$.

(d.i) Show that $27,\ p,\ q$ and 125 are four consecutive terms in a geometric sequence. [4]

(d.ii) Find the value of p and the value of q. [5]

7. [Maximum mark: 9] 21N.2.SL.TZ0.6

The sum of the first n terms of a geometric sequence is given by

$S_n = \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r$.

(a) Find the first term of the sequence, u_1. [2]

(b) Find S_∞. [3]

(c) Find the least value of n such that $S_\infty - S_n < 0.001$. [4]

8. [Maximum mark: 5] 21M.1.SL.TZ1.3

Consider an arithmetic sequence where $u_8 = S_8 = 8$. Find the value of the first term, u_1, and the value of the common difference, d. [5]

9. [Maximum mark: 5] 20N.1.AHL.TZ0.H_5

The first term in an arithmetic sequence is 4 and the fifth term is $\log_2 625$.

Find the common difference of the sequence, expressing your answer in the form $\log_2 p$, where $p \in \mathbb{Q}$. [5]

10. [Maximum mark: 8] 20N.2.SL.TZ0.S_6

An infinite geometric series has first term $u_1 = a$ and second term $u_2 = \frac{1}{4}a^2 - 3a$, where $a > 0$.

(a) Find the common ratio in terms of a. [2]

(b) Find the values of a for which the sum to infinity of the series exists. [3]

(c) Find the value of a when $S_\infty = 76$. [3]

11. [Maximum mark: 5] 19N.2.AHL.TZ0.H_1

A geometric sequence has $u_4 = -70$ and $u_7 = 8.75$. Find the second term of the sequence. [5]

12. [Maximum mark: 7] 19M.2.SL.TZ1.S_7

The first terms of an infinite geometric sequence, u_n, are 2, 6, 18, 54, …

The first terms of a second infinite geometric sequence, v_n, are 2, −6, 18, −54, …

The terms of a third sequence, w_n, are defined as $w_n = u_n + v_n$.

The finite series, $\sum_{k=1}^{225} w_k$, can also be written in the form $\sum_{k=0}^{m} 4r^k$.

(a) Write down the first three **non-zero** terms of w_n. [3]

(b.i) Find the value of r. [2]

(b.ii) Find the value of m. [2]

13. [Maximum mark: 5] 18N.2.AHL.TZ0.H_1

Consider a geometric sequence with a first term of 4 and a fourth term of −2.916.

(a) Find the common ratio of this sequence. [3]

(b) Find the sum to infinity of this sequence. [2]