Review - prob. distr. [97 marks]

1. [Maximum mark: 8] 18N.2.AHL.TZ0.H_3

It is known that 56 % of Infiglow batteries have a life of less than 16 hours, and 94 % have a life less than 17 hours. It can be assumed that battery life is modelled by the normal distribution $\mathrm{N}\left(\mu,\ \sigma^2\right)$.

(a) Find the value of μ and the value of σ. [6]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

use of inverse normal (implied by ±0.1509… or ±1.554…) ***(M1)***

$P(X < 16) = 0.56$

$\Rightarrow \frac{16-\mu}{\sigma} = 0.1509\ldots$ ***(A1)***

$P(X < 17) = 0.94$

$\Rightarrow \frac{17-\mu}{\sigma} = 1.554\ldots$ ***(A1)***

attempt to solve a pair of simultaneous equations ***(M1)***

$\mu = 15.9,\ \sigma = 0.712$ ***A1A1***

[6 marks]

(b) Find the probability that a randomly selected Infiglow battery will have a life of at least 15 hours. [2]

Markscheme

correctly shaded diagram or intent to find $P(X \geq 15)$ ***(M1)***

$= 0.895$ ***A1***

Note: Accept answers rounding to 0.89 or 0.90. Award ***M1A0*** for the answer 0.9.

[2 marks]

2. [Maximum mark: 6] 18N.1.SL.TZ0.T_14

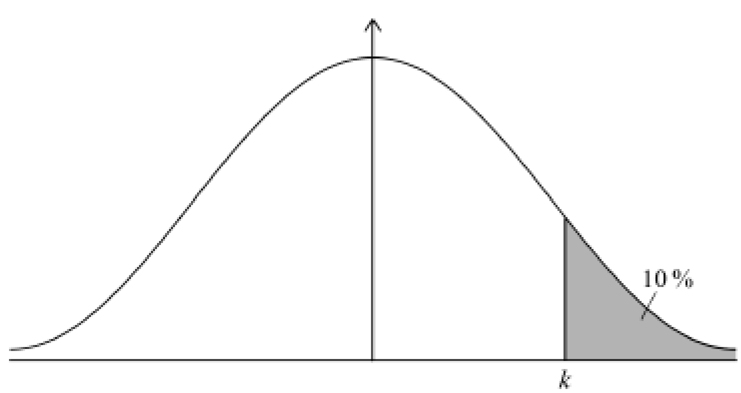
The marks achieved by students taking a college entrance test follow a normal distribution with mean 300 and standard deviation 100.

In this test, 10 % of the students achieved a mark greater than *k*.

(a) Find the value of *k*. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



(M1)

Note: Award ***(M1)*** for diagram that shows the correct shaded area and percentage, k has to be greater than the mean.

OR

Award ***(M1)*** for P(mark > k) = 0.1 or P(mark ≤ k) = 0.9 seen.

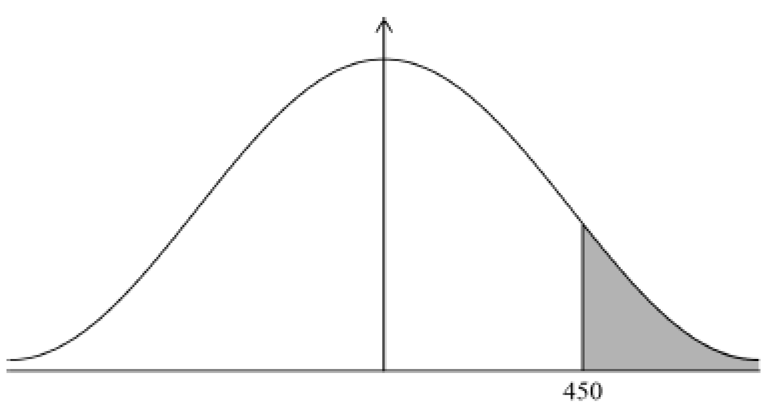
428 (428.155…) ***(A1) (C2)***

[2 marks]

Marron College accepts only those students who achieve a mark of at least 450 on the test.

(b) Find the probability that a randomly chosen student will be accepted by Marron College. [2]

Markscheme



(M1)

Note: Award ***(M1)*** for diagram that shows the correct shaded area and the value 450 labelled to the right of the mean.

OR

Award ***(M1)*** for P(mark ≥ 450) seen.

