Review - prob. distr. [97 marks]

1. [Maximum mark: 8] 18N.2.AHL.TZ0.H_3

It is known that 56 % of Infiglow batteries have a life of less than 16 hours, and 94 % have a life less than 17 hours. It can be assumed that battery life is modelled by the normal distribution $\mathrm{N}\left(\mu,\ \sigma^2\right)$.

(a) Find the value of μ and the value of σ. [6]

(b) Find the probability that a randomly selected Infiglow battery will have a life of at least 15 hours. [2]

2. [Maximum mark: 6] 18N.1.SL.TZ0.T_14

The marks achieved by students taking a college entrance test follow a normal distribution with mean 300 and standard deviation 100.

In this test, 10 % of the students achieved a mark greater than k.

(a) Find the value of k. [2]

Marron College accepts only those students who achieve a mark of at least 450 on the test.

(b) Find the probability that a randomly chosen student will be accepted by Marron College. [2]

(c) Given that Naomi attends Marron College, find the probability that she achieved a mark of at least 500 on the test. [2]

3. [Maximum mark: 5] 19M.2.AHL.TZ2.H_2

Timmy owns a shop. His daily income from selling his goods can be modelled as a normal distribution, with a mean daily income of $820, and a standard deviation of $230. To make a profit, Timmy's daily income needs to be greater than $1000.

(a) Calculate the probability that, on a randomly selected day, Timmy makes a profit. [2]

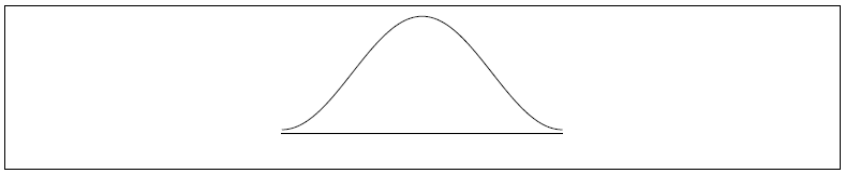
(b) The shop is open for 24 days every month.

Calculate the probability that, in a randomly selected month, Timmy makes a profit on between 5 and 10 days (inclusive). [3]

4. [Maximum mark: 6] 19M.1.SL.TZ2.T_14

The price per kilogram of tomatoes, in euro, sold in various markets in a city is found to be normally distributed with a mean of 3.22 and a standard deviation of 0.84.

(a.i) On the following diagram, shade the region representing the probability that the price of a kilogram of tomatoes, chosen at random, will be higher than 3.22 euro.



[1]

(a.ii) Find the price that is two standard deviations above the mean price. [1]

(b) Find the probability that the price of a kilogram of tomatoes, chosen at random, will be between 2.00 and 3.00 euro. [2]

(c) To stimulate reasonable pricing, the city offers a free permit to the sellers whose price of a kilogram of tomatoes is in the

lowest 20 %.

Find the highest price that a seller can charge and still receive a free permit. [2]

5. [Maximum mark: 7] 19M.1.AHL.TZ1.H_6

Let X be a random variable which follows a normal distribution with mean μ.
Given that $\mathrm{P}(X < \mu - 5) = 0.2$, find

(a) $\mathrm{P}(X > \mu + 5)$. [2]

(b) $\mathrm{P}(X < \mu + 5 \mid X > \mu - 5)$. [5]

6. [Maximum mark: 4] 20N.2.AHL.TZ0.H_2

Jenna is a keen book reader. The number of books she reads during one week can be modelled by a Poisson distribution with mean 2.6.

Determine the expected number of weeks in one year, of 52 weeks, during which Jenna reads at least four books. [4]

7. [Maximum mark: 14] 20N.2.AHL.TZ0.H_9

The weights, in grams, of individual packets of coffee can be modelled by a normal distribution, with mean $102\,\mathrm{g}$ and standard deviation $8\,\mathrm{g}$.

(a) Find the probability that a randomly selected packet has a weight less than $100\,\mathrm{g}$. [2]

(b) The probability that a randomly selected packet has a weight greater than w grams is 0.444. Find the value of w. [2]

(c) A packet is randomly selected. Given that the packet has a weight greater than $105\,\text{g}$, find the probability that it has a weight greater than $110\,\text{g}$. [3]

(d) From a random sample of 500 packets, determine the number of packets that would be expected to have a weight lying within 1.5 standard deviations of the mean. [3]

(e) Packets are delivered to supermarkets in batches of 80. Determine the probability that at least 20 packets from a randomly selected batch have a weight less than $95\,\text{g}$. [4]

8. [Maximum mark: 6] 19N.1.SL.TZ0.T_12

The Malthouse Charity Run is a 5 kilometre race. The time taken for each runner to complete the race was recorded. The data was found to be normally distributed with a mean time of 28 minutes and a standard deviation of 5 minutes.

