Chi squared test [68 marks]

1. [Maximum mark: 5] SPM.1.SL.TZ0.6

As part of a study into healthy lifestyles, Jing visited Surrey Hills University. Jing recorded a person's position in the university and how frequently they ate a salad. Results are shown in the table.

	Salad meals per week			
	0	**1 – 2**	**3 – 4**	**> 4**
Students	45	26	18	6
Professors	15	8	5	12
Staff and Administration	16	13	10	6

Jing conducted a χ^2 test for independence at a 5 % level of significance.

(a) State the null hypothesis. [1]

Markscheme

number of salad meals per week is independent of a person's position in the university ***A1***

Note: Accept "not associated" instead of independent.

[1 mark]

(b) Calculate the p-value for this test. [2]

Markscheme

0.0201 (0.0201118…) ***A2***

[2 marks]

(c) State, giving a reason, whether the null hypothesis should be accepted. [2]

Markscheme

$0.0201 < 0.05$ ***R1***

the null hypothesis is rejected ***A1***

Note: Award ***(R1)*** for a correct comparison of their p-value to the test level, award ***(A1)*** for the correct interpretation from that comparison.

Do not award ***(R0)(A1)***.

[2 marks]

2. [Maximum mark: 16] EXN.2.SL.TZ0.4

A dice manufacturer claims that for a novelty die he produces the probability of scoring the numbers 1 to 5 are all equal, and the probability of a 6 is two times the probability of scoring any of the other numbers.

(a) Find the probability of scoring a six when rolling the novelty die. [3]

Markscheme

*This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

Let the probability of scoring $1, \ \ldots, \ 5$ be p,

$5p + 2p = 1 \Rightarrow p = \frac{1}{7}$ **(M1)(A1)**

Probability of $6 = \frac{2}{7}$ **A1**

[3 marks]

(b) Find the probability of scoring more than 2 sixes when this die is rolled 5 times. [4]

Markscheme

Let the number of sixes be X

$X \sim \mathrm{B}\left(5, \frac{2}{7}\right)$ **(M1)**

$\mathrm{P}(X > 2) = \mathrm{P}(X \geq 3)$ or $\mathrm{P}(X > 2) = 1 - \mathrm{P}(X \leq 2)$ **(M1)**

$= 0.145 \ (0.144701\ldots)$ **(M1)A1**

[4 marks]

To test the manufacture’s claim one of the novelty dice is rolled 350 times and the numbers scored on the die are shown in the table below.

Number scored	Frequency
1	32
2	57
3	47
4	58
5	54
6	102

(c.i) Find the expected frequency for each of the numbers if the manufacturer’s claim is true. [2]

Markscheme

Expected frequency is $350 \times p$ or $350 \times 2p$ **(M1)**

Number scored	Frequency	Expected frequency
1	32	50
2	57	50
3	47	50
4	58	50
5	54	50
6	102	100

A1

[2 marks]

A χ^2 goodness of fit test is to be used with a 5% significance level.

(c.ii) Write down the null and alternative hypotheses. [2]

Markscheme

H_0 : The manufacture's claim is correct **A1**
H_1 : The manufacturer's claim is not correct **A1**

[2 marks]

(c.iii) State the degrees of freedom for the test. [1]

Markscheme

Degrees of freedom $= 5$ **A1**

[1 mark]

(c.iv) Determine the conclusion of the test, clearly justifying your answer. [4]

Markscheme

p-value $= 0.0984\ (0.0984037\ldots)$ **(M1)A1**

$0.0984 > 0.05$ **R1**

Hence insufficient evidence to reject the manufacture's claim. **A1**

[4 marks]

3. [Maximum mark: 5] 22N.1.SL.TZ0.4

Sergio is interested in whether an adult's favourite breakfast berry depends on their income level. He obtains the following data for 341 adults and decides to carry out a χ^2 test for independence, at the 10% significance level.

		Income level		
		Low	Medium	High
Favourite berry	Strawberry	21	39	30
	Blueberry	39	67	42
	Other berry	32	45	26

(a) Write down the null hypothesis. [1]

Markscheme

The favourite breakfast/berry (of adults) is independent of (their) income (level). ***A1***

[1 mark]

(b) Find the value of the χ^2 statistic. [2]

Markscheme

$\chi^2 = 2.27 \ \ (2.26821\ldots)$ ***A2***

[2 marks]

The critical value of this χ^2 test is 7.78.

(c) Write down Sergio's conclusion to the test in context. Justify your answer. [2]

Markscheme

EITHER

$2.27 < 7.78$ **OR** $2.27 <$ critical value ***R1***

OR

$0.687 > 0.1$ (using p-value)

THEN

(Do not reject H_0)

Insufficient evidence (at the 10% significance level) that the favourite berry depends on income level. ***A1***

Note: Do not award ***R0A1***. Accept "χ^2" in place of their "2.27", provided an answer was seen in part (b). Their conclusion must be consistent with their χ^2 (or a correct p-value) and their hypothesis.