0.0668 (0.0668072…, 6.68 %, 6.68072… %) ***(A1) (C2)***

[2 marks]

(c) Given that Naomi attends Marron College, find the probability that she achieved a mark of at least 500 on the test. [2]

Markscheme

$\frac{0.0228}{0.0668}\ \left(\frac{0.0227500\ldots}{0.0668072\ldots}\right)$ ***(M1)***

Note: Award ***(M1)*** for 0.0228 (0.0227500…) seen. Accept 1 − 0.97725.

= 0.341 (0.340532…, 34.1 %, 34.0532…%) ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (b), provided answer is between zero and 1.

[2 marks]

3. [Maximum mark: 5] 19M.2.AHL.TZ2.H_2

Timmy owns a shop. His daily income from selling his goods can be modelled as a normal distribution, with a mean daily income of $820, and a standard deviation of $230. To make a profit, Timmy's daily income needs to be greater than $1000.

(a) Calculate the probability that, on a randomly selected day, Timmy makes a profit. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$X \sim N(820, 230^2)$ ***(M1)***

Note: Award ***M1*** for an attempt to use normal distribution. Accept labelled normal graph.

$\Rightarrow P(X > 1000) = 0.217$ ***A1***

[2 marks]

(b) The shop is open for 24 days every month.

Calculate the probability that, in a randomly selected month, Timmy makes a profit on between 5 and 10 days (inclusive). [3]

Markscheme

$Y \sim B(24, 0.217...)$ ***(M1)***

Note: Award ***M1*** for recognition of binomial distribution with parameters.

$P(Y \leq 10) - P(Y \leq 4)$ ***(M1)***

Note: Award ***M1*** for an attempt to find $P(5 \leq Y \leq 10)$ or $P(Y \leq 10) - P(Y \leq 4)$.

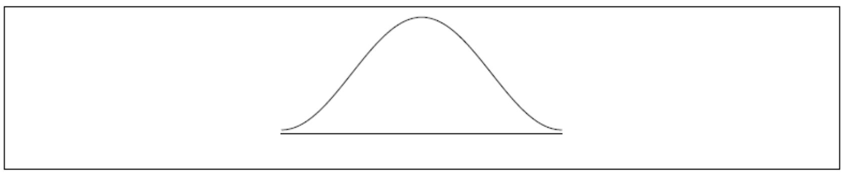
$= 0.613$ ***A1***

[3 marks]

4. [Maximum mark: 6] 19M.1.SL.TZ2.T_14

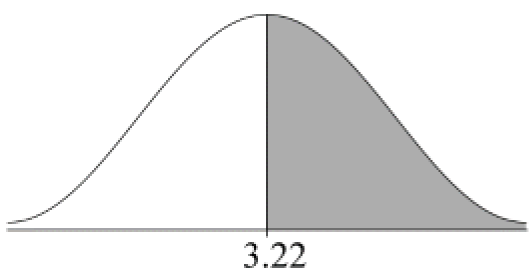
The price per kilogram of tomatoes, in euro, sold in various markets in a city is found to be normally distributed with a mean of 3.22 and a standard deviation of 0.84.

(a.i) On the following diagram, shade the region representing the probability that the price of a kilogram of tomatoes, chosen at random, will be higher than 3.22 euro.



[1]

Markscheme



(A1) (C1)

Note: Award ***(A1)*** for vertical line drawn at the mean (3.22 does not have to be seen) and correct region shaded.

[1 mark]

(a.ii) Find the price that is two standard deviations above the mean price. [1]

Markscheme

4.90 ***(A1) (C1)***

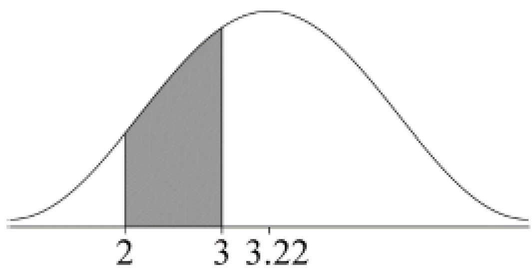
[1 mark]

(b) Find the probability that the price of a kilogram of tomatoes, chosen at random, will be between 2.00 and 3.00 euro. [2]

Markscheme

0.323 (0.323499…; 32.3 %) ***(A2) (C2)***

Note: If final answer is incorrect, ***(M1)(A0)*** may be awarded for correct shaded area shown on a sketch, below, or for a correct probability statement "P(2 ≤ X ≤ 3)" (accept other variables for X or "price" and strict inequalities).



