Mixed Practice

1 Let $f(x) = \dfrac{2x+1}{(3x-2)(x+2)}$.

a State the equation of the vertical asymptotes.
b Find the coordinates of the axis intercepts.
c Sketch the graph of $y = f(x)$.

2 Let $f(x) = x - 2 - \dfrac{8}{x-4}$.

a State the equation of
  i the vertical asymptote
  ii the oblique asymptote.
b Find the coordinates of the axis intercepts.
c Sketch the graph of $y = f(x)$.

3 Find the set of values of x for which $6x + x^2 - 2x^3 < 0$.

4 **a** Show that $(x + 2)$ is a factor of $x^3 - 3x^2 - 6x + 8$.
b Hence solve the inequality $x^3 - 1 \geqslant 3(x^2 + 2x - 3)$.

5 Solve the inequality $2x^4 - 5x^2 + x + 1 < 0$.

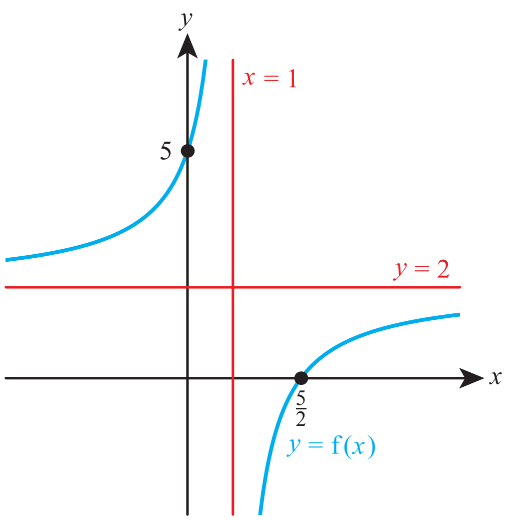
6 Solve the inequality $\ln x \leqslant e^{\sin x}$ for $0 < x \leqslant 10$.

7 **a** Sketch the graph of $y = |\cos 3x|$ for $0 \leqslant x \leqslant \pi$.
b Solve $|\cos 3x| = \dfrac{1}{2}$ for $0 \leqslant x \leqslant \pi$.

8 **a** On the same axes, sketch the graphs of $y = |4 + x|$ and $y = |5 - 3x|$, labelling any axis intercepts.
b Hence solve the inequality $|4 + x| \leqslant |5 - 3x|$.

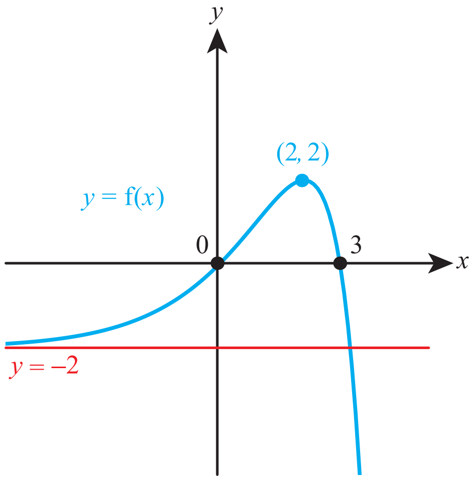
9 **a** On the same axes, sketch the graphs of $y = |5x + 1|$ and $y = 3 - x$, labelling any axis intercepts.
b Hence solve the inequality $3 - x > |5x + 1|$.

10 The graph of $y = f(x)$ is shown below.



Labelling any axis intercepts and asymptotes, on separate axes sketch the graph of
a $y = |f(x)|$
b $y = f(|x|)$.

11 The graph of $y = \mathrm{f}(x)$ is shown below.



Labelling any x-axis intercepts, turning points and asymptotes, on separate axes sketch the graph of

a $y = \dfrac{1}{\mathrm{f}(x)}$

b $y = [\mathrm{f}(x)]^2$

c $y = \mathrm{f}(2x - 1)$.

12 The function f is defined by $\mathrm{f}(x) = 3^x + 3^{-x}$. Deterine algebraically whether f is even, odd or neither.

13 The function f is defined by $\mathrm{f}(x) = -x^2 + 6x - 4,\ x \geqslant k$.

a Find the smallest value of k such that f has an inverse function.

b For this value of k, find $\mathrm{f}^{-1}(x)$ and state its domain.

