1. [9 *points*]

Use the definition of $\frac{dy}{dx}$ to show that:

(a) If $y = 2x^2 + x$, then $\frac{dy}{dx} = 4x + 1$. [3]

(b) If $y = \frac{3}{x}$, then $\frac{dy}{dx} = -\frac{3}{x^2}$. [3]

(c) If $y = 4\sqrt{x}$, then $\frac{dy}{dx} = \frac{2}{\sqrt{x}}$. [3]

(a)

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 + x + \Delta x - 2x^2 - x}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{2x^2} + 4x\Delta x + (\Delta x)^2 + \cancel{x} + \Delta x - \cancel{2x^2} - \cancel{x}}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{4x\cancel{\Delta x} + (\Delta x)^{\cancel{2}} + \cancel{\Delta x}}{\cancel{\Delta x}} = \lim_{\Delta x \to 0} 4x + \Delta x + 1 = 4x + 1$$

(b)

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\frac{3}{x+\Delta x} - \frac{3}{x}}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{3x - 3(x+\Delta x)}{\Delta x(x)(x+\Delta x)} =$$

$$= \lim_{\Delta x \to 0} \frac{-3\cancel{\Delta x}}{\cancel{\Delta x}(x)(x+\Delta x)} =$$

$$= \lim_{\Delta x \to 0} \frac{-3}{x^2 + x\Delta x} = -\frac{3}{x^2}$$

(c)

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{4\sqrt{x+\Delta x} - 4\sqrt{x}}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{4(\sqrt{x+\Delta x} - \sqrt{x})(\sqrt{x+\Delta x} + \sqrt{x})}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})} =$$

$$= \lim_{\Delta x \to 0} \frac{4(\cancel{x} + \Delta x - \cancel{x})}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})} =$$

$$= \lim_{\Delta x \to 0} \frac{4\cancel{\Delta x}}{\cancel{\Delta x}(\sqrt{x+\Delta x} + \sqrt{x})} =$$

$$= \lim_{\Delta x \to 0} \frac{4}{(\sqrt{x+\Delta x} + \sqrt{x})} = \frac{4}{2\sqrt{x}} = \frac{2}{\sqrt{x}}$$

2. [11 *points*]

Find the gradient function y' of the following functions. Simplify your answers.

(a) $y = \dfrac{2}{x^2} + 3x - 1$ [2]

(b) $y = x^5 \sin(2x)\cos(3x)$ [3]

(c) $y = \sqrt{e^{2x} + \sin^2 x}$ [3]

(d) $y = \dfrac{x^3 - \pi \ln 2}{x^2 + e}$ [3]

(a) $y' = -\dfrac{4}{x^3} + 3$

(b) $y' = 5x^4 \sin(2x)\cos(3x) + 2x^5 \cos(2x)\cos(3x) - 3x^5 \sin(2x)\sin(3x)$

(c) $y' = \dfrac{2e^{2x} + 2\sin x \cos x}{2\sqrt{e^{2x} + \sin^2 x}} = \dfrac{e^{2x} + \sin x \cos x}{\sqrt{e^{2x} + \sin^2 x}}$

(d) $y' = \dfrac{3x^2(x^2+e) - (x^3 - \pi \ln 2)(2x)}{(x^2+e)^2} = \dfrac{x^4 + 3ex^2 + 2\pi x \ln 2}{(x^2+e)^2}$