

1. The volume of a sphere is $V = \sqrt{\frac{S^3}{36\pi}}$, where S is its surface area.

The surface area of a sphere is 500 cm^2 .

(a) Calculate the volume of the sphere. Give your answer correct to **two decimal places**.

(3)

(b) Write down your answer to (a) correct to the nearest integer.

(1)

(c) Write down your answer to (b) in the form $a \times 10^n$, where $1 \leq a < 10$ and $n \in \mathbb{Z}$.

(2)

(Total 6 marks)

2. Let $x = 7.94$.

(a) Calculate the value of $\frac{2x+1}{x^3}$.

(b) (i) Give your answer correct to **three** decimal places.

(ii) Write your answer to (b)(i) as a percentage.

(c) Give your answer to part (b)(i) in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$.

(Total 6 marks)

3. U is the set of all the **positive** integers less than or equal to 12.

A , B and C are subsets of U .

$$A = \{1, 2, 3, 4, 6, 12\}$$

$$B = \{\text{odd integers}\}$$

$$C = \{5, 6, 8\}$$

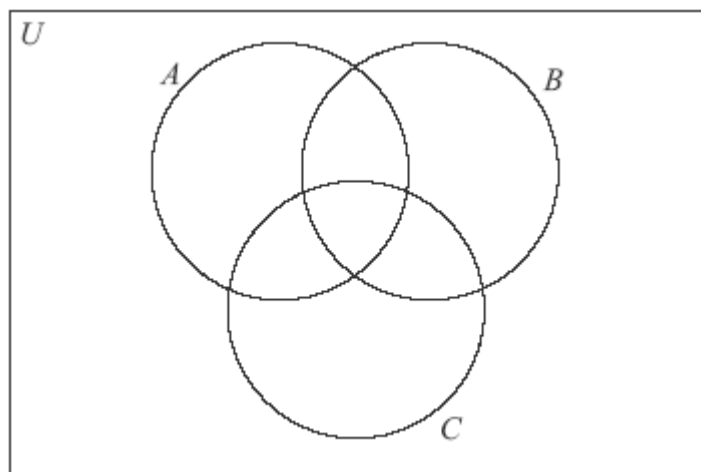
(a) Write down the number of elements in $A \cap C$.

(1)

(b) List the elements of B .

(1)

(c) Complete the following Venn diagram with **all** the elements of

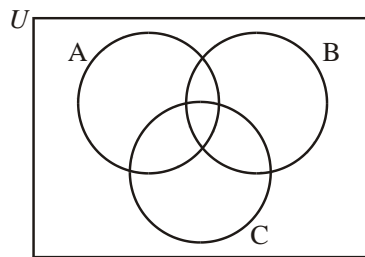


(4)

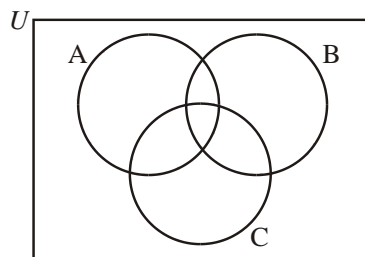
(Total 6 marks)

4. Shade the given region on the corresponding Venn Diagram.

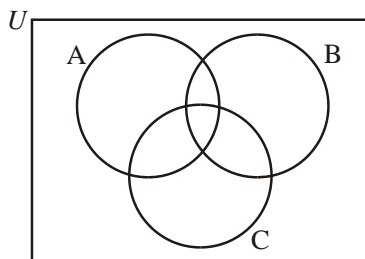
(a) $A \cap B$



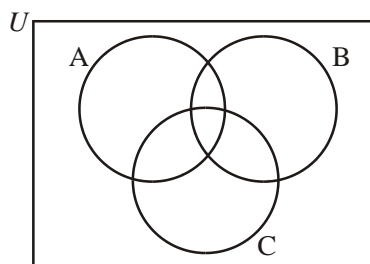
(b) $C \cup B$



(c) $(A \cup B \cup C)'$



(d) $A \cap C'$



(Total 8 marks)

