Mixed Practice

1 Given that $\sec\theta = 3$, find the exact value of $\cos 2\theta$.

2 **a** Show that $\cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right) \equiv \sqrt{2}\cos x$.

b Hence solve the equation $\cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right) = \sqrt{2}\sin x$ for $0 \leqslant x < \pi$.

3 **a** Show that

$$\sin\left(x + \frac{\pi}{3}\right) + \cos\left(x + \frac{\pi}{6}\right) \equiv \sqrt{3}\cos x.$$

b Hence solve the equation

$$\sin\left(x + \frac{\pi}{3}\right) + \cos\left(x + \frac{\pi}{6}\right) = \sin x$$

for $0 < x < 2\pi$.

4 An acute angle θ has $\tan\theta = 3$. Find the exact value of

a $\tan 2\theta$

b $\sec\theta$.

5 Given that $\cos A = \frac{1}{2}$ and $\cos B = \frac{1}{3}$, find the exact value of $\cos(A + B)$.

6 Solve the equation $(\arcsin x)^2 = \frac{\pi^2}{9}$.

7 Find the exact value of $\tan 105°$.

8 Solve the equation

$$\sin\left(x + \frac{\pi}{3}\right) = \sin\left(x - \frac{\pi}{3}\right)$$

for $0 < x < 2\pi$.

9 Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0° \leqslant x < 360°$.

Mathematics HL May 2015 Paper 1 TZ2 Q3

10 **a** Given that $\arctan\frac{1}{2} - \arctan\frac{1}{3} = \arctan a$, $a \in \mathbb{Q}^+$, find the value of a.

b Hence, or otherwise, solve the equation $\arcsin x = \arctan a$.

Mathematics HL November 2011 Paper 2 Q4

11 **a** Use the formulae for $\sin 2\theta$ and $\cos 2\theta$ to derive a formula for $\tan 2\theta$ in terms of $\tan\theta$.

b Hence find the exact value of $\tan 112.5°$.

12 Solve the equation $\sin\left(x + \frac{\pi}{6}\right) + \sin\left(x - \frac{\pi}{6}\right) = 3\cos x$ for $0 \leqslant x \leqslant \pi$.

13 Given that $\cos y = \sin(x + y)$, show that $\tan y = \sec x - \tan x$.

14 Prove that $\cot 2x = \frac{\cot^2 x - 1}{2\cot x}$.

15 **a** Prove that $\operatorname{cosec} 2x - \cot 2x \equiv \tan x$.

b Hence find the exact value of $\tan\left(\frac{3}{8}\pi\right)$.

16 **a** Prove the following identities:

i $\cos^4\theta - \sin^4\theta = \cos 2\theta$

ii $\sin^2 2\theta(\cot^2\theta - \tan^2\theta) \equiv 4(\cos^4\theta - \sin^4\theta)$.

b Hence solve the equation $\sin^2 2\theta(\cot^2\theta - \tan^2\theta) = 2$ for $0 \leqslant \theta \leqslant 2\pi$.

17 **a** Write $3\sin x + \sqrt{3}\cos x$ in the form $R\sin(x+\theta)$ where $R > 0$ and $\theta \in \left(0, \frac{\pi}{2}\right)$.

b Hence solve the equation $3\sin x + \sqrt{3}\cos x = 3$ for $-\pi < x < \pi$.

18 If x satisfies the equation $\sin\left(x + \frac{\pi}{3}\right) = 2\sin x \sin\left(\frac{\pi}{3}\right)$, show that $11\tan x = a + b\sqrt{3}$, where $a, b \in \mathbb{Z}^+$.

Mathematics HL May 2010 Paper 1 TZ2 Q6

19 Let $f(x) = \frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$.

a For what values of x does $f(x)$ not exist?

b Simplify the expression $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$.

Mathematics HL May 2012 Paper 1 TZ1 Q5

20 **a** Show that $\cos(x+y) + \cos(x-y) = 2\cos x \cos y$.

b Hence solve the equation $\cos 3x + \cos x = 3\cos 2x$ for $0 \leqslant \theta < 2\pi$.

21 Find the exact value of:

a $\tan(\arctan 3 - \arctan 2)$

b $\tan\left(2\arctan\left(\frac{1}{2}\right)\right)$.