[2 marks]

4. [Maximum mark: 9] EXM.1.SL.TZ0.9

Six coins are tossed simultaneously 320 times, with the following results.

0 tail	5 times
1 tail	40 times
2 tails	86 times
3 tails	89 times
4 tails	67 times
5 tails	29 times
6 tails	4 times

At the 5% level of significance, test the hypothesis that all the coins are fair. [9]

Markscheme

Let H_0 be the hypothesis that all coins are fair, ***(C1)***

and let H_1 be the hypothesis that not all coins are fair. ***(C1)***

Let T be the number of tails obtained, T is binomially distributed. ***(M1)***

T	0	1	2	3	4	5	6
f_0	5	40	86	89	67	29	4
f_e	**5**	**30**	**75**	**100**	**75**	**30**	**5**

(A3)

Notes: Award ***(A2)*** if one entry on the third row is incorrect. Award ***(A1)*** if two entries on the third row are incorrect. Award ***(A0)*** if three or more entries on the third row are incorrect.

$$\chi^2_{\text{calc}} = \frac{(5-5)^2}{5} + \frac{(40-30)^2}{30} + \frac{(86-75)^2}{75} + \frac{(89-100)^2}{100} + \frac{(67-75)^2}{75} + \frac{(29-30)^2}{30} + \frac{(4-5)^2}{5}$$

$= 7.24$ ***(A1)***

Also $\chi^2_{0.05,\ 6} = 12.592$ ***(A1)***

Since $7.24 < 12.592$, H_0 cannot be rejected. ***(R1)***

[9 marks]

5. [Maximum mark: 6] 23M.1.SL.TZ2.6

A company that owns many restaurants wants to determine if there are differences in the quality of the food cooked for three different meals: breakfast, lunch and dinner.

Their quality assurance team randomly selects 500 items of food to inspect. The quality of this food is classified as perfect, satisfactory, or poor. The data is summarized in the following table.

		Quality			
		Perfect	Satisfactory	Poor	Total
Meal	Breakfast	101	124	7	232
	Lunch	68	81	5	154
	Dinner	35	69	10	114
	Total	204	274	22	500

An item of food is chosen at random from these 500.

(a) Find the probability that its quality is not perfect, given that it is from breakfast. [2]

Markscheme

$0.565 \ \left(0.564655\ldots,\ \frac{131}{232},\ 56.4655\ldots\%\right)$ ***A1A1***

Note: Award ***A1*** for correct numerator, ***A1*** for correct denominator.

[2 marks]

A χ^2 test at the 5% significance level is carried out to determine if there is significant evidence of a difference in the quality of the food cooked for the three meals.

The critical value for this test is 9.488.

The hypotheses for this test are:

H_0:The quality of the food and the type of meal are independent.
H_1:The quality of the food and the type of meal are not independent.

(b) Find the χ^2 statistic. [2]

Markscheme

$11.0\ (11.0212\ldots)$ ***A2***

Note: Award **A1** for a final answer of 11 if no unrounded answer is seen.

[2 marks]

(c) State, with justification, the conclusion for this test. [2]

Markscheme

EITHER

$11.0 > 9.88\ (11.0212\ldots > 9.488)$ ***R1***

OR

$0.0263 < 0.05\ (0.0263264\ldots < 0.05)$ ***R1***

THEN

EITHER

(there is significant evidence to) reject H_0 ***A1***

OR

(there is significant evidence that) the (food) quality and the type of meal are not independent ***A1***

Note: Do not award ***R0A1***.

Award ***R1*** for $\chi^2_{\text{calc}} > \chi^2_{\text{crit}}$, provided the calculated value is explicitly seen in part (b).

Accept "p-value $<$ significance level" provided their p-value is seen and their p-value is between 0 and 1.

[2 marks]

6. [Maximum mark: 9] EXM.1.SL.TZ0.11

A calculator generates a random sequence of digits. A sample of 200 digits is randomly selected from the first 100 000 digits of the sequence. The following table gives the number of times each digit occurs in this sample.

digit	0	1	2	3	4	5	6	7	8	9
frequency	17	21	15	19	25	27	19	23	18	16

It is claimed that all digits have the same probability of appearing in the sequence.

(a) Test this claim at the 5% level of significance. [7]

Markscheme

H_0: The sequence contains equal numbers of each digit. ***(A1)***

H_1: The sequence does not contain equal numbers of each digit. ***(A1)***

$\chi^2_{\text{calc}} = \frac{(9+1+25+1+25+49+1+9+4+16)}{20} = 7$ ***(M1)(A1)***

The number of degrees of freedom is 9. ***(A1)***

$\chi^2_{0.95;\ 5} = 16.919$ ***(A1)***

$\chi^2_{\text{calc}} < 16.919$. Hence H_0 is accepted. ***(A1)***

[7 marks]

(b) Explain what is meant by the 5% level of significance. [2]

Markscheme

The probability of rejecting H_0 when it is true *(A1)*

is 0.05. *(A1)*

Note: Award ***(A1)(A1)*** for "the probability of a type I error is 0.05."