5. Let $x = 6.4 \times 10^7$ and $y = 1.6 \times 10^8$.

Find

(a) $\frac{x}{y}$

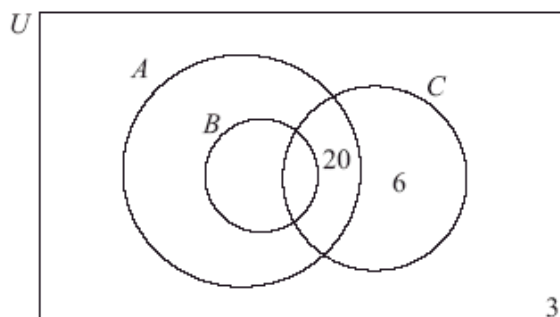
(b) $y - 2x$,

giving your answers in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(Total 8 marks)

6. The Venn diagram below represents the students studying Mathematics (A), Further Mathematics (B) and Physics (C) in a school.

50 students study Mathematics
 38 study Physics
 20 study Mathematics and Physics but not Further Mathematics
 10 study Further Mathematics but not Physics
 12 study Further Mathematics and Physics
 6 study Physics but not Mathematics
 3 study none of these three subjects.



- (a) Complete the Venn diagram. (3)
- (b) Write down the number of students who study Mathematics but not Further Mathematics. (1)
- (c) Write down the total number of students in the school. (1)
- (d) Write down $n(B \cup C)$. (2)

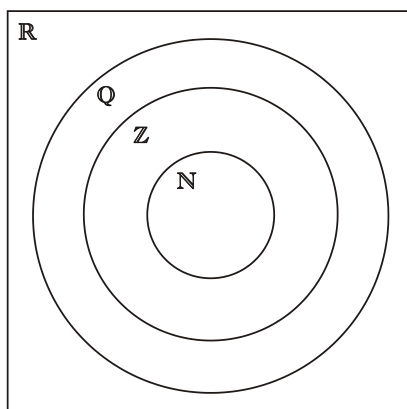
(Total 7 marks)

7. The Venn diagram below shows the universal set of real numbers \mathbb{R} and some of its important subsets:

\mathbb{Q} : the rational numbers,
 \mathbb{Z} : the integers,
 \mathbb{N} : the natural numbers.

Write the following numbers in the correct position in the diagram.

$$-1, 1, \pi, \frac{7}{16}, 3.333\bar{3}, \sqrt{3}.$$



(Total 6 marks)

8. Give all answers in this question to the nearest whole currency unit.

In January 2008 Larry had 90 000 USD to invest for his retirement in January 2011.

He invested 40 000 USD in US government bonds which paid 4 % per annum **simple interest**.

- (a) Calculate the value of Larry's investment in government bonds in January 2011. (3)

Larry changed this investment into South African rand (ZAR) at an exchange rate of 1 USD = 18.624 ZAR.

- (b) Calculate the amount that Larry received in ZAR from the exchange. (2)

He changed the remaining 50 000 USD to South African rand (ZAR) in January 2008. The exchange rate between USD and ZAR was 1 USD = 10.608 ZAR. There was 2.5 % commission charged on the exchange.

- (c) Calculate the value, **in USD**, of the commission Larry paid. (2)

- (d) Show that the amount that Larry had to invest is 517 000 ZAR, correct to the nearest thousand ZAR. (3)

In January 2008, Larry deposited this money into a bank account that paid interest at a nominal annual rate of 12 %, **compounded monthly**.

- (e) Find the value of the money in Larry's bank account in January 2011. (3)
(Total 13 marks)

9. Let $U = \{-4, -\frac{2}{3}, 1, \pi, 13, 26.7, 69, 10^{33}\}$.

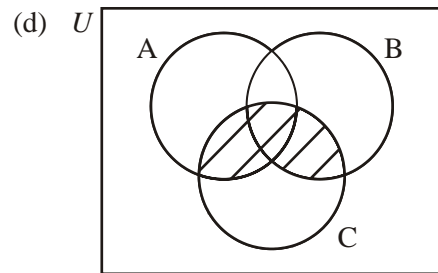
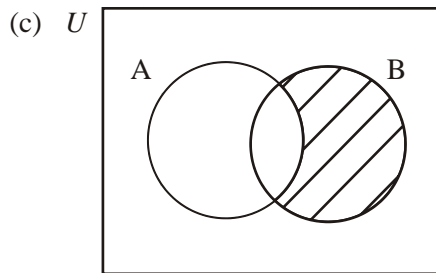
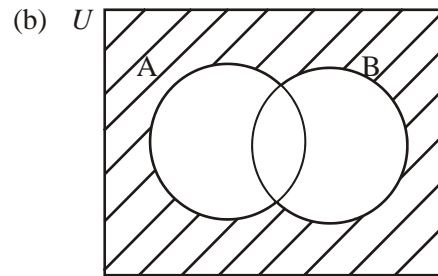
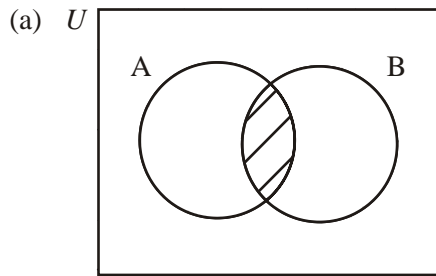
A is the set of all the integers in U .