14 The functions f and g are defined by $\mathrm{f}(x) = ax^2 + bx + c,\ x \in \mathbb{R}$ and $\mathrm{g}(x) = p \sin x + qx + r,\ x \in \mathbb{R}$ where a, b, c, p, q, r are real constants.

a Given that f is an even function, show that $b = 0$.

b Given that g is an odd function, find the value of r.

The functions h is both odd and even, with domain $\mathbb{R}$.

c Find $\mathrm{h}(x)$.

Mathematics HL May 2015 Paper 1 TZ1 Q5

15 a i Find the set of values of k for which the equation $kx^2 - 2(k + 1)x + 7 - 3k = 0$ has real roots.

ii Hence determine the range of the function $\mathrm{f}(x) = \dfrac{2x - 7}{x^2 - 2x - 3}$.

b Sketch the graph of $y = \mathrm{f}(x)$ labelling any vertical asymptotes.

16 a Sketch the graph of $y = |2|x| - 3|$. State the coordinates of any axis intercepts.

b Solve the equation $|2|x| - 3| = 2$.

17 The function f is defined by $\mathrm{f}(x) = (x - a)(x - b)$. On separate axes, sketch the graph of $y = \mathrm{f}(|x|)$ in the case where

a $0 < b < a$ **b** $b < 0 < a$ **c** $b < a < 0$.

18 a Describe a sequence of two transformations that map the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}\left(\dfrac{x - 6}{3}\right)$.

b Describe a different sequence of two transformations that has the same effect as in part **a**.

19 Let $f(x) = x^2 - 3$.

a On the same axes, sketch the graphs of $y = |f(x)|$ and $y = \frac{1}{f(x)}$.

b Hence solve the inequality $|f(x)| \leqslant \frac{1}{f(x)}$.

20 Given $f(x) = |x + a| + |x + b|$, where $a, b \neq 0$, find the condition on a and b such that f is an even function.

21 The function f is defined by $f(x) = e^{2x} - 8e^x + 7, x \leqslant k$.

a Find the largest value of k such that f has an inverse function.

b For this value of k, find $f^{-1}(x)$ and state its domain.

22 The function f is defined by $f(x) = xe^{\frac{x}{2}}, x \geqslant k$.

a Find $f'(x)$ and $f''(x)$.

b Find the smallest value of k such that f has an inverse function.

c For this value of k, find the domain of f^{-1}.

23 Let $f(x) = \frac{3x}{x^2 + 1}$.

a **i** Show algebraically that f is an odd function.

ii What type of symmetry does this mean the graph of $y = f(x)$ must have?

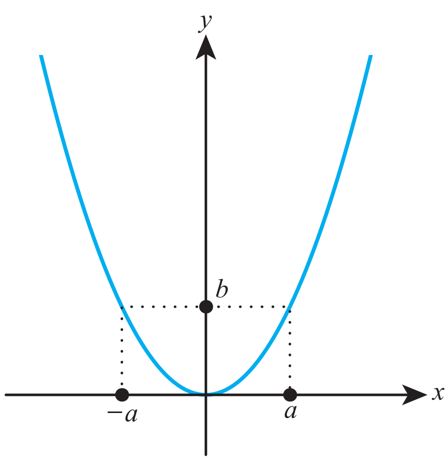
b **i** If the line $y = k$ intersects the curve, show that $4k^2 - 9 \leqslant 0$.

ii Hence find the coordinates of the turning points of the curve.

c Sketch the graph of $y = |f(x)|$.

d Solve the inequality $|f(x)| \geqslant |x|$.

24 The diagram below shows the graph of the function $y = f(x)$, defined for all $x \in \mathbb{R}$, where $b > a > 0$.



Consider the function $g(x) = \frac{1}{f(x-a) - b}$.

a Find the largest possible domain of the function g.

b Sketch the graph of $y = g(x)$. Indicate any asymptotes and local maxima or minima, and write down their equations and coordinates.

Mathematics HL May 2011 Paper 1 TZ1 Q10

25 The function f is defined by $f(x)=\dfrac{2x-1}{x+2}$, with domain $D=\{x: -1 \leqslant x \leqslant 8\}$.

a Express $f(x)$ in the form $A+\dfrac{B}{x+2}$, where A and $B \in \mathbb{Z}$.
b Hence show that $f'(x) > 0$ on D.
c State the range of f.
d **i** Find an expression for $f^{-1}(x)$.
ii Sketch the graph of $f(x)$, showing the points of intersection with both axes.
iii On the same diagram, sketch the graph of $y = f^{-1}(x)$.
e **i** On a different diagram, sketch the graph of $y = f(|x|)$ where $x \in D$.
ii Find all the solutions of the equation $f(|x|) = -\dfrac{1}{4}$.