22 Let θ be the acute angle between the lines with equations $y = x$ and $y = 2x$.
Find the exact value of $\tan\theta$.

23 **a** Use the identity for $\tan(A+B)$ to express $\tan 3x$ in terms of $\tan x$.

b Hence solve the equation $\tan x + \tan 3x = 0$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

24 **a** Show that $\sin\left(2x + \frac{\pi}{2}\right) \equiv \cos 2x$.

b Hence solve the equation $\sin 3x = \cos 2x$ for $0 \leqslant x \leqslant \frac{\pi}{2}$.

c Show that $\sin 3x \equiv 3\sin x - 4\sin^3 x$ and hence express $\cos 2x - \sin 3x$ in terms of $\sin x$.

Let $f(s) = 4s^3 - 2s^2 - 3s + 1$.

d Show that $(s-1)$ is a factor of $f(s)$ and factorize $f(s)$ completely.

e Hence find the exact value of $\sin\left(\frac{\pi}{10}\right)$.

25 Compactness is a measure of how compact an enclosed region is. The compactness, C, of an enclosed region can be defined by $C = \frac{4A}{\pi d^2}$ where A is the area of the region and d is the maximum distance between any two points in the region.

For a circular region, $C = 1$.

Consider a regular polygon of n sides constructed such that its vertices lie on the circumference of a circle of diameter x units.

a If $n > 2$ and even, show that $C = \frac{n}{2\pi}\sin\frac{2\pi}{n}$.

If $n > 1$ and odd, it can be shown that $C = \frac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)}$.

b Find the regular polygon with the least number of sides for which the compactness is more than 0.99.

c Comment briefly on whether C is a good measure of compactness.

Mathematics HL November 2014 Paper 2 Q9

26 **a** Given that $\arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right) = \arctan\left(\frac{1}{p}\right)$, where $p \in \mathbb{Z}^+$, find p.

b Hence find the value of $\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right)$.

Mathematics HL May 2013 Paper 1 TZ2 Q10

3 a $\frac{\sqrt{6}-\sqrt{2}}{4}$ b $\frac{-\sqrt{6}+\sqrt{2}}{4}$

4 a $\frac{\sqrt{6}-\sqrt{2}}{4}$ b $\frac{\sqrt{6}+\sqrt{2}}{4}$

5 a $2-\sqrt{3}$ b $-2+\sqrt{3}$

6 a $-\frac{1}{\sqrt{3}}$ b 1

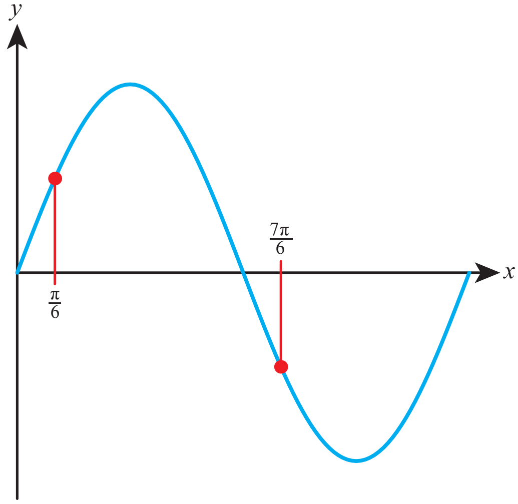
7 a $-2-\sqrt{3}$ b $2+\sqrt{3}$

8 a $-\sin x$ b $\cos x$

9 a $\cos x$ b $\sin x$

10 a $-\tan x$ b $\tan x$

11 b



14 a $\sqrt{2}\cos x$ b 60°, 300°

17 a $\frac{3}{4}$ b $\frac{13}{9}$

18 3

19 a $\frac{2\sqrt{2}}{3}$ b $\frac{6\sqrt{2}-4}{15}$

20 $\frac{16}{65}$

21 $4-\sqrt{3}$

22 $\tan y = \frac{\tan x - 2}{1 + 2\tan x}$

23 $3, -\frac{1}{3}$

24 $\frac{\sqrt{3}}{2}$

25 $\sqrt{2}$

27 a $2\sin x\cos x$

b $0, \pi, 2\pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

28 $\sqrt{2}-1$

29 b $-\frac{1}{2}, -\frac{1}{3}$

30 b $0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$

32 a $\sqrt{58}$ b $\frac{3}{10-\sqrt{58}}$

33 a $2\sqrt{3}\sin\left(\theta+\frac{\pi}{6}\right)$ b $\frac{3-\sqrt{3}}{12}$ for $x = \frac{\pi}{6}$