B is the set of all the rational numbers in U .

- (a) List all the prime numbers contained in U .
(b) List all the members of A .
(c) List all the members of B .
(d) List all the members of the set $A \cap B$.

(Total 8 marks)

10. Write down an expression to describe the shaded area on the following Venn diagrams:



(Total 8 marks)

11. Give all your numerical answers correct to two decimal places.

On 1 January 2005, Daniel invested 30 000 AUD at an annual **simple** interest rate in a *Regular Saver* account. On 1 January 2007, Daniel had 31 650 AUD in the account.

(a) Calculate the rate of interest.

(3)

On 1 January 2005, Rebecca invested 30 000 AUD in a *Supersaver* account at a nominal annual rate of 2.5 % **compounded annually**.

(b) Calculate the amount in the *Supersaver* account after two years.

(3)

(c) Find the number of complete years since 1 January 2005 it will take for the amount in Rebecca's account to exceed the amount in Daniel's account.

(3)

On 1 January 2007, Daniel reinvested 80 % of the money from the *Regular Saver* account in an *Extra Saver* account at a nominal annual rate of 3 % **compounded quarterly**.

(d) (i) Calculate the amount of money reinvested by Daniel on the 1 January 2007.

(ii) Find the number of complete years it will take for the amount in Daniel's *Extra Saver* account to exceed 30 000 AUD.

(5)

(Total 14 marks)

12. A committee U has three sub-committees: research R , finance F and purchasing P . No member belongs to both finance and purchasing sub-committees. Some members belong to both research and purchasing committees. All members of the finance sub-committee also belong to the research sub-committee.

Draw a Venn diagram, showing the relationship between the sets U , R , F and P .

(Total 4 marks)

13. A rectangle has length 2.6×10^4 and width 1.9×10^4 . Find each of the following, giving your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

- (a) The area of the rectangle;
- (b) The perimeter of the rectangle.

(Total 8 marks)

14. Given \mathbb{Z} the set of integers, \mathbb{Q} the set of rational numbers, \mathbb{R} the set of real numbers.

- (a) Write down an element that belongs to $\mathbb{R} \cap \mathbb{Z}$.
- (b) Write down an element that belongs to $\mathbb{Q} \cap \mathbb{Z}'$.
- (c) Write down an element that belongs to \mathbb{Q}' .
- (d) Use a Venn diagram to represent the sets \mathbb{Z} , \mathbb{Q} and \mathbb{R} .

(Total 6 marks)

15. Susi travels from Singapore to Thailand and changes 1500 Singapore dollars (SGD) to Thai baht (THB). The exchange rate is 1 SGD buys 21.03464 THB.

- (a) Calculate the number of Thai baht Susi buys. Give your answer **correct to the nearest baht**.

(2)

Susi leaves Thailand and travels to Indonesia. She has 20 000 THB and uses these to buy Indonesian rupiah (IDR). The exchange rate is 3.28352 THB buys 1000 IDR.

- (b) Calculate the **total** number of Indonesian rupiah Susi receives, **correct to the nearest thousand rupiah**.

(2)

Susi wants to find the approximate exchange rate between Singapore dollars and Indonesian rupiah and uses the exchange rates for Thai baht to do this.

- (c) Calculate Susi's exchange rate between Singapore dollars and Indonesian rupiah. Give your answer in the form 1 SGD buys x IDR, where x is given correct to the nearest rupiah.