Mathematics HL May 2013 Paper 1 TZ2 Q12

26 Let $f(x)=\dfrac{x^2+7x+10}{x+1}$.

a Find the equation of the oblique asymptote.
b By finding a condition on k such that $f(x) = k$ has real solutions, or otherwise, find the coordiantes of the turning points of f.
c On the same axes, sketch the grap of $y = f(x)$ and $y = 2x + 7$.
d Hence solve the inequality $\dfrac{x^2+7x+10}{x+1} < 2x+7$.
e On a separate set of axes, sketch the graph of $y = |f(x)|$, labelling the coordinates of all axis intercepts.
f State the complete set of values of c for which $|f(x)| = c$ has two solutions.

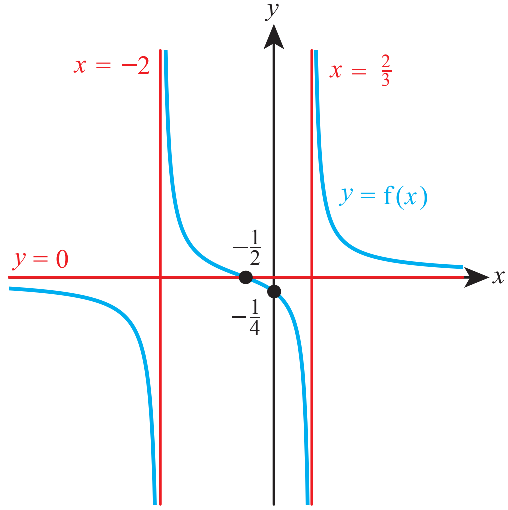
27 The function f is defined by $f(x)=\dfrac{ax^2+bx+c}{dx+e}$ and the function g is defined by $g(x)=\dfrac{1}{f(x)}$. $f(x)$ has an oblique asymptote $y = x + 1$ and $g(x)$ has vertical asymptotes $x=\dfrac{3}{2}$ and $x = -4$. Solve the equation $f(x) = g(x)$.

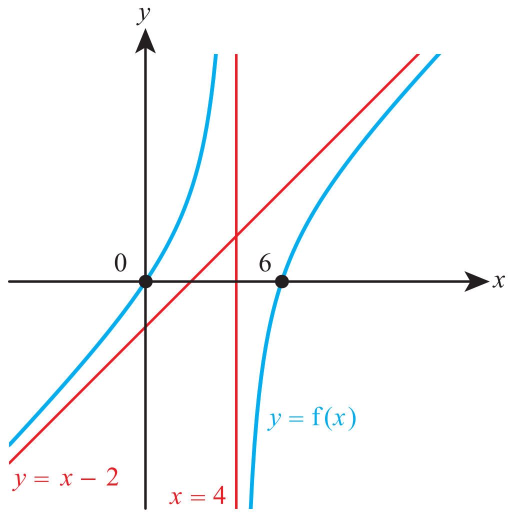
28 Find the value of c for which the function $f(x)=\dfrac{3x-5}{x+c}$ is self-inverse.

Exercise 7E

1 a Neither b Even
2 a Neither b Odd
3 a Neither b Even
4 a Even b Neither
5 a Odd b Even
6 a Neither b Even
7 a Odd b Neither
8 a Even b Neither
9 a $x \leqslant 2$ b $x \geqslant 2$
10 a $x \geqslant 4$ b $x \leqslant -2$
11 a $x \leqslant 1$ b $x \geqslant 3$
12 a Yes b Yes
13 a No b Yes
14 a No b Yes
15 Odd
16 Neither
17 Odd
18 a $k = -4$
b $f^{-1}(x) = -4 + \sqrt{x-3},\ x \geqslant 3$
19 a $x \leqslant \frac{3}{2}$
b $f^{-1}(x) = \frac{3}{2} - \sqrt{x + \frac{5}{4}},\ x \geqslant -\frac{5}{4}$
20 b $x \in \mathbb{R}$
21 b Neither
23 Even
24 a $-3 \leqslant x \leqslant -1$
b $-6 \leqslant x \leqslant -2$
25 a $k = 2$
b $x \geqslant -11$
26 a $x \leqslant \ln 4$
b $x \leqslant 4 - 4\ln 4$
27 b $x \neq \frac{2}{3}$
28 $a = -1$
30 $c = 2$

