Normal distribution 1 [33 marks]

1. [Maximum mark: 5] 23M.1.AHL.TZ1.9

On a specific day, the speed of cars as they pass a speed camera can be modelled by a normal distribution with a mean of $67.3\ \mathrm{km\ h^{-1}}$.

A speed of $75.7\ \mathrm{km\ h^{-1}}$ is two standard deviations from the mean.

(a) Find the standard deviation for the speed of the cars. [2]

Markscheme

attempt to find the difference between 75.7 and 67.3 *(M1)*

$\frac{75.7-67.3}{2}$

$4.2\ (\mathrm{km\ h^{-1}})$ ***A1***

[2 marks]

It is found that 82% of cars on this road travel at speeds between $p\ \mathrm{km\ h^{-1}}$ and $q\ \mathrm{km\ h^{-1}}$, where $p < q$. This interval includes cars travelling at a speed of $74\ \mathrm{km\ h^{-1}}$.

(b) Show that the region of the normal distribution between p and q is **not** symmetrical about the mean. [3]

Markscheme

METHOD 1 (Comparing areas above and below the mean)

$\mathrm{P}(67.3 <$ speed $< 74)$ **OR** Normal CDF$(67.3,\ 74,\ 67.3,\ 4.2)$ **OR** sketch of normal distribution with 67.3 and 74 labelled and shaded between *(M1)*

area of region between mean and q is at least $0.445\ (0.444670\ldots)$ ***A1***

Hence no more than $0.375\ (0.375329\ldots)$ between mean and p ***R1***

The region between p and q is not symmetrical ***AG***

METHOD 2 (Comparing areas in the tails)

attempt to calculate probability that speed $< p$ and speed $> q$ with $q = 74$ *(M1)*

$\mathrm{P}(\text{speed} < 74) = 0.944670\ldots$

$\mathrm{P}(\text{speed} < p) = (0.944670\ldots - 0.82 =)\ 0.124670\ldots$

$\mathrm{P}(\text{speed} > q) = (1 - 0.944670\ldots =)\ 0.0553295\ldots$ ***A1***

if $q \geq 74$, then $\mathrm{P}(\text{speed} > q) \geq 0.124670$ and $\mathrm{P}(\text{speed} < p) \geq 0.124670$ so

$\mathrm{P}(\text{speed} > q)$ will never equal $\mathrm{P}(\text{speed} < p)$ ***R1***

the region between p and q is not symmetrical ***AG***

METHOD 3 (Assumption of symmetry comparing speeds)

attempt to calculate area below q assuming distribution is symmetrical *(M1)*

e.g. $\mathrm{P}(\text{speed} < q) = 0.82 + {}^{1}\!/_{2} \times 0.18\ \ (0.91)$

EITHER

$(q =)\ 72.9\ \ (72.9311\ldots)$ ***A1***

$72.9 < 74$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

OR

$\mathrm{P}(\text{speed} < 74) = 0.945\ \ (0.944670\ldots)$ ***A1***

$0.945 > 0.91$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

METHOD 4 (Assumption of symmetry comparing areas)

attempt to calculate symmetrical area with 74 as a boundary ***(M1)***

$\mathrm{P}(60.6 < \text{speed} < 74)$ **OR** Normal CDF$(60.6,\ 74,\ 67.3,\ 4.2)$ **OR**

$\mathrm{P}(67.3 < \text{speed} < 74)$ **OR** Normal CDF(67.3, 74, 67.3, 4.2)

EITHER

$0.889\ \ (0.889340\ldots)$ ***A1***

$0.889 > 0.82$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

OR

$0.445\ (0.444670\ldots)$ ***A1***

$0.4459 > 0.82 \div 2$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

[3 marks]

2. [Maximum mark: 6] 23M.1.AHL.TZ2.5

The lengths of the seeds from a particular mango tree are approximated by a normal distribution with a mean of $4\ \text{cm}$ and a standard deviation of $0.25\ \text{cm}$.

