Implicit differentiation [47 marks]

1. [Maximum mark: 7] EXN.2.AHL.TZ0.6

The curve C has equation $\mathrm{e}^{2y} = x^3 + y$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2\mathrm{e}^{2y} - 1}$. [3]

(b) The tangent to C at the point P is parallel to the y-axis.

Find the x-coordinate of P. [4]

2. [Maximum mark: 8] 21N.2.AHL.TZ0.8

Consider the curve C given by $y = x - xy\ln(xy)$ where $x > 0,\ y > 0$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)(1 + \ln(xy)) = 1$. [3]

(b) Hence find the equation of the tangent to C at the point where $x = 1$. [5]

3. [Maximum mark: 15] 20N.1.AHL.TZ0.H_11

Consider the curve C defined by $y^2 = \sin(xy)\ ,\ y \neq 0$.

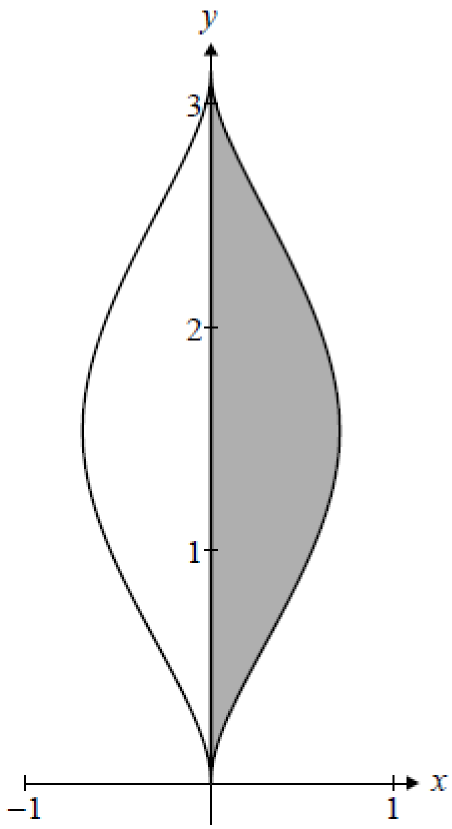
(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y\cos(xy)}{2y - x\cos(xy)}$. [5]

(b) Prove that, when $\frac{\mathrm{d}y}{\mathrm{d}x} = 0\ ,\ y = \pm 1$. [5]

(c) Hence find the coordinates of all points on C, for $0 < x < 4\pi$, where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. [5]

4. [Maximum mark: 8] 19N.2.AHL.TZ0.H_11

The following diagram shows part of the graph of $2x^2 = \sin^3 y$ for $0 \leqslant y \leqslant \pi$.



(a.i) Using implicit differentiation, find an expression for $\frac{dy}{dx}$. [4]

(a.ii) Find the equation of the tangent to the curve at the point $\left(\frac{1}{4}, \frac{5\pi}{6}\right)$. [4]

5. [Maximum mark: 9] 19M.1.AHL.TZ1.H_7

Find the coordinates of the points on the curve $y^3 + 3xy^2 - x^3 = 27$ at which $\frac{dy}{dx} = 0$. [9]