

Mixed Practice



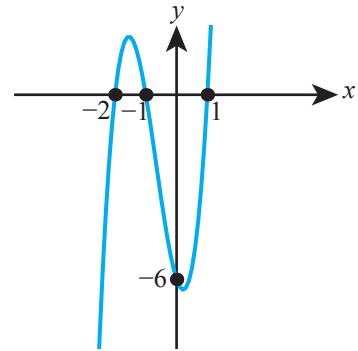
1 a Factorize $f(x) = 3x^3 - 2x^2 - x$.

b Hence sketch the graph of $y = f(x)$, showing all the axis intercepts.

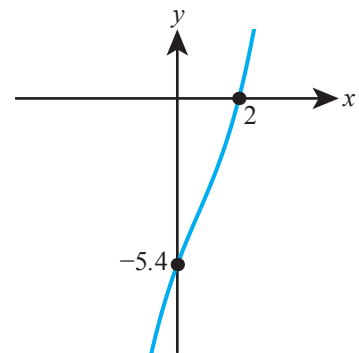


2 Sketch the graph of $y = (x - 3)(x + 1)(5 - 2x)$ showing all the axis intercepts.

3 The diagram shows the graph of $y = f(x)$, where f is a cubic polynomial. Find an expression for $f(x)$, giving your answer in a fully expanded form.



4 The diagram shows the graph of $y = 0.3(x - p)(x^2 + k^2)$, where $k > 0$. Find the values of p and k .



5 A polynomial is given by $f(x) = (ax + b)^3$. The remainder when $f(x)$ is divided by $(x - 2)$ is 8 and the remainder when it is divided by $(x + 3)$ is -27 . Find the values of a and b .

6 $f(x) = x^3 + 4x^2 + ax + b$ has a factor of $(x - 1)$ and leaves a remainder of 17 when divided by $(x - 2)$. Find the constants a and b .

7 When $f(x) = x^4 + px^2 - x + q$ is divided by $(x - 3)$ the remainder is 52 and $(x + 1)$ is a factor of $f(x)$. Find the values of p and q .

8 The quadratic equation $5x^2 + bx + c = 0$ has real coefficients and one of its roots is $4 + 7i$. Find the values of b and c .

9 The sum of the roots of the equation $4x^3 - ax^2 + 5x + 3 = 0$ is $\frac{3}{2}$. Find the value of a .

10 The mean of the roots of the equation $kx^4 + 8x^3 - 3x + 1 = 0$ is $-\frac{1}{2}$. Find the value of k .

11 The equation $4x^4 - 40ax^3 + 140a^2x^2 - 200a^3x + 1536 = 0$ has roots a , $2a$, $3a$ and $4a$. Find the possible values of a .

12 The equation $3x^3 + 2x^2 - x + 5 = 0$ has roots a , b and c .

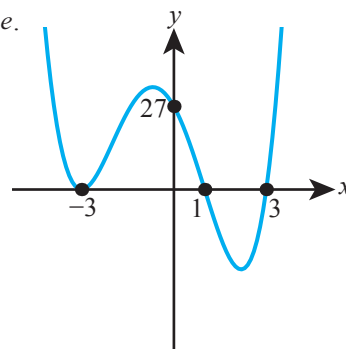
a Write down the value of abc .

b Find the value of $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}$.

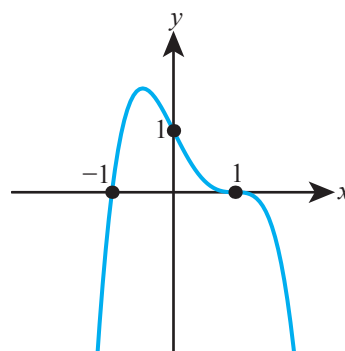
- 13** The same remainder is found when $2x^3 + kx^2 + 6x + 32$ and $x^4 - 6x^2 - k^2x + 9$ are divided by $x + 1$. Find the possible values of k .

Mathematics HL May 2012 Paper 1 TZ2 Q1

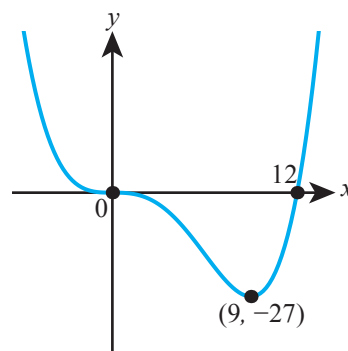
- 14** The diagram shows the graph with equation $y = ax^4 + bx^3 + cx^2 + dx + e$. Find the values of a , b , c , d and e .