[2 marks]

7. [Maximum mark: 11] EXM.1.SL.TZ0.8

In an effort to study the level of intelligence of students entering college, a psychologist collected data from 4000 students who were given a standard test. The predictive norms for this particular test were computed from a very large population of scores having a normal distribution with mean 100 and standard deviation of 10. The psychologist wishes to determine whether the 4000 test scores he obtained also came from a normal distribution with mean 100 and standard deviation 10. He prepared the following table (expected frequencies are rounded to the nearest integer):

Score	observed frequencies	expected frequencies	Score	observed frequencies	expected frequencies
≤ 70.5	20	6	100.5–110.5	1450	
70.5–80.5	90	96	110.5–120.5	499	507
80.5–90.5	575		120.5–130.5	80	76
90.5–100.5	1282		≥ 130.5	4	4

(a) Copy and complete the table, showing how you arrived at your answers. [5]

Markscheme

To calculate expected frequencies, we multiply 4000 by the probability of each cell:

$$p\left(80.5 \leqslant X \leqslant 90.5\right) = p\left(\tfrac{80.5-100}{10} \leqslant Z \leqslant \tfrac{90.5-100}{10}\right) \quad \textit{(M1)}$$

$$= p\left(-19.5 \leqslant Z \leqslant -0.95\right)$$

$$= 0.1711 - 0.0256$$

$$= 0.1455$$

Therefore, the expected frequency $= 4000 \times 0.1455$ *(M1)*

≈ 582 *(A1)*

Similarly: $p\left(90.5 \leqslant X \leqslant 100.5\right) = 0.5199 - 0.1711$

$= 0.3488$

Frequency $= 4000 \times 0.3488$

≈ 1396 *(A1)*

And $p\,(100.5 \leqslant X \leqslant 110.5) = 0.8531 - 0.5199$

$= 0.3332$

Frequency $= 4000 \times 0.3332$

≈ 1333 ***(A1)***

[5 marks]

(b) Test the hypothesis at the 5% level of significance. [6]

Markscheme

To test the goodness of fit of the normal distribution, we use the χ^2 distribution. Since the last cell has an expected frequency less than 5, it is combined with the cell preceding it. There are therefore 7 – 1 = 6 degrees of freedom. ***(C1)***

$$\chi^2 = \frac{(20-6)^2}{6} + \frac{(90-96)^2}{96} + \frac{(575-582)^2}{582} + \frac{(1282-1396)^2}{1396} + \frac{(1450-1333)^2}{1333} + \frac{(499-507)^2}{507} + \frac{(84-80)}{80}^2$$ ***(M1)***

= 53.03 ***(A1)***

H_0: Distribution is Normal with $\mu = 100$ and $\sigma = 10.$

H_1: Distribution is not Normal with $\mu = 100$ and $\sigma = 10.$ ***(M1)***

$\chi^2_{(0.95,\ 6)} = 14.07$

Since $\chi^2 = 53.0 > \chi^2_{\text{critical}} = 14.07$, we reject H_0 ***(A1)***

Or use of p-value

Therefore, we have enough evidence to suggest that the normal distribution with mean 100 and standard deviation 10 does not fit the data well. ***(R1)***

Note: If a candidate has not combined the last 2 cells, award ***(C0)(M1)(A0)(M1)(A1)(R1)*** (or as appropriate).

[6 marks]

8. [Maximum mark: 7] 22M.1.SL.TZ1.7

Leo is investigating whether a six-sided die is fair. He rolls the die 60 times and records the observed frequencies in the following table:

Number on die	1	2	3	4	5	6
Observed frequency	8	7	6	15	12	12

Leo carries out a χ^2 goodness of fit test at a 5% significance level.

(a) Write down the null and alternative hypotheses. [1]

Markscheme

H_0 : The die is fair **OR** $\mathrm{P}(\text{any number}) = \frac{1}{6}$ **OR** probabilities are equal

H_1 : The die is not fair **OR** $P(\text{any number}) \neq \frac{1}{6}$ **OR** probabilities are not equal *A1*

[1 mark]

(b) Write down the degrees of freedom. [1]

Markscheme

5 *A1*

[1 mark]

(c) Write down the expected frequency of rolling a 1. [1]

Markscheme

10 *A1*

[1 mark]

(d) Find the p-value for the test. [2]

Markscheme

(p-value $=$) $0.287 \quad (0.28724163\ldots)$ *A2*

[2 marks]

(e) State the conclusion of the test. Give a reason for your answer. [2]

Markscheme

$0.287 > 0.05$ *R1*

EITHER

Insufficient evidence to reject the null hypothesis *A1*

OR

Insufficient evidence to reject that the die is fair *A1*

Note: Do not award ***R0A1***. Condone "*accept* the null hypothesis" or "the die is fair". Their conclusion must be consistent with their p-value and their hypothesis.

[2 marks]