[2 marks]

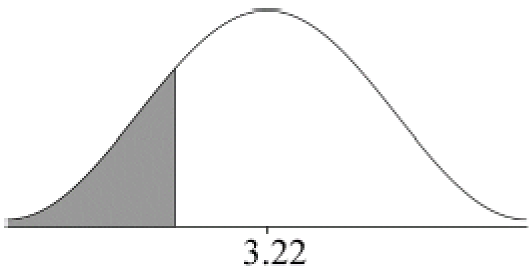
(c) To stimulate reasonable pricing, the city offers a free permit to the sellers whose price of a kilogram of tomatoes is in the lowest 20 %.

Find the highest price that a seller can charge and still receive a free permit. [2]

Markscheme

2.51 (2.51303…) ***(A2) (C2)***

Note: If final answer is incorrect, ***(M1)(A0)*** may be awarded for correct shaded area shown on a sketch, below, or for a correct probability statement "P(X ≤ a) = 0.2" (accept other variables and strict inequalities).



[2 marks]

5. [Maximum mark: 7] 19M.1.AHL.TZ1.H_6

Let X be a random variable which follows a normal distribution with mean μ.
Given that $\mathrm{P}\left(X<\mu-5\right)=0.2$, find

(a) $\mathrm{P}\left(X>\mu+5\right)$. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

use of symmetry *eg* diagram ***(M1)***

$\mathrm{P}\left(X>\mu+5\right)=0.2$ ***A1***

[2 marks]

(b) $\mathrm{P}\left(X<\mu+5\mid X>\mu-5\right)$. [5]

Markscheme

EITHER

$\mathrm{P}\left(X<\mu+5\mid X>\mu-5\right)=\frac{\mathrm{P}(X>\mu-5\cap X<\mu+5)}{\mathrm{P}(X>\mu-5)}$ ***(M1)***

$= \frac{P(\mu-5<X<\mu+5)}{P(X>\mu-5)}$ ***(A1)***

$= \frac{0.6}{0.8}$ ***A1A1***

Note: ***A1*** for denominator is independent of the previous ***A*** marks.

OR

use of diagram ***(M1)***

Note: Only award ***(M1)*** if the region $\mu - 5 < X < \mu + 5$ is indicated and used.

$P(X > \mu - 5) = 0.8 \quad P(\mu - 5 < X < \mu + 5) = 0.6$ ***(A1)***

Note: Probabilities can be shown on the diagram.

$= \frac{0.6}{0.8}$ ***M1A1***

THEN

$= \frac{3}{4} = (0.75)$ ***A1***

[5 marks]

6. [Maximum mark: 4] 20N.2.AHL.TZ0.H_2

Jenna is a keen book reader. The number of books she reads during one week can be modelled by a Poisson distribution with mean 2.6.

Determine the expected number of weeks in one year, of 52 weeks, during which Jenna reads at least four books. [4]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

let X be the random variable “number of books Jenna reads per week.”

then $X \sim \text{Po}(2.6)$

$\text{P}(X \geq 4) = 0.264 \quad (0.263998\ldots)$ ***(M1)(A1)***

$0.263998\ldots \times 52$ ***(M1)***

$= 13.7$ ***A1***

Note: Accept 14 weeks.

[4 marks]

7. [Maximum mark: 14] 20N.2.AHL.TZ0.H_9

The weights, in grams, of individual packets of coffee can be modelled by a normal distribution, with mean $102\,\text{g}$ and standard deviation $8\,\text{g}$.

