1. The weights of the oranges produced by a farm may be assumed to be normally distributed with mean 205 grams and standard deviation 10 grams.

(a) Find the probability that a randomly chosen orange weighs more than 200 grams.

(2)

(b) Five of these oranges are selected at random to be put into a bag. Find the probability that the combined weight of the five oranges is less than 1 kilogram.

(4)

(c) The farm also produces lemons whose weights may be assumed to be normally distributed with mean 75 grams and standard deviation 3 grams. Find the probability that the weight of a randomly chosen orange is more than three times the weight of a randomly chosen lemon.

(5)
(Total 11 marks)

(a) $z = \frac{200-205}{10} = -0.5$ (M1)

probability = 0.691 (accept 0.692) A1

Note: Award M1A0 for 0.309 or 0.308

(b) let X be the total weight of the 5 oranges

then $E(X) = 5 \times 205 = 1025$ (A1)

$\text{Var}(X) = 5 \times 100 = 500$ (M1)(A1)

$P(X < 1000) = 0.132$ (A1)

(c) let $Y = B - 3C$ where B is the weight of a random orange and C the weight of a random lemon (M1)

$E(Y) = 205 - 3 \times 75 = -20$ (A1)

$\text{Var}(Y) = 100 + 9 \times 9 = 181$ (M1)(A1)

$P(Y > 0) = 0.0686$ A1

Note: Award A1 for 0.0681 obtained from tables

[11]

2. A shop sells apples and pears. The weights, in grams, of the apples may be assumed to have a N $(200, 15^2)$ distribution and the weights of the pears, in grams, may be assumed to have a N $(120, 10^2)$ distribution.

(a) Find the probability that the weight of a randomly chosen apple is more than double the weight of a randomly chosen pear.

(8)

(b) A shopper buys 3 apples and 4 pears. Find the probability that the total weight is greater than 1000 grams.

(6)
(Total 14 marks)

(a) Let X, Y (grams) denote respectively the weights of a randomly chosen apple, pear.
Then

$X - 2Y$ is N $(200 - 2 \times 120,\ 15^2 + 4 \times 10^2)$, (M1)(A1)(A1)

i.e. $N(-40, 25^2)$ A1

We require

$P(X > 2Y) = P(X - 2Y > 0)$ (M1)(A1)

$= 0.0548$ A2

(b) Let $T = X_1 + X_2 + X_3 + Y_1 + Y_2 + Y_3 + Y_4$ (grams) denote the total weight.

Then

T is N $(3 \times 200 + 4 \times 120,\ 3 \times 15^2 + 4 \times 10^2)$, (M1)(A1)(A1)
i.e. $N(1080, 1075)$ A1
$P(T > 1000) = 0.993$ A2

[14]

3. (a) The random variable Y is such that $E(2Y + 3) = 6$ and $Var(2 - 3Y) = 11$.

Calculate

(i) $E(Y)$;

(ii) $Var(Y)$;

(iii) $E(Y^2)$.

(6)

(b) Independent random variables R and S are such that

$$R \sim N(5, 1) \text{ and } S \sim N(8, 2).$$

The random variable V is defined by $V = 3S - 4R$.

Calculate $P(V > 5)$.

(6)
(Total 12 marks)

(a) (i) $E(2Y + 3) = 6$
$2E(Y) + 3 = 6$ M1
$E(Y) = \frac{3}{2}$ A1

(ii) $Var(2 - 3Y) = 11$
$Var(-3Y) = 11$ (M1)
$9 Var(Y) = 11$
$Var(Y) = \frac{11}{9}$ A1

(iii) $E(Y^2) = Var(Y) + [E(Y)]^2$ M1
$= \frac{11}{9} + \frac{9}{4}$
$= \frac{125}{36}$ A1 N0

(b) $E(V) = E(3S - 4R)$
$= 3E(S) - 4E(R)$ M1
$= 24 - 20 = 4$ A1
$Var(3S - 4R) = 9 Var(S) + 16 Var(R)$, since R and S are independent random variables M1
$= 18 + 16 = 34$ A1

$V \sim N(4, 34)$
$P(V > 5) = 0.432$ A2 N0

[12]