- 15** The diagram shows the graph of $y = f(x)$, where $f(x)$ is a polynomial of order 4. Find an expression for $f(x)$, giving your answer in an expanded form.



- 16** The diagram shows a graph of a polynomial of order 4. Find the expression for the polynomial, giving your answer in an expanded form.



- 17** a Show that $(x - 2)$ is a factor of $f(x) = 2x^3 - 5x^2 + x + 2$.
 b Factorize $f(x)$ completely.
 c Hence sketch the graph $y = f(x)$, showing all the axis intercepts.
- 18** a Show that $(x + 1)$ is a factor of $f(x) = x^3 - 4x^2 + x + 6$.
 b Factorize $f(x)$.
 c Sketch the graph of $y = f(x)$.

- 19** Sketch the graph of $y = (x - a)^2(x - b)(x - c)$ where $b < 0 < a < c$.

- 20** Given that one of the roots of the equation $x^3 + ax^2 - 7x + 15 = 0$ is -3 , find all the roots.

- 21** One of the roots of the equation $ax^3 + bx^2 + 177x - 210 = 0$ is 2 , and the sum of the other two roots is 12 . Find the values of a and b .



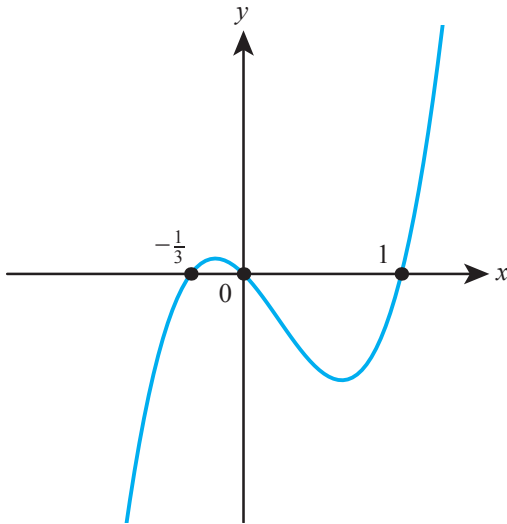


- 22** One of the roots of the equation $3x^3 - 12x^2 + 16x - 8 = 0$ is an integer between 0 and 3 inclusive. Find the other two roots.
- 23** The roots p and q of the equation $ax^2 + bx + c = 0$ satisfy $\frac{1}{p} + \frac{1}{q} = 0$. Show that $b + 3c = 0$.
- 24** Let $f(x) = (ax + b)^4$. The remainder when $f(x)$ is divided by $(x - 2)$ is 16 and the remainder when it is divided by $(x + 1)$ is 81. Find the possible values of a and b .
- 25** The quartic equation $x^4 + px^3 + 14x^2 - 18x + q = 0$ has real coefficients and two of its roots are $3i$ and $1 - 2i$. Find the values of p and q .
- 26** The quadratic equation $3x^2 - 4x + 7 = 0$ has roots p and q .
- Find the value of $p^2 + q^2$.
 - Find a quadratic equation with integer coefficients and roots p^2 and q^2 .
- 27** Let $g(x) = 3x^5 - 6x^4 + 13x^2 - 2x + 18$.
- Write down the sum of the roots of the equation $g(x) = 0$.
 - A new polynomial is defined by $h(x) = g(x - 4)$. Find the sum of the roots of the equation $h(x) = 0$.
- 28** Let $f(x) = 5x^4 + 2x^3 - x^2 - x + 3$.
- Write down the sum and the product of the roots of the equation $f(x) = 0$.
 - Find the product of the roots of the equation $f(3x) = 0$.
- 29** The function $f(x) = 4x^3 + 2ax - 7a$, $a \in \mathbb{R}$, leaves a remainder of -10 when divided by $(x - a)$.
- Find the value of a .
 - Show that for this value of a there is a unique real solution to the equation $f(x) = 0$.
- Mathematics HL May 2011 Paper 2 TZ1 Q4
- 30** The equation $5x^3 + 48x^2 + 100x + 2 = a$ has roots r_1, r_2 and r_3 . Given that $r_1 + r_2 + r_3 + r_1 r_2 r_3 = 0$, find the value of a .
- Mathematics HL May 2014 Paper 1 TZ1 Q4
- 31 a** Find the exact solutions of the equation $x^2 - 4x + 5 = 0$.
- b** Given that $x^2 - 4x + 5$ is a factor of $x^4 - 4x^3 + 8x^2 + ax + b$, find the values of a and b .
- 32** The polynomial $x^2 - 4x + 3$ is a factor of the polynomial $x^3 + ax^2 + 27x + b$. Find the values of a and b .
- 33 a** Given that a polynomial $f(x)$ can be written as $f(x) = (x - a)^2 g(x)$, show that $f'(x)$ has a factor $(x - a)$.
- b** The polynomial $2x^4 + bx^3 + 11x^2 - 12x + e$ has a factor $(x - 2)^2$. Find the values of b and e .
- 34** The roots of the equation $6x^3 - 19x^2 + cx + d = 0$ form a geometric sequence with the second term equal to 1. Find the values of c and d .
- 35** The cubic equation $x^3 + px^2 + qx + c = 0$, has roots α, β, γ . By expanding $(x - \alpha)(x - \beta)(x - \gamma)$ show that
- $p = -(\alpha + \beta + \gamma)$
 - $q = \alpha\beta + \beta\gamma + \gamma\alpha$
 - $c = -\alpha\beta\gamma$.
- It is now given that $p = -6$ and $q = 18$ for parts **b** and **c** below.
- In the case that the three roots α, β, γ form an arithmetic sequence, show that one of the roots is 2.
 - Hence determine the value of c .
 - In another case the three roots α, β, γ form a geometric sequence. Determine the value of c .
- Mathematics HL May 2015 Paper 1 TZ2 Q12