(a) Find the probability that a randomly selected packet has a weight less than $100\,\text{g}$. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$X \sim \text{N}(102,\ 8^2)$

$\text{P}(X < 100) = 0.401$ ***(M1)A1***

[2 marks]

(b) The probability that a randomly selected packet has a weight greater than w grams is 0.444. Find the value of w. [2]

Markscheme

$\mathrm{P}(X > w) = 0.444$ ***(M1)***

$\Rightarrow w = 103\ (\mathrm{g})$ ***A1***

[2 marks]

(c) A packet is randomly selected. Given that the packet has a weight greater than 105 g, find the probability that it has a weight greater than 110 g. [3]

Markscheme

$\mathrm{P}(X > 100 |\ X > 105) = \frac{\mathrm{P}(X>100 \cap X>105)}{\mathrm{P}(X>105)}$ ***(M1)***

$= \frac{\mathrm{P}(X>100)}{\mathrm{P}(X>105)}$ ***(A1)***

$= \frac{0.15865\ldots}{0.35383\ldots}$

$= 0.448$ ***A1***

[3 marks]

(d) From a random sample of 500 packets, determine the number of packets that would be expected to have a weight lying within 1.5 standard deviations of the mean. [3]

Markscheme

EITHER

$\mathrm{P}(90 < X < 114) = 0.866\ldots$ ***(A1)***

OR

$\mathrm{P}(-1.5 < Z < 1.5) = 0.866\ldots$ ***(A1)***

THEN

$0.866\ldots \times 500$ ***(M1)***

$= 433$ ***A1***

[3 marks]

(e) Packets are delivered to supermarkets in batches of 80. Determine the probability that at least 20 packets from a randomly selected batch have a weight less than 95 g. [4]

Markscheme

$p = \mathrm{P}(X < 95) = 0.19078\ldots$ ***(A1)***

recognising $Y \sim \mathrm{B}(80,\ p)$ ***(M1)***

now using $Y \sim \mathrm{B}(80,\ 0.19078\ldots)$ ***(M1)***

$\mathrm{P}(Y \geq 20) = 0.116$ ***A1***

[4 marks]

8. [Maximum mark: 6] 19N.1.SL.TZ0.T_12

The Malthouse Charity Run is a 5 kilometre race. The time taken for each runner to complete the race was recorded. The data was found to be normally distributed with a mean time of 28 minutes and a standard deviation of 5 minutes.

A runner who completed the race is chosen at random.

(a) Write down the probability that the runner completed the race in more than 28 minutes. [1]

Markscheme

$0.5\ \left(\frac{1}{2},\ 50\%\right)$ ***(A1) (C1)***

[1 mark]

(b) Calculate the probability that the runner completed the race in less than 26 minutes. [2]

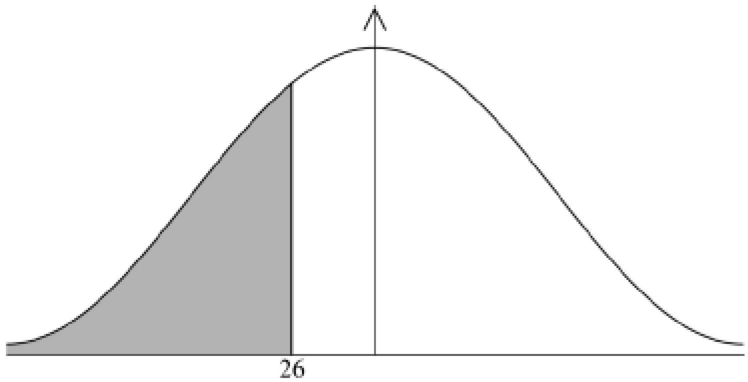
Markscheme

$\mathrm{P}(X \leq 26)$ ***(M1)***

Note: Award ***(M1)*** for a correct mathematical statement.

OR

Award ***(M1)*** for a diagram that shows the value 26 labelled to the left of the mean and the correct shaded region.



3.45 $(0.344578\ldots,\ 34.5\%)$ ***(A1) (C2)***

[2 marks]

(c) It is known that 20% of the runners took more than 28 minutes and less than k minutes to complete the race.

Find the value of k. [3]

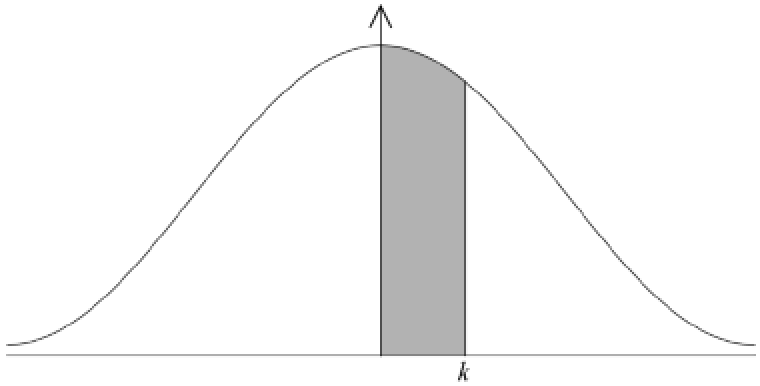
Markscheme

0.7 **OR** 0.3 (seen) ***(A1)***

Note: Award ***(A1)*** for 0.7 or 0.3 seen.

