MATH HL TEST

EXPONENTS – LOGARITHMS

(without GDC)

by Christos Nikolaidis

Na	ame:			Marks:/80
Da	ate: 12 – 12 – 201	Grade:		
		Question	ns	
1.	[Maximum mark: 7] Let $\log_2 3 = a$, $\log_2 3 = a$	5 = b. Express the following	owing in terms of a	a and b :
	(a) $\log_2 60$	(b) $\log_4 0.2$	(c) $\log_3 25$	[2+3+2 marks]
• • •				

2.	[Maximum mark: 5] Solve the exponential equation
	$2^{x+5} = \frac{3 \cdot (5^x)}{2^{x-2}}.$
	Give your answer in the form $x = \frac{\ln a}{\ln b}$, where $a, b \in Q$.

3.	[Maximum mark: 6] Solve the exponential equation
	$2^{x+5} = 8 + \frac{3}{2^{x-2}}.$
	Give your answer in the form $x = a + \log_2 b$, $a, b \in \mathbb{Z}$.

4.	Fin	Find the values of (a) $A = (\log_2 3)(\log_3 4)(\log_4 5) \cdots (\log_{15} 16)$, as an integer															/	[4 marks]								
		(b) $B = \log_2 3 + \log_2 3^2 + \log_2 3^3 + \dots + \log_2 3^{20}$ in the form $a \log_2 3$													b			arks								
• •																	 	 	 	 						
• •																	 	 	 	 						•
• •																	 	 	 	 						•
• •																	 	 	 	 						•
• •																	 	 	 	 						•
• •																	 	 	 	 						•
• •																	 	 	 	 						
• •																	 	 	 	 						•
• •																	 	 	 	 						
• •																	 	 	 	 						
• •																	 	 	 	 						•
• •																	 	 	 	 						•
							· • •										 	 	 	 						•
							· • •										 	 	 	 						•
• •																	 	 	 	 						•
• •							· • •										 	 	 	 						
							· • •										 	 	 	 						•
																	 	 	 	 						•
• •							· • •										 	 	 	 						
• •							· • •										 	 	 	 						

5.	[Maximum mark: 7] Solve the logarithmic equa	ition
		$\log_{\sqrt{5}}(4 - \frac{x}{5}) - 4\log_5(\frac{x}{5}) = 2$
• • •		

6.	[Maximum mark: 6] Solve the logarithmic equation
	$\log_x 2 - 3\log_2 x = 2$
• •	
• •	
• •	
• •	
• •	
• •	

7.	[Maximum mark: 6] Solve the equation
	$\ln(e^{4x}-8)=2x+\ln 7$

8.	[Maximum mark: 7]	
	It is given that $\log_a(x^2y) = p$ and $\log_a(\frac{x}{y^2}) = q$	
	(a) Find $\log_a x$ and $\log_a y$ in terms of p and q .	[5 marks]
	(b) Hence find $\log_x y$ in terms of p and q .	[2 marks]

9.	[Maximum mark: 6] The mass m kg of a radio-active substance at time t hours is given by $m = 8e$ The half life time of the substance is 3 hours													
	The half life time of the substance is 3 hours. (a) Find the initial mass of the substance. [1]													
	(b) Show that $k = \frac{\ln 2}{3}$	[3 marks]												
	(c) Given that the mass of the substance after t hours is 1kg, find the value of t.	[2 marks]												

10. [Maximum mark: 6] Solve the equation	
	$8(3^x) + 5(2^x) - 6^x = 40$

11. [Maximum mark: 16]	
Let $f(x) = 2e^{5x} - 3$	
(a) Write down the domain and the range of f .	[2 marks]
(b) Find f^{-1}	[3 marks]
(c) Sketch the graphs of <i>f</i> . Indicate intercepts and asymptotes.	[3 marks]
(d) Sketch the graphs of f^{-1} . Indicate intercepts and asymptotes.	[2 marks]
(e) Sketch the graphs of $\frac{1}{f}$. Indicate intercepts and asymptotes.	[3 marks]
(f) Sketch the graphs of $\frac{1}{f^{-1}}$. Indicate intercepts and asymptotes.	[3 marks]

 	 	 	 	• •	 	•	 	•	 		 •	 	•	 	 	 		 		 	 	
 	 	 	 •		 	•	 		 	•		 	•	 		 		 		 	 	
 	 	 	 •		 	•	 		 			 	•	 	 	 		 		 	 	
 	 	 			 		 		 		 •	 		 	 	 		 		 	 	
 	 	 	 •		 	•	 		 			 		 		 		 		 	 	
 	 	 		• •	 		 	•	 	•	 •	 		 	 •	 		 		 	 	
 	 	 	 •		 		 		 	•		 		 	 	 		 	٠.	 	 	
 	 	 	 •		 		 		 	•		 		 	 	 		 	٠.	 	 	
 	 	 	 •		 	•	 		 	•		 	•	 	 	 		 		 	 	
 	 	 	 		 	•	 		 	•	 •	 		 		 		 		 	 	
 	 	 	 		 	•	 		 	•	 •	 		 		 		 		 	 	
 	 	 	 	• •	 		 	•	 		 •	 		 	 •	 		 		 	 	
 	 	 			 		 		 	•		 		 	 	 		 		 	 	
 	 	 	 		 	•	 		 	•	 •	 		 		 		 		 	 	
 	 	 	 •		 		 		 	•		 		 	 	 		 		 	 	
 	 	 			 		 		 	•		 		 	 	 		 		 	 	
 	 	 	 •		 		 		 	•		 		 	 	 		 		 	 	
 	 	 	 •		 	•	 		 			 		 	 	 		 		 	 	
 	 	 			 		 		 	•		 		 	 	 		 		 	 	
 	 	 	 •		 		 		 	•		 		 	 	 		 		 	 	
 	 	 	 •		 	•	 		 		 •	 		 	 	 		 		 	 	
 	 	 	 •		 	•	 		 			 		 		 		 		 	 	
 	 	 			 	•	 		 	•	 •	 		 	 	 		 		 	 	
 	 	 			 		 		 	•		 		 	 	 		 		 	 	