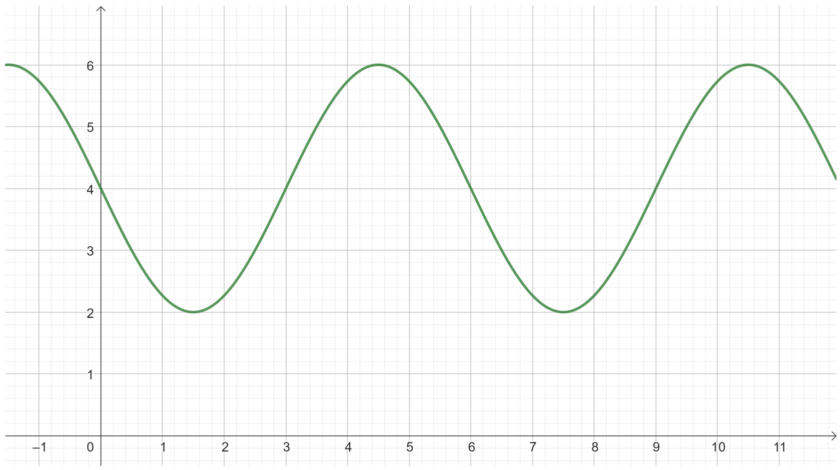
**1.** [6 *points*]

The following diagram shows the graph of $f(x) = a\sin(bx) + d$.




The graph has a $y$-intercept at 4, a minimum at $(1.5, 2)$ and a maximum at $(4.5, 6)$.

(a) Find the values of $a, b$ and $d$.

(b) The domain of $f(x)$ is restricted to $1.5 \leqslant x \leqslant k$, where $k$ is the largest possible value for which the inverse function exists. State the value of $k$ and find the inverse function.

(a) $a = -2, \quad b = \frac{\pi}{3}$ and $d = 4$.

(b) $k = 4.5$

For inverse we have:

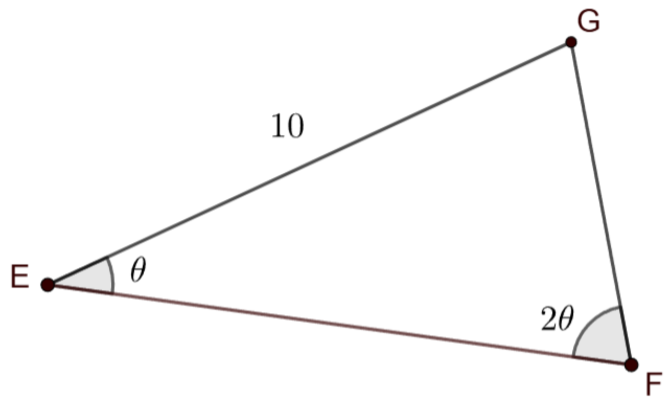
$$x = -2\sin\left(\frac{\pi}{3}y\right) + 4$$

Rearranging we get:

$$f^{-1}(x) = \frac{3}{\pi}\arcsin\left(\frac{4-x}{2}\right)$$

**2.** [9 *points*]

Consider the following triangle:




with $EG = 10$ and $\angle GFE = 2\angle FEG$. Let $\sin\theta = x$.

(a) Express $\sin 2\theta$ in terms of $x$.

(b) Hence express $GF$ in terms of $x$.

(c) Show that $\sin\angle EGF = 3x - 4x^3$.

(d) Hence express the area of the triangle in terms of $x$ and find the greatest possible area of the triangle.

(a) $\theta$ must be acute, so (using Pythagorean identity) we get:

$$\cos\theta = \sqrt{1 - x^2}$$

So $\sin 2\theta = 2x\sqrt{1 - x^2}$

(b) Using sine rule:

$$\frac{10}{2x\sqrt{1 - x^2}} = \frac{GF}{x}$$

which gives $GF = \dfrac{5}{\sqrt{1 - x^2}}$.

(c) We have:

$$\sin\angle EGF = \sin(\pi - 3\theta) = \sin 3\theta$$

We also have:

$$\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2x^2$$

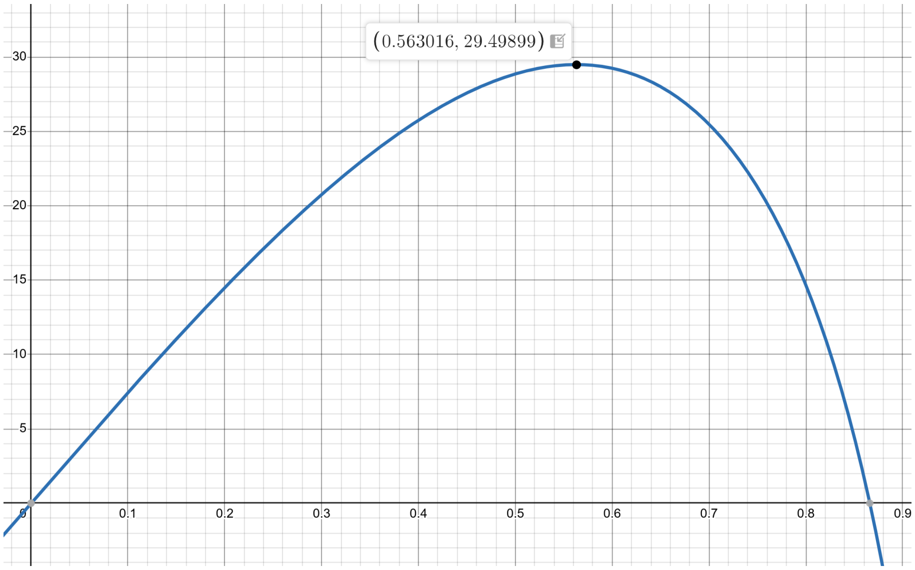
So:

$$\sin 3\theta = \sin(\theta + 2\theta) = \sin\theta\cos 2\theta + \sin 2\theta\cos\theta = x(1 - 2x^2) + 2x\sqrt{1 - x^2}\sqrt{1 - x^2} = x - 2x^3 + 2x - 2x^3 = 3x - 4x^3$$

(d)

$$A(x) = \frac{1}{2} \times 10 \times \frac{5}{\sqrt{1-x^2}} \times (3x - 4x^3) = \frac{25(3x-4x^3)}{\sqrt{1-x^2}}$$

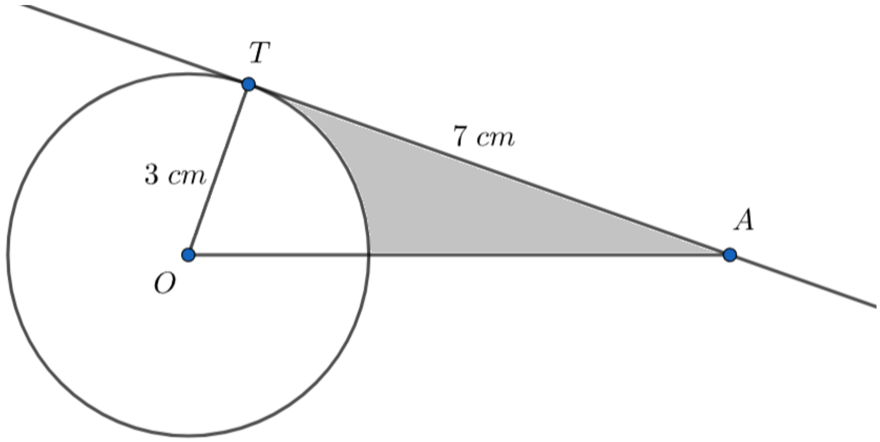
We can now sketch the area function to get:




We have $A_{max} \approx 29.5$.

**3.** [4 *points*]

Consider a circle with radius 3 $cm$ and a tangent to this circle drawn from point $A$, Let $T$ be the point of tangency and let $AT = 7\ cm$. The following diagrams shows the above information.




Find the shaded area. Give your answer correct to 4 significant figures.

$$\angle AOT = \arctan\frac{7}{3} = 1.1659...$$

$$A_{\triangle AOT} = \frac{3 \times 7}{2} = 10.5$$

$$A_{sector} = \frac{1.1659... \times 3^2}{2} = 5.24657...$$

$$A_{shaded} = 10.5 - 5.24657... \approx 5.253$$

**4.** [6 *points*]

Tomasz is at a point $T_1$ on level ground. He sees a tower which is 145 metres away on bearing of 040. The angle of elevation from Tomasz to the top of the tower is 25°. Tomasz walks 120 metres West to a point $T_2$.

(a) Find the distance from $T_2$ to the foot of the tower.

(b) Find the bearing of the tower from $T_2$.

(c) Find the angle of elevation from $T_2$ to the top of the tower.

(a) Using cosine rule:

$$T_2F = \sqrt{145^2 + 120^2 - 2 \times 145 \times 120 \times \cos 130^\circ} \approx 240\ m$$

(b) Using cosine rule again:

$$\theta = \arccos\left(\frac{120^2 + 240.40...^2 - 145^2}{2 \times 120 \times 240.40...}\right) \approx 27.5^\circ$$

So we have $90^\circ - 27.51889...^\circ = 62.48110...^\circ$. Bearing: 062.

(c) The height of the tower, $h$, is:

$$h = 145 \times \tan 25^\circ = 67.6146...$$

The angle of elevation is then:

$$\alpha = \arctan \frac{67.6146...}{240.40...} \approx 15.7^\circ$$