$\mathrm{P}(\text{time} < 7) = 0.7$ **OR** $\mathrm{P}(\text{time} > k) = 0.3$ ***(M1)***

Note: Award ***(M1)*** for a correct mathematical statement.
OR
Award ***(M1)*** for a diagram that shows k greater than the mean and shading in the region below k, above k, or between k and the mean.



$(k =)\ 30.6\ (30.6220\ldots)$ (minutes) ***(A1) (C3)***

Note: Accept "30 minutes and 37 seconds" or (from 3 sf k value) "30 minutes and 36 seconds".

[3 marks]

9. [Maximum mark: 6] 19N.2.AHL.TZ0.H_2

The number of marathons that Audrey runs in any given year can be modelled by a Poisson distribution with mean 1.3 .

(a) Calculate the probability that Audrey will run at least two marathons in a particular year. [2]

Markscheme

$X \sim \text{Po}\,(1.3)$

$\text{P}\,(X \geqslant 2) = 0.373$ ***(M1)A1***

[2 marks]

(b) Find the probability that she will run at least two marathons in exactly four out of the following five years. [4]

Markscheme

$V \sim \mathrm{B}(5, 0.373)$ ***(M1)A1***

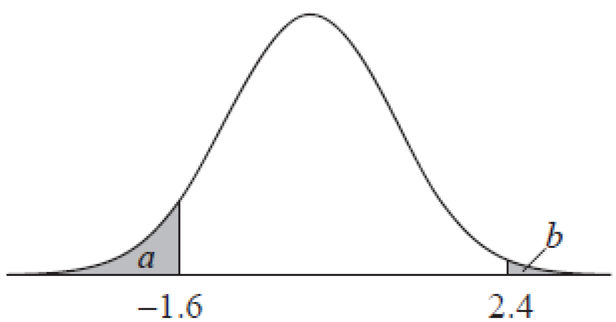
Note: Award ***(M1)*** for recognition of binomial or equivalent, ***A1*** for correct parameters.

$\mathrm{P}(V = 4) = 0.0608$ ***(M1)A1***

[4 marks]

10. [Maximum mark: 13] 19M.1.SL.TZ1.S_9

A random variable Z is normally distributed with mean 0 and standard deviation 1. It is known that $\mathrm{P}(z < -1.6) = a$ and $\mathrm{P}(z > 2.4) = b$. This is shown in the following diagram.



(a) Find $\mathrm{P}(-1.6 < z < 2.4)$. Write your answer in terms of a and b. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

recognizing area under curve = 1 ***(M1)***

eg $a + x + b = 1$, $100 - a - b$, $1 - a + b$

$\mathrm{P}(-1.6 < z < 2.4) = 1 - a - b \ \ (= 1 - (a + b))$ ***A1 N2***

[2 marks]

(b) Given that $z > -1.6$, find the probability that z < 2.4 . Write your answer in terms of a and b. [4]

Markscheme

$\mathrm{P}(z > -1.6) = 1 - a$ (seen anywhere) ***(A1)***

recognizing conditional probability ***(M1)***

eg $\mathrm{P}(A|B)$, $\mathrm{P}(B|A)$

correct working ***(A1)***

eg $\frac{\mathrm{P}(z<2.4 \cap z>-1.6)}{\mathrm{P}(z>-1.6)}$, $\frac{\mathrm{P}(-1.6<z<2.4)}{\mathrm{P}(z>-1.6)}$

$\mathrm{P}(z < 2.4 | z > -1.6) = \frac{1-a-b}{1-a}$ ***A1 N4***

Note: Do not award the final ***A1*** if correct answer is seen followed by incorrect simplification.

[4 marks]

A second random variable X is normally distributed with mean m and standard deviation s.

It is known that P(x < 1) = a.