(2)

(Total 6 marks)

16. (a) Given $x = 2.6 \times 10^4$ and $y = 5.0 \times 10^{-8}$, calculate the value of $w = x \times y$. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.
- (b) Which **two** of the following statements about the nature of x , y and w above are **incorrect**?
- (i) $x \in \mathbb{N}$
 - (ii) $y \in \mathbb{Z}$
 - (iii) $y \in \mathbb{Q}$
 - (iv) $w < y$
 - (v) $x + y \in \mathbb{R}$
 - (vi) $\frac{1}{w} < x$

(Total 8 marks)

17. The total weight of 256 identical pencils is 4.24 kg. Calculate the weight of one pencil, in kg.
- (a) Give your answer exactly.
 - (b) Give your answer correct to three significant figures.
 - (c) Write your answer to part (b) in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(Total 8 marks)

18. The sets U , P , R and S are defined as follows:

$$U = \{\text{all quadrilaterals}\}$$

$$P = \{\text{all parallelograms}\}$$

$$R = \{\text{all rectangles}\}$$

$$S = \{\text{all squares}\}$$

- (a) Draw a Venn Diagram illustrating the relationships of the above sets.

(4)

- (b) Draw a separate Venn Diagram for each of the examples below. Indicate by shading each of the following:

(i) $(P \cup S)'$

(ii) $(R \cup S) \cap P$

(4)
(Total 8 marks)

19. The population of big cats in Africa is increasing at a rate of 5 % per year. At the beginning of 2004 the population was 10 000.

(a) Write down the population of big cats at the beginning of 2005. (1)

(b) Find the population of big cats at the beginning of 2010. (2)

(c) Find the number of years, from the beginning of 2004, it will take the population of big cats to exceed 50 000. (3)

(Total 6 marks)

20. Consider the numbers $\sqrt{3}$, 6, $2\frac{1}{2}$, π , -5 , and the sets \mathbb{N} , \mathbb{Z} , and \mathbb{Q} . Complete the following table by placing a tick in the appropriate box if the number is an element of the set.

	$\sqrt{3}$	6	$2\frac{1}{2}$	π	-5
\mathbb{N}					
\mathbb{Z}					
\mathbb{Q}					

(Total 6 marks)

21. The sets A , B and C are subsets of U . They are defined as follows:

$$U = \{\text{positive integers less than 16}\}$$

$$A = \{\text{prime numbers}\}$$

$$B = \{\text{factors of 36}\}$$

$$C = \{\text{multiples of 4}\}$$

(a) List the elements (if any) of the following:

(i) A ;

(ii) B ;

(iii) C ;

(iv) $A \cap B \cap C$.

(4)

- (b) (i) Draw a Venn diagram showing the relationship between the sets U, A, B and C .
(ii) Write the elements of sets U, A, B and C in the appropriate places on the Venn diagram. (4)
- (c) From the Venn diagram, list the elements of each of the following
(i) $A \cap (B \cup C)$;
(ii) $(A \cap B)'$;
(iii) $(A \cap B)' \cap C$. (3)
- (Total 11 marks)

22. One day the number of customers at three cafés, “Alan’s Diner” (A), “Sarah’s Snackbar” (S) and “Pete’s Eats” (P) was recorded and are given below.

17 were customers of Pete’s Eats only
27 were customers of Sarah’s Snackbar only
15 were customers of Alan’s Diner only
10 were customers of Pete’s Eats **and** Sarah’s Snackbar **but not** Alan’s Diner
8 were customers of Pete’s Eats **and** Alan’s Diner **but not** Sarah’s Snackbar

- (a) Draw a Venn Diagram, using sets labelled A, S and P , that shows this information. (3)

There were 48 customers of Pete’s Eats that day.

- (b) Calculate the number of people who were customers of all three cafés. (2)

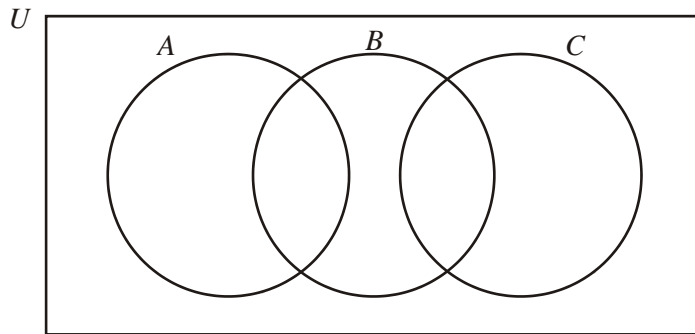
There were 50 customers of Sarah’s Snackbar that day.