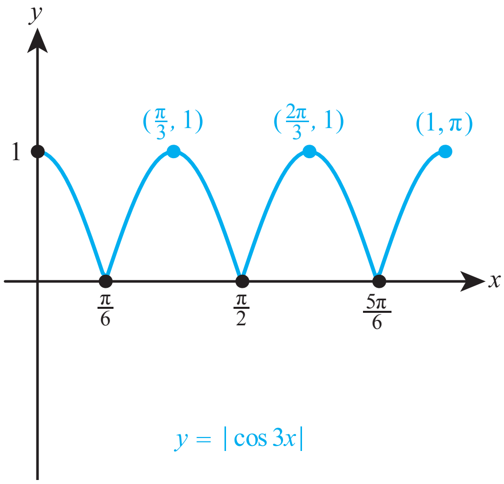
Chapter 7 Mixed Practice

1 a $x = \frac{2}{3},\ x = -2$
b $\left(-\frac{1}{2}, 0\right)$ and $\left(0, -\frac{1}{4}\right)$
c


2 a i $x = 4$ ii $y = x - 2$
b (0, 0) and (6, 0)
c


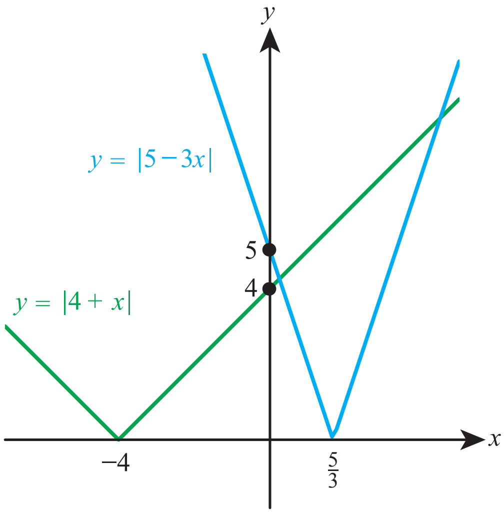
3 $\left]-\frac{3}{2}, 0\right[\cup \left]2, \infty\right[$
4 b $-2 \leqslant x \leqslant 1$ or $x \geqslant 4$
5 $-1.62 \leqslant x \leqslant -0.366$ or $0.618 \leqslant x \leqslant 1.37$
6 $0 < x \leqslant 3.04$ or $7.01 \leqslant x \leqslant 8.56$

7 a



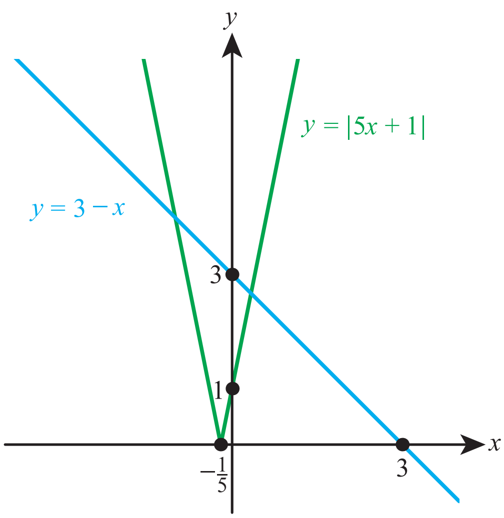
b $x = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}$

8 a



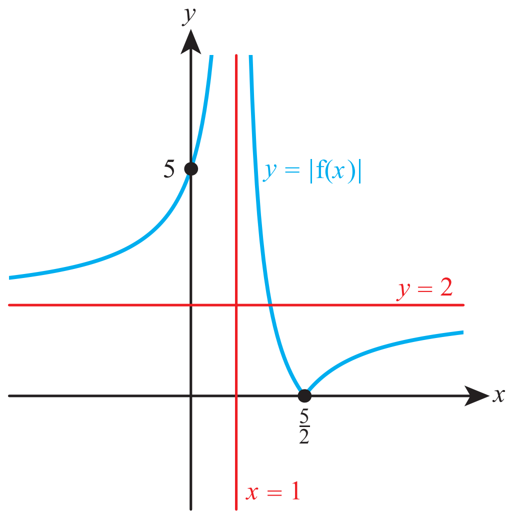
b $x \leqslant \frac{1}{4}$ or $x \geqslant \frac{9}{2}$