(c) Write down the standardized value for $x = 1$. [1]

Markscheme

$z = -1.6$ (may be seen in part (d)) ***A1 N1***

Note: Depending on the candidate's interpretation of the question, they may give $\frac{1-m}{s}$ as the answer to part (c). Such answers should be awarded the first ***(M1)*** in part (d), even when part (d) is left blank. If the candidate goes on to show $z = -1.6$ as part of their working in part (d), the ***A1*** in part (c) may be awarded.

[1 mark]

(d) It is also known that P(x > 2) = b.

Find s. [6]

Markscheme

attempt to standardize x (do not accept $\frac{x-\mu}{\sigma}$) ***(M1)***

eg $\frac{1-m}{s}$ (may be seen in part (c)), $\frac{m-2}{s}$, $\frac{x-m}{\sigma}$

correct equation with each z-value ***(A1)(A1)***

eg $-1.6 = \frac{1-m}{s}$, $2.4 = \frac{2-m}{s}$, $m + 2.4s = 2$

valid approach (to set up equation in one variable) ***M1***

eg $2.4 = \frac{2-(1.6s+1)}{s}$, $\frac{1-m}{-1.6} = \frac{2-m}{2.4}$

correct working ***(A1)***

eg $1.6s + 1 = 2 - 2.4s$, $4s = 1$, $m = \frac{7}{5}$

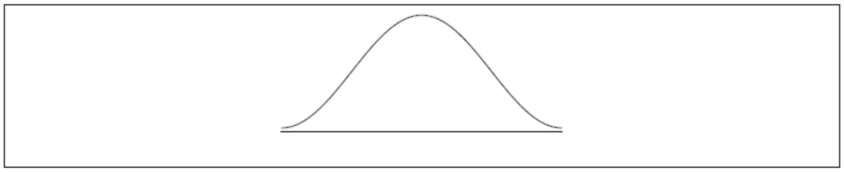
$s = \frac{1}{4}$ ***A1 N2***

[6 marks]

11. [Maximum mark: 6] 19M.1.SL.TZ2.T_14

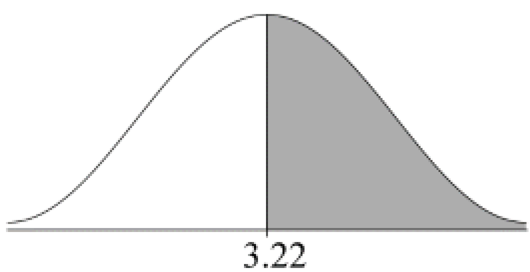
The price per kilogram of tomatoes, in euro, sold in various markets in a city is found to be normally distributed with a mean of 3.22 and a standard deviation of 0.84.

(a.i) On the following diagram, shade the region representing the probability that the price of a kilogram of tomatoes, chosen at random, will be higher than 3.22 euro.



[1]

Markscheme



(A1) (C1)

Note: Award ***(A1)*** for vertical line drawn at the mean (3.22 does not have to be seen) and correct region shaded.

[1 mark]

(a.ii) Find the price that is two standard deviations above the mean price.

[1]

Markscheme

4.90 ***(A1) (C1)***

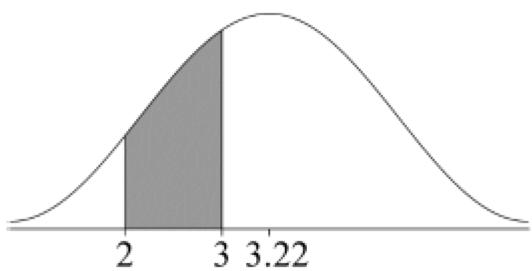
[1 mark]

(b) Find the probability that the price of a kilogram of tomatoes, chosen at random, will be between 2.00 and 3.00 euro. [2]

Markscheme

0.323 (0.323499…; 32.3 %) ***(A2) (C2)***

Note: If final answer is incorrect, ***(M1)(A0)*** may be awarded for correct shaded area shown on a sketch, below, or for a correct probability statement "P(2 ≤ X ≤ 3)" (accept other variables for X or "price" and strict inequalities).



[2 marks]

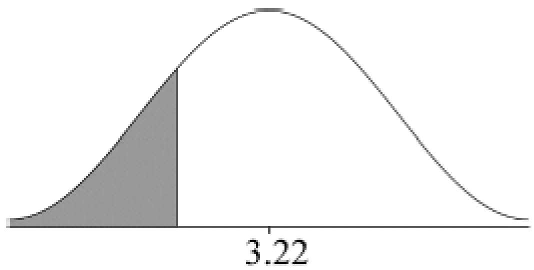
(c) To stimulate reasonable pricing, the city offers a free permit to the sellers whose price of a kilogram of tomatoes is in the lowest 20 %.