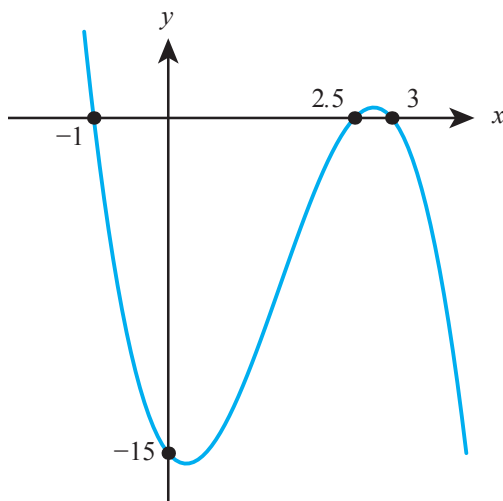
Chapter 6 Mixed Practice

1 a $x(x-1)(3x+1)$

b



2



3 $3(x+1)(x+2)(x-1)$

4 $p = 2, k = 3$

5 $a = 1, b = 0$

6 $a = -2, b = -3$

7 $p = -3, q = 1$

8 $b = -40, c = 325$

9 6

10 4

11 ± 2

12 a $-\frac{5}{3}$

b $\frac{2}{5}$

13 -4, 5

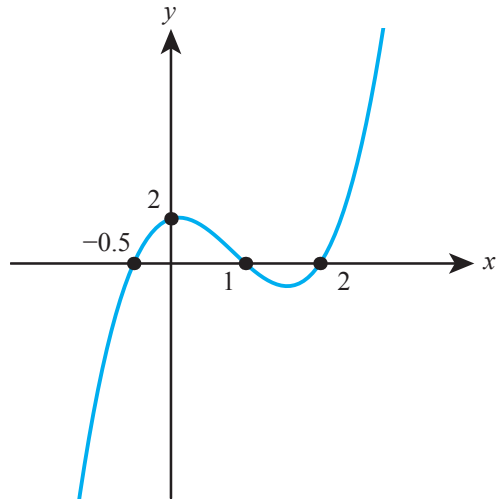
14 $a = 1, b = 2, c = -12, d = -18, e = 27$