9 a

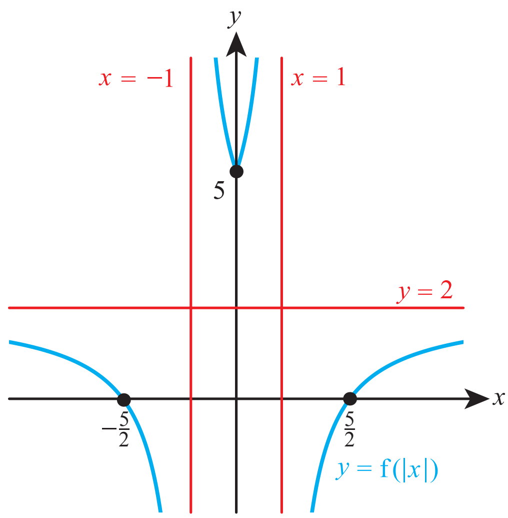


b $-1 < x < \frac{1}{3}$

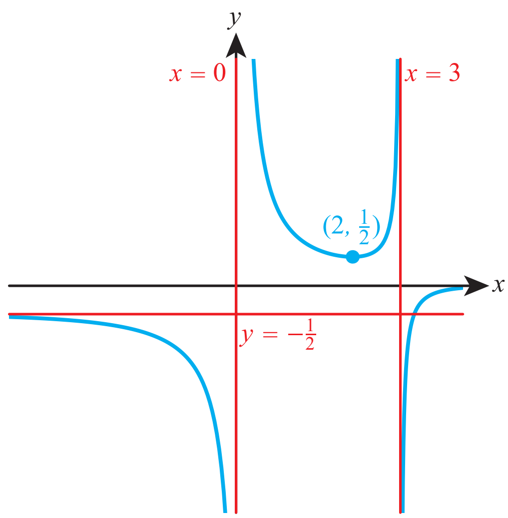
10 a



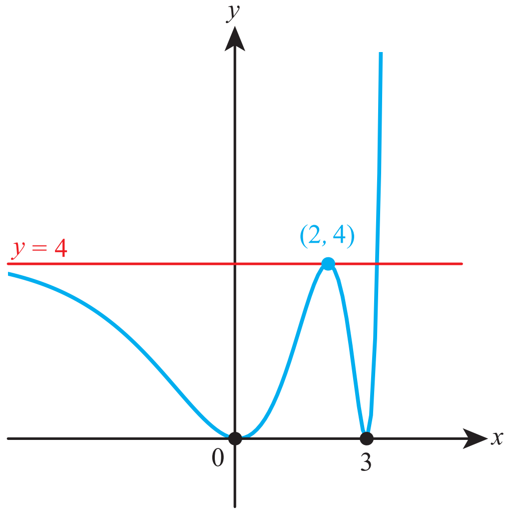
b



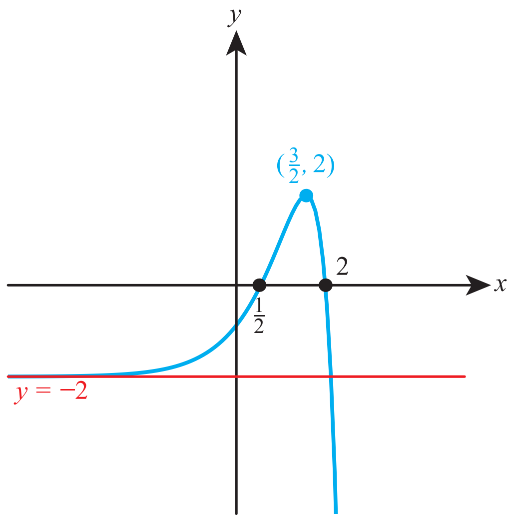
11 a



b



c



12 Even

13 a $k = 3$

b $f^{-1}(x) = 3 + \sqrt{5 - x},\ x \leqslant 5$

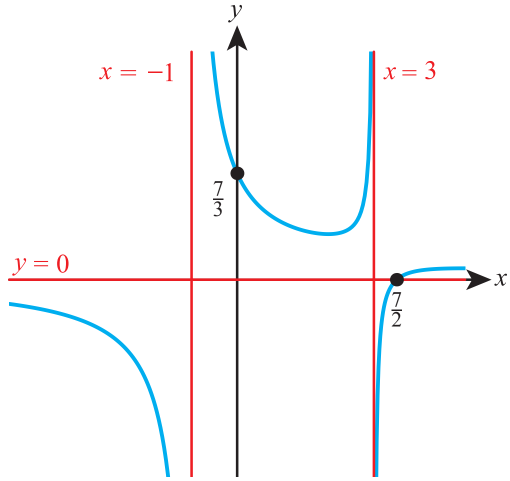