A seed from this mango tree is chosen at random.

(a) Calculate the probability that the length of the seed is less than $3.7\ \text{cm}$. [2]

Markscheme

$X \sim \mathrm{N}\left(4,\ 0.25^2\right)$

EITHER

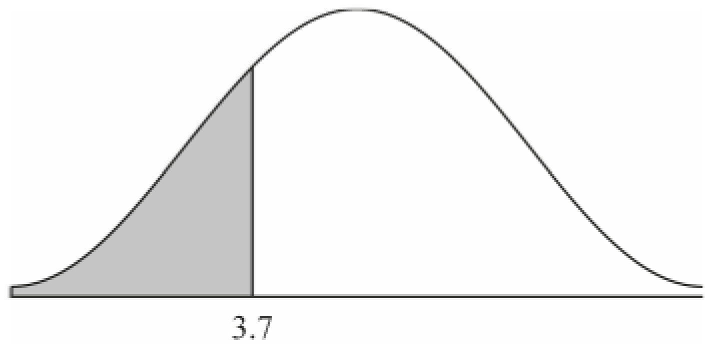
correct probability expression ***(M1)***

$\mathrm{P}(X < 3.7)$

Note: Accept a weak or strict inequality, and any label instead of X, e.g. length or L.

OR

normal curve with vertical line, left of mean, labelled 3.7, and shaded region *(M1)*



THEN

$0.115\ (0.115069\ldots,\ 11.5\%)$ ***A1***

Note: Award ***M1A0*** for 0.12 if no previous working.

[2 marks]

It is known that 30% of the seeds have a length greater than k cm.

(b) Find the value of k. [2]

Markscheme

EITHER

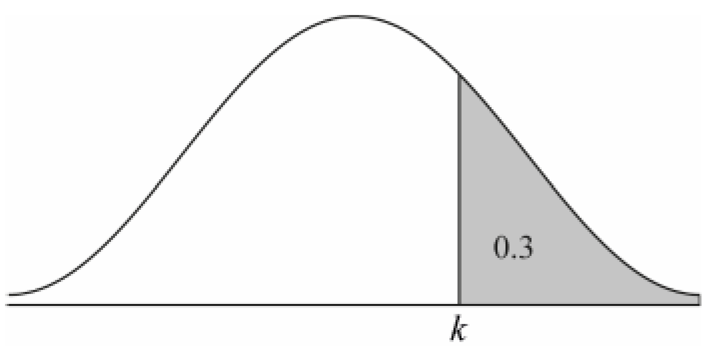
Correct probability expression *(M1)*

$\mathrm{P}(X < k) = 0.7$ **OR** $\mathrm{P}(X > k) = 0.3$

Note: Accept a weak or strict inequality, and any label instead of X e.g., length or L.

OR

normal curve with vertical line to the right of the mean and shaded region, correctly labelled either 0.3 or 0.7 ***(M1)***



THEN

$(k =)\ 4.13\ (4.13110\ldots)$ ***A1***

Note: Award ***M1A0*** for 4.1 if no previous working.

[2 marks]

For a seed of length d cm, chosen at random, $\mathrm{P}(4 - m < d < 4 + m) = 0.6$.

(c) Find the value of m. [2]

Markscheme

EITHER

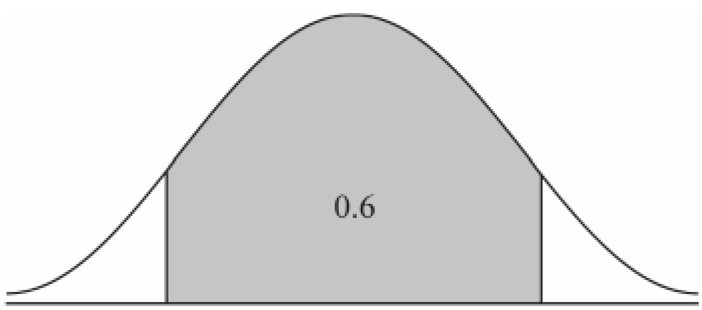
correct probability equation ***(M1)***

$\mathrm{P}(\text{length} < 4 + m) = 0.8$ **OR** $\mathrm{P}(\text{length} < 4 - m) = 0.2$

Note: Accept any letter instead of "length" e.g., X or L.