Find the highest price that a seller can charge and still receive a free permit. [2]

Markscheme

2.51 (2.51303…) ***(A2) (C2)***

Note: If final answer is incorrect, ***(M1)(A0)*** may be awarded for correct shaded area shown on a sketch, below, or for a correct probability statement "P(X ≤ a) = 0.2" (accept other variables and strict inequalities).



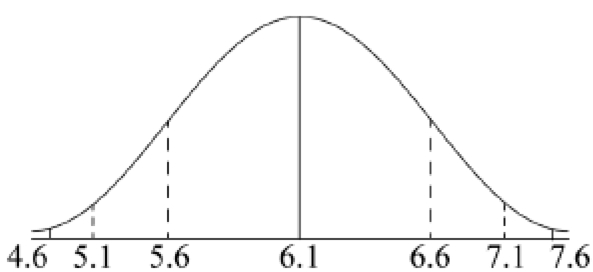
[2 marks]

12. [Maximum mark: 16] 21M.2.AHL.TZ2.2

It is known that the weights of male Persian cats are normally distributed with mean $6.1\,\mathrm{kg}$ and variance $0.5^2\,\mathrm{kg}^2$.

(a) Sketch a diagram showing the above information. [2]

Markscheme



A1A1

Note: Award ***A1*** for a normal curve with mean labelled 6.1 or μ, ***A1*** for indication of SD (0.5) : marks on horizontal axis at 5.6 and/or 6.6 **OR** $\mu - 0.5$ and/or $\mu + 0.5$ on the correct side and approximately correct position.

[2 marks]

(b) Find the proportion of male Persian cats weighing between $5.5\,\text{kg}$ and $6.5\,\text{kg}$. [2]

Markscheme

$X \sim \text{N}(6.1,\ 0.5^2)$

$\text{P}(5.5 < X < 6.5)$ **OR** labelled sketch of region ***(M1)***

$= 0.673\ \ (0.673074\ldots)$ ***A1***

[2 marks]

A group of 80 male Persian cats are drawn from this population.

(c) Determine the expected number of cats in this group that have a weight of less than $5.3\,\text{kg}$. [3]

Markscheme

$(\text{P}(X < 5.3) =)\ 0.0547992\ldots$ ***(A1)***

$0.0547992\ldots \times 80$ ***(M1)***

$= 4.38\ \ (4.38393\ldots)$ ***A1***

[3 marks]

The male cats are now joined by 80 female Persian cats. The female cats are drawn from a population whose weights are normally distributed with mean $4.5\,\text{kg}$ and standard deviation $0.45\,\text{kg}$.

Ten female cats are chosen at random.

(d.i) Find the probability that exactly one of them weighs over $4.62\,\text{kg}$. [4]

Markscheme

$Y \sim \mathrm{N}(4.5,\ 0.45^2)$,

$(\mathrm{P}(Y > 4.62) =)\ 0.394862\ldots$ *(A1)*

use of binomial seen or implied *(M1)*

using $\mathrm{B}(10,\ 0.394862\ldots)$ *(M1)*

$0.0430\ \ (0.0429664\ldots)$ *A1*

[4 marks]

(d.ii) Let N be the number of cats weighing over $4.62\,\text{kg}$.

Find the variance of N. [1]

Markscheme

$np(1-p) = 2.39\ \ (2.38946\ldots)$ *A1*

[1 mark]

(e) A cat is selected at random from all 160 cats.

Find the probability that the cat was female, given that its weight was over $4.7\,\text{kg}$. [4]

Markscheme

$\mathrm{P}(F \cap (W > 4.7)) = 0.5 \times 0.3284\ \ (= 0.1642)$ *(A1)* attempt use of tree diagram **OR** use of

$\mathrm{P}(F|W > 4.7) = \frac{\mathrm{P}(F\cap(W>4.7))}{\mathrm{P}(W>4.7)}$ ***(M1)***

$\frac{0.5\times0.3284}{0.5\times0.9974+0.5\times0.3284}$ ***(A1)***

$= 0.248\ \ (0.247669\ldots)$ ***A1***

[4 marks]