14 b $r = 0$

c $k(x) = 0$

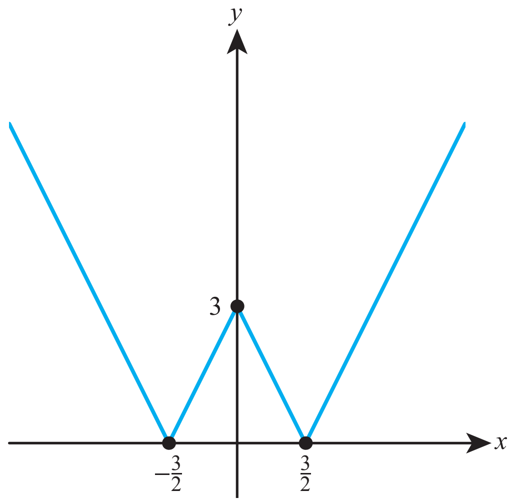
15 a i $k \leqslant \frac{1}{4}$ or $k \geqslant 1$

ii $f(x) \leqslant \frac{1}{4}$ or $f(x) \geqslant 1$

b



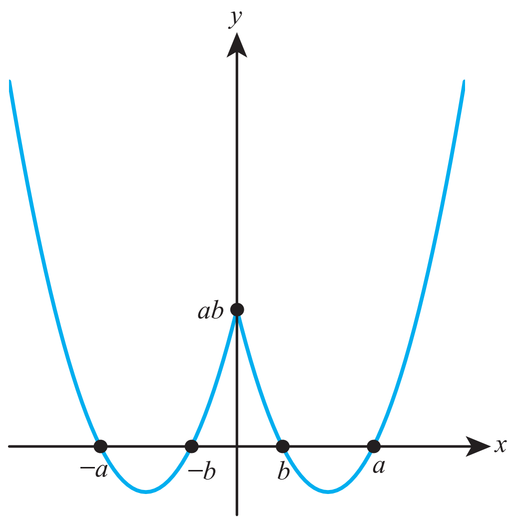
16 a



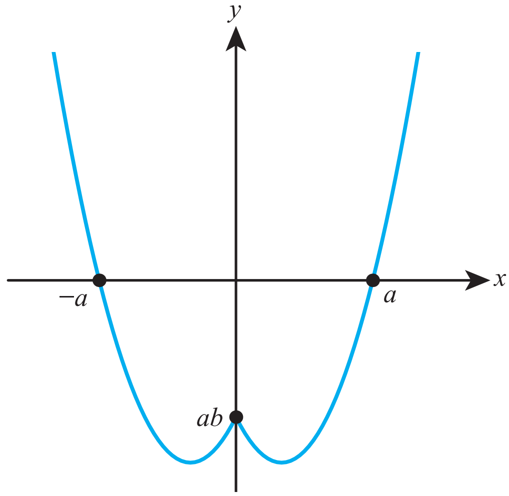
$\left(-\frac{3}{2}, 0\right), \left(\frac{3}{2}, 0\right), (0, 3)$