OR

normal curve with vertical lines symmetrical about the mean line with a correct indication of an area of 0.6 or 0.2 or 0.8 ***(M1)***



THEN

$0.210 \ (0.210405\ldots)$ ***A1***

Note: Award ***(M1)A0*** for an answer of 3.7895 or 4.2105 seen without working. Condone 0.21 seen and award ***(M1)A1***.

[2 marks]

3. [Maximum mark: 5] 22N.1.SL.TZ0.8

Roy is a member of a motorsport club and regularly drives around the Port Campbell racetrack.

The times he takes to complete a lap are normally distributed with mean 59 seconds and standard deviation 3 seconds.

(a) Find the probability that Roy completes a lap in less than 55 seconds. [2]

Markscheme

$\mathrm{P}(T < 55)$ ***(M1)***

$0.0912\ \ (0.0912112\ldots)$ ***A1***

Note: Award ***M1*** for a correct calculator notation such as $\text{normal cdf}(0,\ 55,\ 59,\ 3)$ or $\text{normal cdf}\left(-1^{99},\ 55,\ 59,\ 3\right)$.

[2 marks]

Roy will complete a 20 lap race. It is expected that 8.6 of the laps will take more than t seconds.

(b) Find the value of t. [3]

Markscheme

correct use of expected value

$8.6 = 20 \times p$ **OR** $(p =)\ 0.43$ seen ***(M1)***

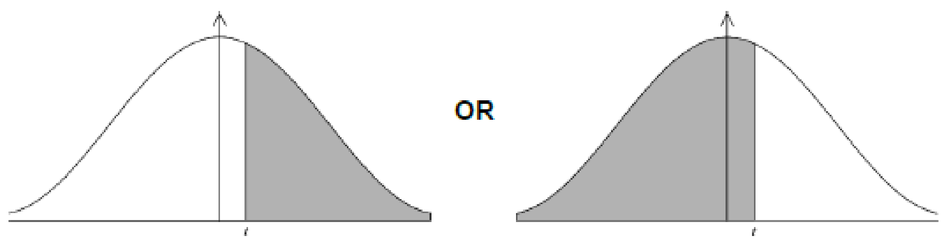
EITHER

correct probability statement

$\mathrm{P}(T > t) = 0.43$ **OR** $P(T < t) = 0.57$ ***(M1)***

OR

t indicated on sketch to communicate correct area ***(M1)***



THEN

$(t =)\ 59.5\ (\text{seconds})\ (59.5291\ldots)$ ***A1***

[3 marks]

4. [Maximum mark: 6] 22M.1.SL.TZ1.8

A factory produces bags of sugar with a labelled weight of $500\ \mathrm{g}$. The weights of the bags are normally distributed with a mean of $500\ \mathrm{g}$ and a standard deviation of $3\ \mathrm{g}$.

(a) Write down the percentage of bags that weigh more than $500\ \mathrm{g}$. [1]

Markscheme

50% ***A1***

Note: Do not accept 0.5 or $\frac{1}{2}$.

[1 mark]

A bag that weighs less than $495\,\mathrm{g}$ is rejected by the factory for being underweight.

(b) Find the probability that a randomly chosen bag is rejected for being underweight. [2]

Markscheme

$0.0478 \ \ (0.0477903\ldots, \ 4.78\%)$ ***A2***

[2 marks]

(c) A bag that weighs more than k grams is rejected by the factory for being overweight. The factory rejects 2% of bags for being overweight.

Find the value of k. [3]

Markscheme

$\mathrm{P}(X < k) = 0.98$ **OR** $\mathrm{P}(X > k) = 0.02$ ***(M1)***

Note: Award ***(M1)*** for a sketch with correct region identified.