34 $\frac{3}{11}$

Chapter 3 Mixed Practice

1 $-\frac{7}{9}$

2 b $\frac{\pi}{4}$

3 b $\frac{\pi}{3}, \frac{4\pi}{3}$

4 a $-\frac{3}{4}$ b $\sqrt{10}$

5 $\frac{1 \pm 2\sqrt{6}}{6}$

6 $\pm\frac{\sqrt{3}}{2}$

7 $-2-\sqrt{3}$

8 $\frac{\pi}{2}$ or $\frac{3\pi}{2}$

9 0°, 60°, 120°, 180°, 240°, 300°

10 a $\frac{1}{7}$ b $\frac{1}{\sqrt{50}}$

11 b $-\sqrt{2}-1$

12 $\frac{\pi}{3}$

15 b $1+\sqrt{2}$

16 b $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

17 a $2\sqrt{3}\sin\left(x+\frac{\pi}{6}\right)$ b $\frac{\pi}{6}, \frac{\pi}{2}$

18 $a = 6, b = 1$

19 a $\frac{n\pi}{2}, n \in \mathbb{Z}$ b 2

20 b $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

21 a $\frac{1}{7}$ b $\frac{4}{3}$

22 $\frac{1}{3}$

23 a $\frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$ b $-\frac{\pi}{4}, 0, \frac{\pi}{4}$

24 b $\frac{\pi}{10}, \frac{\pi}{2}$

$1 - 3\sin x - 2\sin^2 2x + 4\sin^3 x$

d $(s-1)\left(2s + \frac{1}{2} - \frac{\sqrt{5}}{2}\right)\left(2s + \frac{1}{2} + \frac{\sqrt{5}}{2}\right)$

e $\frac{-1+\sqrt{5}}{4}$

25 b 21

c e.g. As n increases, the compactness of a polygon with n sides gets closer to that of a circle ($c = 1$).

26 a $p = 3$ b $\frac{\pi}{4}$

Chapter 4 Prior Knowledge

1 $\frac{3 \pm \sqrt{3}}{3}$

2 $5 - 2\sqrt{5}$

3 a $\frac{\sqrt{2}}{2}$ b $-\frac{1}{2}$

4 a $\sin\frac{3\pi}{10}$ b $\frac{5\pi}{24}$

5 $e^{(\ln 4)x}$

6 $a(x-3)(x+5)$

7 $\frac{1}{1-x}$

Exercise 4A

1 a i b −1

2 a 4i b 5

3 a −9 b −8i

4 a 1 b −1

5 a $x = \pm 3i$ b $x = \pm 6i$

6 a $x = \pm 2\sqrt{2}i$ b $x = \pm 5\sqrt{3}i$

7 a $x = 1 \pm 2i$ b $x = 2 \pm 3i$

8 a $x = -1 \pm \frac{\sqrt{2}}{2}i$ b $x = \frac{1}{3} \pm \frac{\sqrt{5}}{3}i$

9 a $11 + 4i$ b $-4 + 4i$

10 a $1 - 2i$ b $-6 + 10i$

11 a $8 - i$ b $16 + 2i$

12 a $8 + 6i$ b $7 - 24i$

13 a $4 - 2i$ b $-3 + 3i$

14 a $4 + 3i$ b $1 - i$

15 a $\frac{1}{2} + \frac{1}{2}i$ b $-\frac{2}{13} - \frac{23}{13}i$

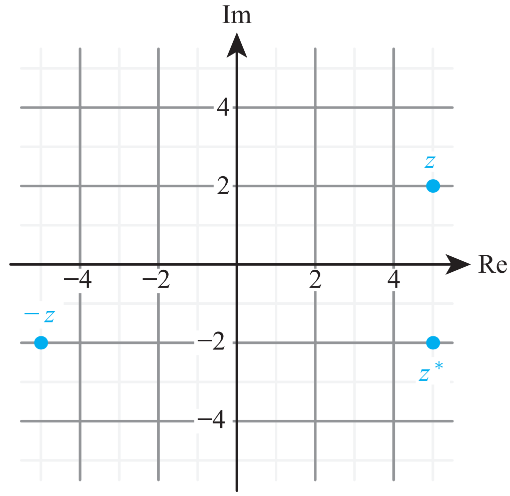
16 a $a = 5, b = 7$ b $a = -3, b = 9$

17 a $a = -\frac{3}{2}, b = \frac{31}{2}$ b $a = \frac{1}{2}, b = -8$

18 a $z = -4 - i$ b $z = 1 + 2i$

19 a $z = \frac{2}{3} + 7i$ b $z = -\frac{5}{3} + \frac{1}{3}i$

20 a



b