15 $-x^4 + 2x^3 - 2x + 1$

16 $\frac{1}{81}x^4 - \frac{4}{27}x^3$

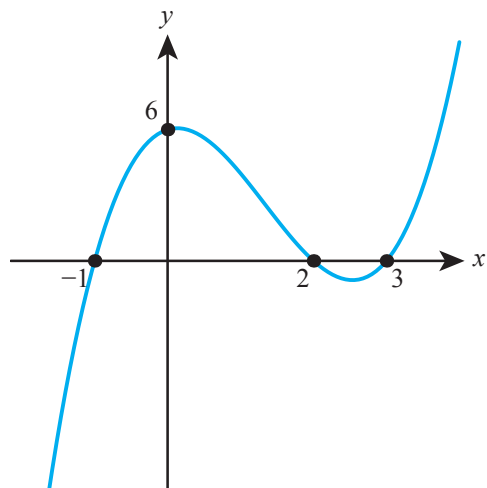
17 b $(x-2)(x-1)(2x+1)$

c

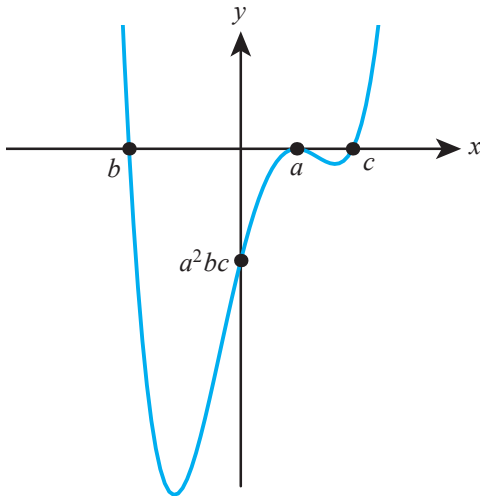


18 b $(x-2)(x+1)(x-3)$

c



19



20 $-3, 2 \pm i$

21 $a = 3, b = -42$

22 $\frac{3 \pm i\sqrt{3}}{3}$

24 $(a, b) = \pm\left(\frac{5}{3}, -\frac{4}{3}\right), \pm\left(-\frac{1}{3}, \frac{8}{3}\right)$

25 $p - 2, q = 45$

26 a $-\frac{26}{9}$

b $9x^2 + 26x + 49 = 0$

27 a 2

b 22

28 a $-\frac{2}{5}, \frac{3}{5}$

b $\frac{1}{135}$

29 a -2

30 50

31 a $2 + i, 2 - i$

b $a = -12, b = 15$

32 $a = -10, b = -18$

33 b $b = -8, e = 12$

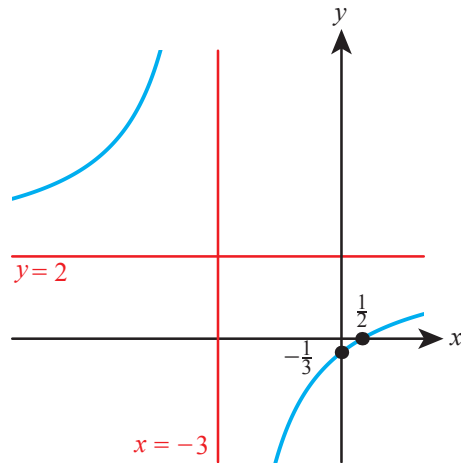
34 $c = 19, d = -6$

35 b ii -20

c -27

Chapter 7 Prior Knowledge

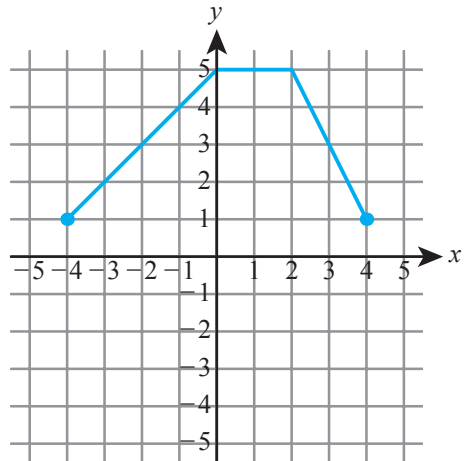
1



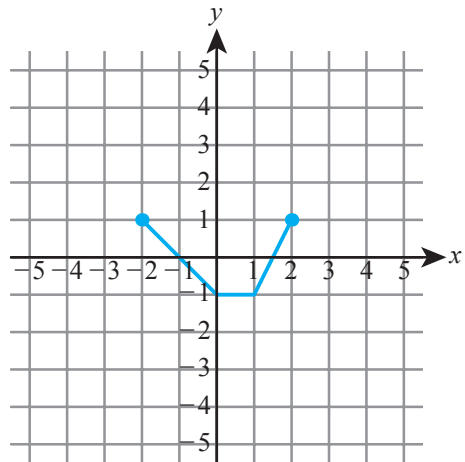
2 $-4 < x < 2$

3 1.40

4 a



b



5 $\frac{3x+1}{2-x}, x \neq 2$