$506\,\mathrm{g}\quad(506.161\ldots)$ ***A2***

[3 marks]

5. [Maximum mark: 5] 22M.1.SL.TZ2.10

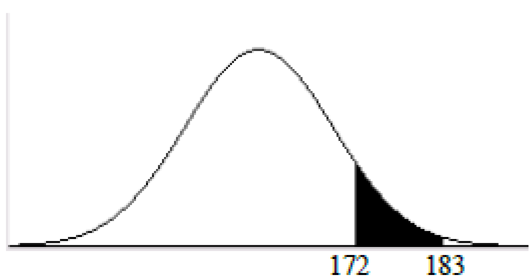
The masses of Fuji apples are normally distributed with a mean of $163\,\mathrm{g}$ and a standard deviation of $6.83\,\mathrm{g}$.

When Fuji apples are picked, they are classified as small, medium, large or extra large depending on their mass. Large apples have a mass of between $172\,\mathrm{g}$ and $183\,\mathrm{g}$.

(a) Determine the probability that a Fuji apple selected at random will be a large apple. [2]

Markscheme

sketch of normal curve with shaded region to the right of the mean and correct values ***(M1)***



$0.0921\quad(0.0920950\ldots)$ ***A1***

[2 marks]

Approximately 68% of Fuji apples have a mass within the medium-sized category, which is between k and 172 g.

(b) Find the value of k. [3]

Markscheme

EITHER

$(\mathrm{P}(x < 172))$

$0.906200\ldots$ ***(A1)***

$(0.906200\ldots - 0.68)$

$0.226200\ldots$ ***(A1)***

OR

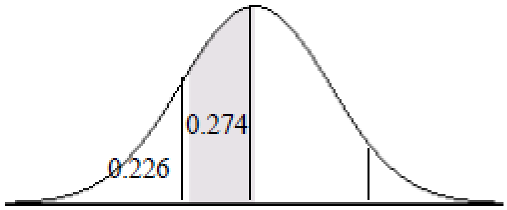
$(\mathrm{P}(163 < x < 172))$

$0.406200\ldots$ ***(A1)***

$0.5 - (0.68 - 0.406200\ldots)$ **OR**
$0.5 + (0.68 - 0.406200\ldots)$

$0.226200\ldots$ **OR** $0.773799\ldots$ ***(A1)***

OR



(A1)(A1)

Note: Award ***A1*** for a normal distribution curve with a vertical line on each side of the mean and a correct probability of either 0.406 or 0.274 or 0.906 shown, ***A1*** for a probability of 0.226 seen.

THEN

$(k =)\ 158\,\text{g}\ \ (157.867\ldots\ \text{g})$ ***A1***

[3 marks]

6. [Maximum mark: 6] 23M.1.AHL.TZ1.11

A shop sells oranges and lemons. The weights of the oranges are assumed to be normally distributed with mean 205 grams and standard deviation 5 grams. The weights of the lemons are assumed to be normally distributed with mean 105 grams and standard deviation 3 grams.

Nelia selects 1 orange and 2 lemons at random and independent of each other. Calculate the probability that the weight of her orange exceeds the combined weight of her lemons. [6]

Markscheme

Let $D = O - L - L$ ***(A1)***

$(\text{mean} =)\ 205 - 105 - 105\ (= -5)$ ***(A1)*** manipulating variances (not standard deviations) ***(M1)***

$(\text{variance} =)\ 25 + 9 + 9\ (= 43)$ **OR** $(\text{SD} =)\ 6.55743\ldots$ ***(A1)***

D~$\text{N}(205 - 105 - 105,\ 25 + 9 + 9)$

attempt to find the probability that $D > 0$ ***(M1)***

$\text{P}(D > 0)$

$= 0.223\ (0.222882\ldots)$ ***A1***

Note: If $D = O - 2L$ is seen or implied, award at most ***(A0)A1(M0)(A0)M1A0***.

[6 marks]

© International Baccalaureate Organization, 2024