b $x = \pm\frac{1}{2}, \pm\frac{5}{2}$

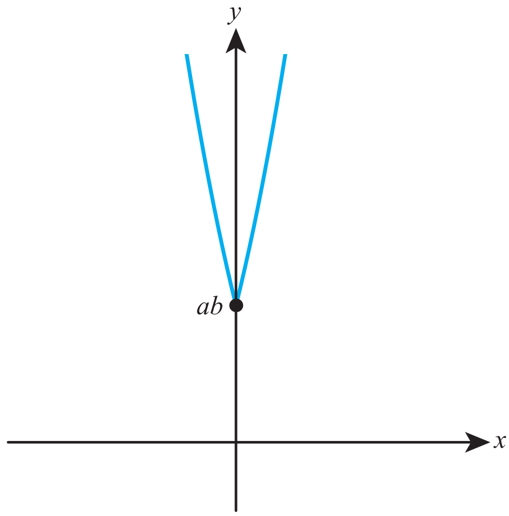
17 a



b



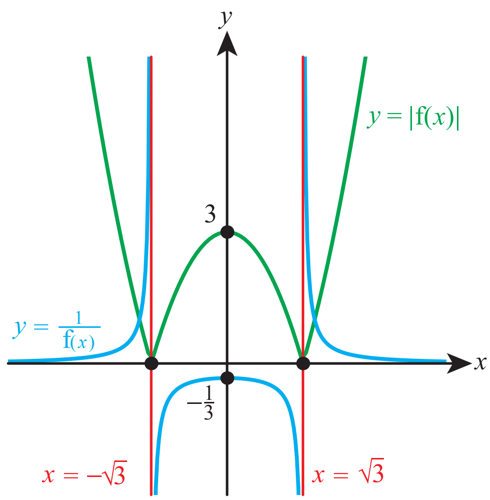
c



18 **a** Horizontal stretch with scale factor 3 followed by horizontal translation by +6

b Horizontal translation by +2 followed by horizontal stretch with scale factor 3

19 **a**



b $-2 \leq x < -\sqrt{3}$ or $\sqrt{3} < x \leq 2$

20 $a = -b$

21 **a** $k = \ln 4$

b $f^{-1}(x) = \ln(4 - \sqrt{x+9}),\ -9 \leqslant x < 7$

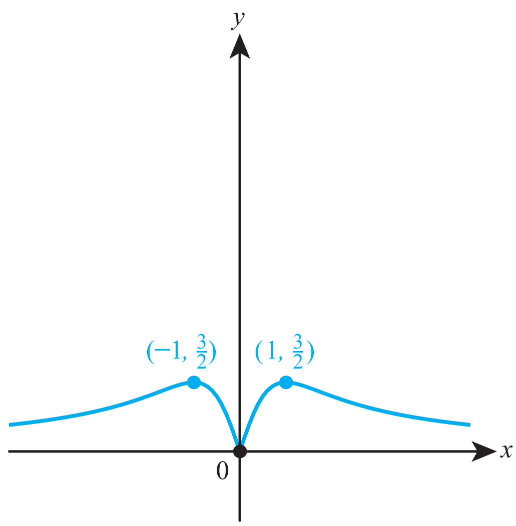
22 **a** $f'(x) = e^{\frac{x}{2}} + \frac{x}{2}e^{\frac{x}{2}},\ f''(x) = e^{\frac{x}{2}} + \frac{x}{4}e^{\frac{x}{2}}$