- (c) Calculate the total number of people who were customers of Alan’s Diner. (3)

- (d) Write down the number of customers of Alan’s Diner that were also customers of Pete’s Eats. (1)

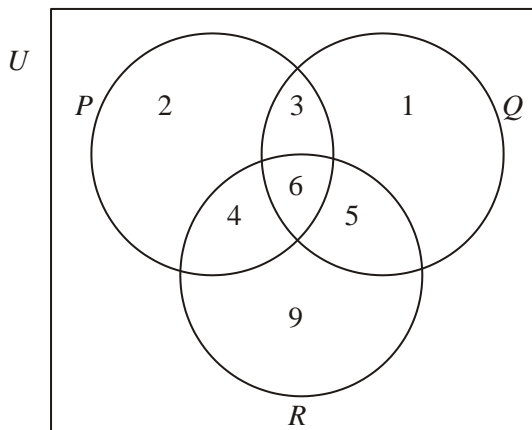
- (e) Find $n[(S \cup P) \cap A']$. (2)
- (Total 11 marks)

23. (a) Shade $(A \cup B) \cap C'$ on the diagram below.



(2)

- (b) In the Venn diagram below, the number of elements in each region is given. Find $n((P \cap Q) \cup R)$.



(2)

- (c) U is the set of positive integers, \mathbb{Z}^+ .
 E is the set of even numbers.
 M is the set of multiples of 3.

- (i) List the first six elements of the set M .
(ii) List the first six elements of the set $E' \cap M$.

(2)

(Total 6 marks)

24. Astrid invests 1200 euros for five years at a nominal annual interest rate of 7.2 %, **compounded monthly**.

- (a) Find the interest Astrid has earned during the five years of her investment.
Give your answer correct to two decimal places.

(3)

Helen invests 1200 euros in an annual **simple interest** scheme for five years. She earns **the same** interest as Astrid.

- (b) Find the simple interest rate of this scheme.

(3)

(Total 6 marks)

25. A school offers three activities, basketball (B), choir (C) and drama (D). Every student must participate in at least one activity.

16 students play basketball only.

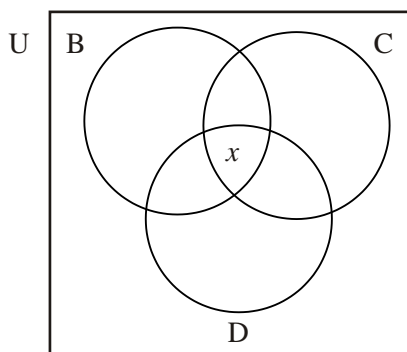
18 students play basketball and sing in the choir but do not do drama.

34 students play basketball and do drama but do not sing in the choir.

27 students are in the choir and do drama but do not play basketball.

- (a) Enter the above information on the Venn diagram below.

(2)



99 of the students play basketball, 88 sing in the choir and 110 do drama.

- (b) Calculate the number of students x participating in all three activities.

(1)

- (c) Calculate the total number of students in the school.

(3)

(Total 6 marks)

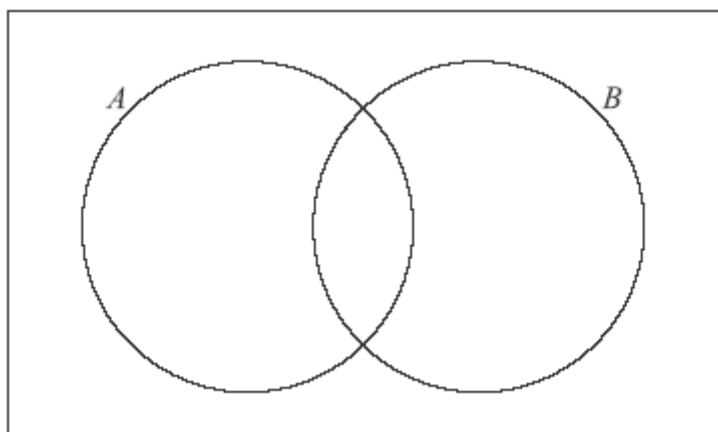
26. A group of 30 students were asked about their favourite topping for toast.

18 liked peanut butter (A)

10 liked jam (B)

6 liked neither

- (a) Show this information on the Venn diagram below.