A runner who completed the race is chosen at random.

(a) Write down the probability that the runner completed the race in more than 28 minutes. [1]

(b) Calculate the probability that the runner completed the race in less than 26 minutes. [2]

(c) It is known that 20% of the runners took more than 28 minutes and less than k minutes to complete the race.

Find the value of k. [3]

9. [Maximum mark: 6] 19N.2.AHL.TZ0.H_2

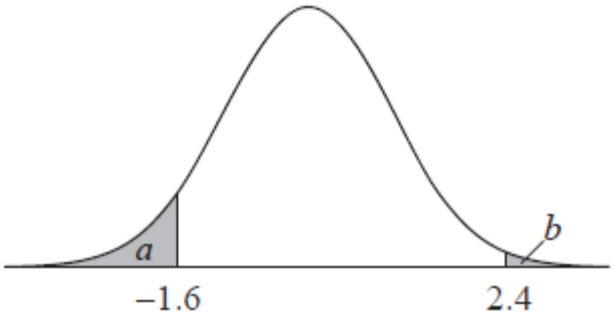
The number of marathons that Audrey runs in any given year can be modelled by a Poisson distribution with mean 1.3 .

(a) Calculate the probability that Audrey will run at least two marathons in a particular year. [2]

(b) Find the probability that she will run at least two marathons in exactly four out of the following five years. [4]

10. [Maximum mark: 13] 19M.1.SL.TZ1.S_9

A random variable Z is normally distributed with mean 0 and standard deviation 1. It is known that $P(z < -1.6) = a$ and $P(z > 2.4) = b$. This is shown in the following diagram.



(a) Find $P(-1.6 < z < 2.4)$. Write your answer in terms of a and b. [2]

(b) Given that $z > -1.6$, find the probability that $z < 2.4$. Write your answer in terms of a and b. [4]

A second random variable X is normally distributed with mean m and standard deviation s.

It is known that $P(x < 1) = a$.

(c) Write down the standardized value for $x = 1$. [1]

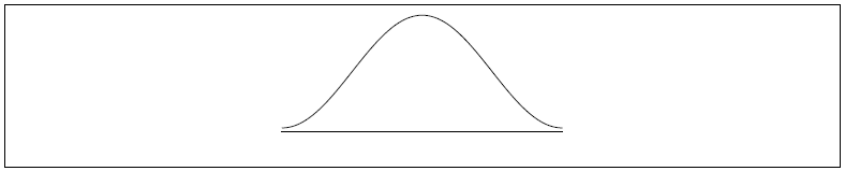
(d) It is also known that $P(x > 2) = b$.

Find s. [6]

11. [Maximum mark: 6] 19M.1.SL.TZ2.T_14

The price per kilogram of tomatoes, in euro, sold in various markets in a city is found to be normally distributed with a mean of 3.22 and a standard deviation of 0.84.

(a.i) On the following diagram, shade the region representing the probability that the price of a kilogram of tomatoes, chosen at random, will be higher than 3.22 euro.



[1]

(a.ii) Find the price that is two standard deviations above the mean price. [1]

(b) Find the probability that the price of a kilogram of tomatoes, chosen at random, will be between 2.00 and 3.00 euro. [2]

(c) To stimulate reasonable pricing, the city offers a free permit to the sellers whose price of a kilogram of tomatoes is in the lowest 20 %.

Find the highest price that a seller can charge and still receive a free permit. [2]

12. [Maximum mark: 16] 21M.2.AHL.TZ2.2

It is known that the weights of male Persian cats are normally distributed with mean $6.1\,\text{kg}$ and variance $0.5^2\,\text{kg}^2$.

(a) Sketch a diagram showing the above information. [2]

(b) Find the proportion of male Persian cats weighing between $5.5\,\text{kg}$ and $6.5\,\text{kg}$. [2]

A group of 80 male Persian cats are drawn from this population.

(c) Determine the expected number of cats in this group that have a weight of less than $5.3\,\text{kg}$. [3]

The male cats are now joined by 80 female Persian cats. The female cats are drawn from a population whose weights are normally distributed with mean $4.5\,\text{kg}$ and standard deviation $0.45\,\text{kg}$.

Ten female cats are chosen at random.

(d.i) Find the probability that exactly one of them weighs over $4.62\,\text{kg}$. [4]

(d.ii) Let N be the number of cats weighing over $4.62\,\text{kg}$.

Find the variance of N. [1]

(e) A cat is selected at random from all 160 cats.

Find the probability that the cat was female, given that its weight was over $4.7\,\text{kg}$. [4]