b $k = -2$ **c** $x \geqslant -2e^{-1}$

23 **a** **ii** Rotation 180° around the origin

b **ii** $\left(1, \frac{3}{2}\right), \left(-1, -\frac{3}{2}\right)$

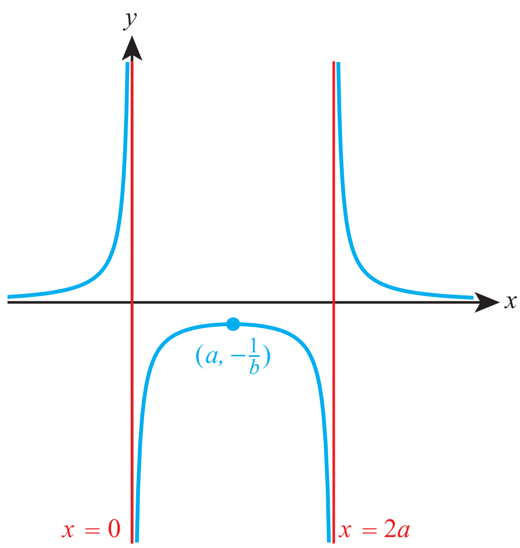
c



d $-\sqrt{2} \leqslant x \leqslant \sqrt{2}$

24 **a** $x \neq 0,\ 2a$

b

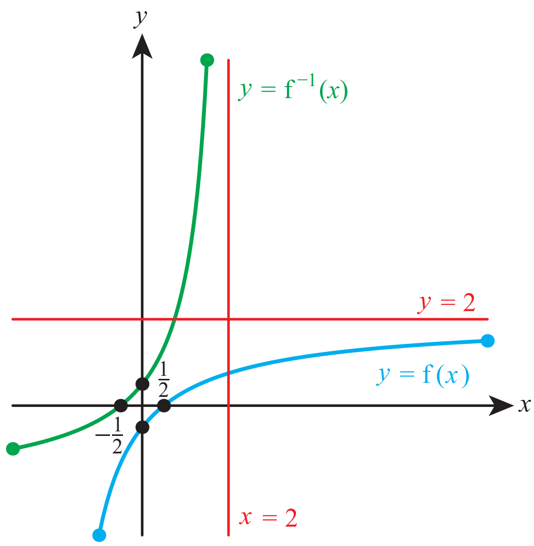


25 **a** $2-\dfrac{5}{x+2}$

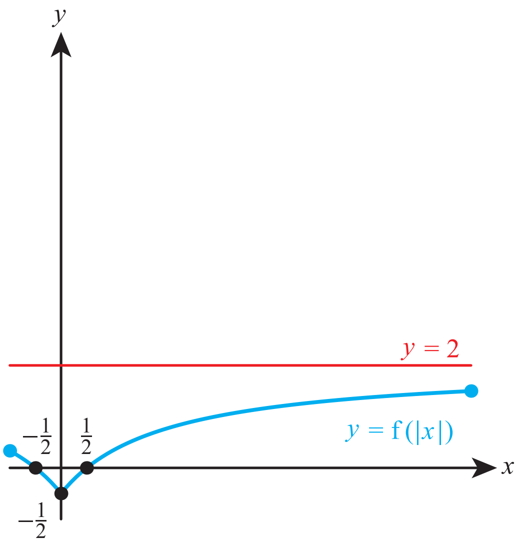
c $-3 \leqslant f(x) \leqslant 1.5$

d **i** $f^{-1}(x)=\dfrac{2x+1}{2-x}$

ii, iii



e **i**

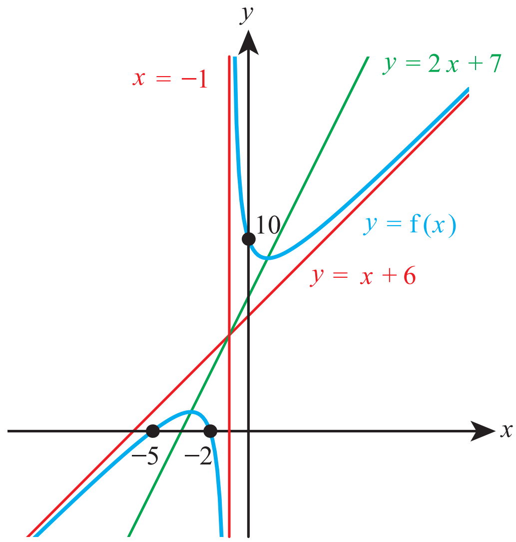


ii $x = \pm\dfrac{2}{9}$

26 **a** $y = x + 6$

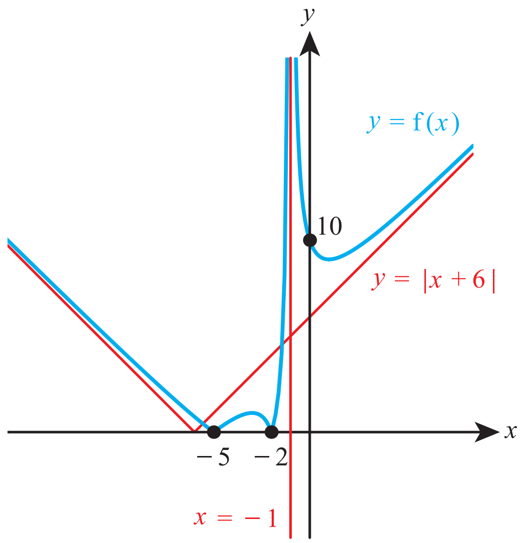
b (–3, 1) and (1, 9)

c



d $-3 < x < -1$ or $x > 1$

e



f $c = 0$ or $1 < c < 9$

27 $x = -4.5, -3.59, 1, 2.09$

28 $c = -3$