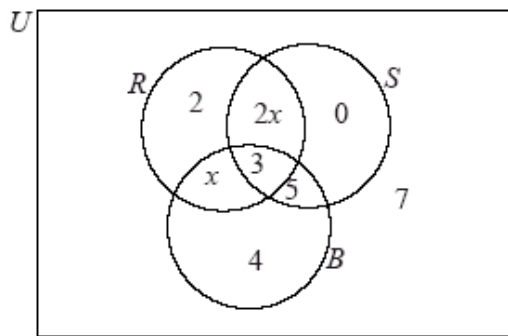
(2)

- (b) Find the number of students who like peanut butter but not jam.

(2)

(Total 4 marks)

27. A survey was carried out in a year 12 class. The pupils were asked which pop groups they like out of the *Rockers* (R), the *Salseros* (S), and the *Bluers* (B). The results are shown in the following diagram.



- (a) Write down $n(R \cap S \cap B)$. (1)
- (b) Find $n(R')$. (2)
- (c) Describe which groups the pupils in the set $S \cap B$ like. (2)
- (d) Use set notation to describe the group of pupils who like the *Rockers* and the *Bluers* but do not like the *Salseros*. (2)

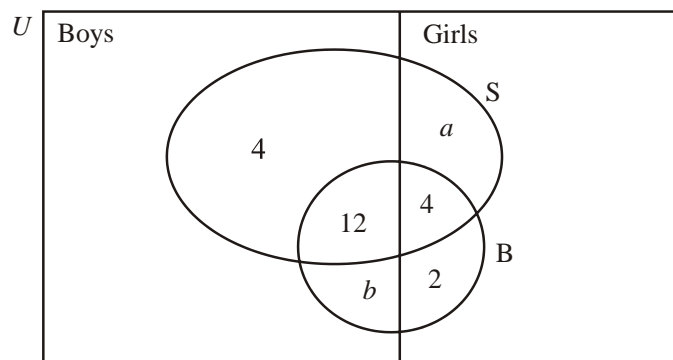
There are 33 pupils in the class.

- (e) (i) Find x . (3)
- (ii) Find the number of pupils who like the *Rockers*. (3)
- (Total 10 marks)**

28. Children in a class of 30 students are asked whether they can swim (S) or ride a bicycle (B).

There are 12 girls in the class. 8 girls can swim, 6 girls can ride a bicycle and 4 girls can do both.

16 boys can swim, 13 boys can ride a bicycle and 12 boys can do both. This information is represented in a Venn diagram.



- (a) Find the values of a and b . (2)
- (b) Calculate the number of students who can do neither. (2)
- (Total 4 marks)**

29. Daniel wants to invest \$25 000 for a total of three years. There are three investment options.

Option One pays simple interest at an annual rate of interest of 6 %.

Option Two pays compound interest at a nominal annual rate of interest of 5 %, compounded **annually**.

Option Three pays compound interest at a nominal annual rate of interest of 4.8 %, compounded **monthly**.

(a) Calculate the value of his investment at the end of the third year for each investment option, **correct to two decimal places**.

(8)

(b) Determine Daniel's best investment option.

(1)

(Total 9 marks)

30. At a certain school there are 90 students studying for their IB diploma. They are required to study at **least one** of the subjects: Physics, Biology or Chemistry.

50 students are studying Physics,

60 students are studying Biology,

55 students are studying Chemistry,

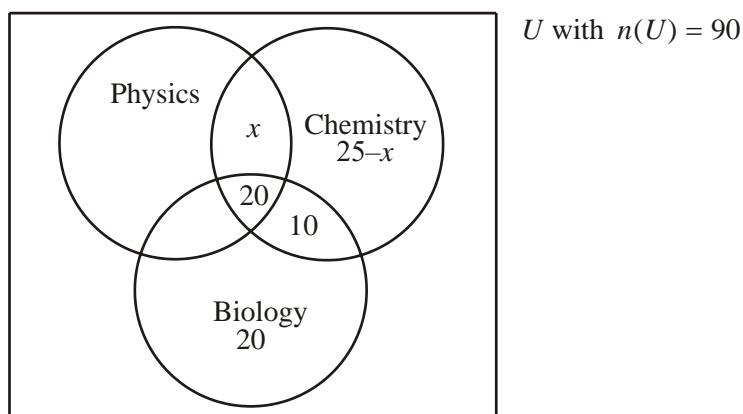
30 students are studying both Physics and Biology,

10 students are studying both Biology and Chemistry but not Physics,

20 students are studying all three subjects.

