

MATH HL
TEST
EXPONENTS – LOGARITHMS
(without GDC)
by Christos Nikolaidis

Name: _____

Date: _____

Marks: ____/100
Grade: _____

Questions

1. [Maximum mark: 9]

Let $\ln x = a$, $\ln y = b$ and $\ln 5 = c$. Express the following in terms of a, b and c :

- (a) $\ln \frac{25\sqrt{x}}{y^3}$ (b) $\log_5 xy$ (c) $\log_y 5e$ [3+3+3 marks]

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2. [Maximum mark: 11]

Solve the exponential equations

(a) $8^{x+3} = \left(\frac{1}{16}\right)^{-x-5}$ (b) $7e^{2x+3} = 1$ (c) $2^{x-1}3^{x+3} = 7^{x+2}$ [3+3+5 marks]

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3. [Maximum mark: 6]

Solve the exponential equation

$$2(4^{x+1}) = 2 + \frac{3}{4^x}$$

Give your answer in the following forms

(i) $x = a - \log_4 b$, where a and b are integers.

(ii) $x = \frac{\ln c}{\ln d}$, where c and d are real numbers.

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5. [Maximum mark: 8]

Solve the logarithmic equations

(a) $2\log_3(x-3) = 2 - \log_{1/3}(x+1)$

(b) $\log_x 2 - 3\log_2 x = 2$ [4+4 marks]

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6. [Maximum mark: 9]

Solve the equations

(a) $x^{\ln x} = e^4$

(b) $\frac{x^{\log x}}{x^2} = \frac{\sqrt{x}}{10}$

[4+5 marks]

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7. *[Maximum mark: 6]*
Solve the equation

$$2^{\log x} + 3(4^{\log x}) = 52$$

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8. [Maximum mark: 6]

Solve the simultaneous equations

$$27^y = 9^{2x+3} \quad \text{and} \quad \log_4 y = 2 \log_{16} x + 2$$

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9. [Maximum mark: 8]

It is given that $\log_a(x^2y) = p$ and $\log_a\left(\frac{x}{y^2}\right) = q$

a) Find $\log_a x$ and $\log_a y$ in terms of p and q . [6 marks]

b) Express $\log_a(xy)$ in terms of p and q . [2 marks]

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10. *[Maximum mark: 8]*

The mass m kg of a radio-active substance at time t hours is given by $m = m_0 e^{-kt}$.

a) If the half live time of the material is 3 hours show that $k = \frac{\ln 2}{3}$

b) If the mass of the substance after 6 hours is 1kg, find its mass after 12 hours..

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11. [Maximum mark: 10]

Let $f(x) = e^{3-2x}$ and $g(x) = \frac{3 - \ln x}{2}$

- a) Show that f and g are inverse to each other.
- b) Find $(f \circ g)(x)$ in the simplest form.
- c) Sketch the graphs of f and g ; Indicate intercepts and asymptotes.
- d) Write down the number of solutions of the equation $f(x) = g(x)$.

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12. [Maximum mark: 12]

Consider the function $f(x) = e^{x+2}$.

- a) Sketch the graph of $f(x)$ by indicating clearly the intercepts and any asymptotes.
- b) Write down the domain and the range of f
- c) Find f^{-1}
- d) On the same axes, sketch the graph of f^{-1} by indicating clearly the intercepts and any asymptotes.
- e) Write down the domain and the range of f^{-1} .

[12 marks]

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