Let x represent the number of students who study both Physics and Chemistry but not Biology. Then $25-x$ is the number who study Chemistry only.

The figure below shows some of this information and can be used for working.



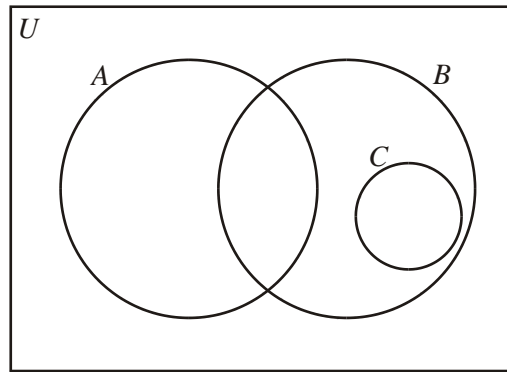
(a) Express the number of students who study Physics only, in terms of x .

(b) Find x .

(c) Determine the number of students studying **at least two** of the subjects.

(Total 6 marks)

31. The following Venn Diagram shows the sets U , A , B and C .



State whether the following statements are true or false for the information illustrated in the Venn Diagram.

- (a) $A \cap C = \emptyset$
- (b) $C \cup B = C$
- (c) $C \subset (A \cup B)$
- (d) $A \subset C'$

(Total 8 marks)

32. Give all answers in this question to the nearest whole currency unit.

Ying and Ruby each have 5000 USD to invest.

Ying invests his 5000 USD in a bank account that pays a nominal annual interest rate of 4.2 % **compounded yearly**. Ruby invests her 5000 USD in an account that offers a fixed interest of 230 USD each year.

- (a) Find the amount of money that Ruby will have in the bank after 3 years. (2)
- (b) Show that Ying will have 7545 USD in the bank at the end of 10 years. (3)
- (c) Find the number of complete years it will take for Ying's investment to first exceed 6500 USD. (3)
- (d) Find the number of complete years it will take for Ying's investment to exceed Ruby's investment. (3)

Ruby moves from the USA to Italy. She transfers 6610 USD into an Italian bank which has an exchange rate of 1 USD = 0.735 euros. The bank charges 1.8 % commission.

- (e) Calculate the amount of money Ruby will invest in the Italian bank after commission. (4)

Ruby returns to the USA for a short holiday. She converts 800 euros at a bank in Chicago and receives 1006.20 USD. The bank advertises an exchange rate of 1 euro = 1.29 USD.

- (f) Calculate the percentage commission Ruby is charged by the bank. (5)
(Total 20 marks)

33. Given a universal set $U = \{\text{cars}\}$, $S = \{\text{sports cars}\}$, $G = \{\text{green sports cars}\}$.

- (a) Draw a Venn diagram to illustrate this information. (3)

- (b) Shade the set $S \cap G'$ on your diagram. (1)

- (c) Write in words the meaning of $S \cap G'$. (2)
(Total 6 marks)

34. The exchange rates between the British pound (GBP) and the United States dollar (USD) and between the USD and the euro (EUR) are given below.

1 GBP	2.034 USD
1 USD	0.632 EUR

- (a) Find the exchange rate between GBP and EUR in the form $1 \text{ GBP} = k \text{ EUR}$, where k is a constant. Give your answer correct to **two decimal places**. (2)

Isabella changes 400 USD into euros and is charged 2 % commission.

- (b) Calculate how many euros she receives. Give your answer correct to **two decimal places**. (4)
(Total 6 marks)

35. A survey of 100 families was carried out, asking about the pets they own. The results are given below.

56 owned dogs (S)
38 owned cats (Q)
22 owned birds (R)
16 owned dogs and cats, but not birds
8 owned birds and cats, but not dogs
3 owned dogs and birds, but not cats
4 owned all three types of pets

- (a) Draw a Venn diagram to represent this information. (5)
- (b) Find the number of families who own no pets. (2)
- (c) Find the percentage of families that own exactly one pet. (3)
- (d) A family is chosen at random. Find the probability that they own a cat, given that they own a bird. (2)

(Total 12 marks)