Mixed questions [60 marks]

1. [Maximum mark: 6]

The displacement, x (cm), of the end of a spring, at time t (seconds), is given by

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 0.$$

At $t = 0, x = 0.75$ and $\frac{dx}{dt} = 0$.

Use Euler's method, with a step length 0.1 seconds, to estimate the value of x when $t = 0.5$. [6]

Markscheme

$\frac{dx}{dt} = y$ *(A1)*

$\frac{dy}{dt} = -10x - 2y$ *(A1)*

Note: Writing $\frac{d^2x}{dt^2} = -10x - 2\frac{dx}{dt}$ is a valid approach and should be awarded *A1A1*.

attempt to use the Euler equations shown by finding either a correct x_{n+1} or y_{n+1} *(M1)*

correct equations for both x_{n+1} and y_{n+1}

$x_{n+1} = x_n + 0.1(y_n), \quad y_{n+1} = y_n + 0.1(-10x_n - 2y_n)$
(accept equivalent notation)

$(t_{n+1} = t_n + 0.1)$

Note: All of the above marks can be implied by a correct second row in a table **OR** by a correct f_1 and f_2 clearly identified for use in Euler's method formula.

T	x	y
0	0.75	0
0.1	**0.75**	**−0.75**
0.2	0.675	−1.35
0.3	0.54	−1.755
0.4	0.3645	−1.944
0.5	0.1701	

so estimate is 0.170 *A2*

Note: Accept 0.17 rounded to 2 sf.

[6 marks]

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2.   [Maximum mark: 17]

A large international sports tournament tests their athletes for banned substances.
They interpret a positive test result as meaning that the athlete uses banned substances.
A negative result means that they do not.

The probability that an athlete uses banned substances is estimated to be 0.06.

If an athlete **uses** banned substances, the probability that they will test positive is 0.71.

If an athlete does **not use** banned substances, the probability that they will test negative is 0.98.

(a)   Using the information given, complete the following tree diagram.
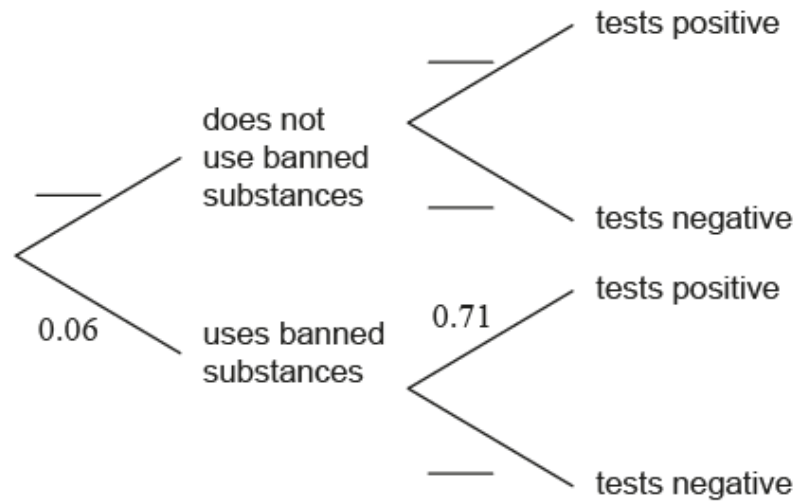
tests positive

does not use banned substances

tests negative

tests positive

0.71

0.06   uses banned substances

tests negative   [2]

(b.i)   Determine the probability that a randomly selected athlete
does not use banned substances and tests negative.   [2]

Markscheme

multiplication of two probabilities along the tree diagram   *(M1)*

0.94×0.98

$= 0.921 \; (0.9212, \; 92.1\%, \; 92.12\%)$ *A1*

[2 marks]

(b.ii)   If two athletes are selected at random, calculate the probability
that both athletes do not use banned substances and both test
negative.   [2]

Markscheme

$(0.9212)^2$ *(A1)*

$= 0.849 \; (0.848609\ldots, \; 84.9\%, \; 84.8609\ldots\%)$ *A1*

[2 marks]

(c.i) Calculate the probability that a randomly selected athlete will receive an **incorrect** test result. [3]

(c.ii) A random sample of 1300 athletes at the tournament are selected for testing. Calculate the expected number of athletes in the sample that will receive an incorrect test result. [2]

Team X are competing in the tournament. There are 20 athletes in this team. It is known that none of the athletes in Team X use banned substances.

(d) Calculate the probability that none of the athletes in Team X
will test positive. [4]

(e) Determine the probability that more than 2 athletes in Team X
will test positive. [2]

3. [Maximum mark: 5]   23M.1.AHL.TZ1.12

Two AC (alternating current) electrical sources with the same frequencies are combined. The voltages from these sources can be expressed as
$V_1 = 6 \sin (at + 30°)$ and $V_2 = 6 \sin (at + 90°)$.

The combined total voltage can be expressed in the form
$V_1 + V_2 = V \sin (at + \theta°)$.

Determine the value of V and the value of θ.   [5]

Markscheme

METHOD 1 Analytical approach

attempt to express V_1 or V_2 in exponential form   *(M1)*

e.g. $V_1 = \text{Im}\left(6e^{i\left(at+\frac{\pi}{6}\right)}\right)$, $V_2 = \text{Im}\left(6e^{i\left(at+\frac{\pi}{2}\right)}\right)$

Note: Accept angles in radians or degrees.

$\left(V_1 + V_2 = \right) 6e^{i \times \frac{\pi}{6}} + 6e^{i \times \frac{\pi}{2}}$ *(A1)*

Note: This mark can be awarded even if seen as part of a correct larger expression.

$$= 10.\,4\mathrm{e}^{1.05\mathrm{i}}\left(6\sqrt{3}\mathrm{e}^{\frac{\mathrm{i}\pi}{3}}\right) \quad \textit{(A1)}$$

so V is $10.\,4\left(10.\,3923\ldots,\ 6\sqrt{3}\right)$ and θ is 60 (degrees)   *A1A1*
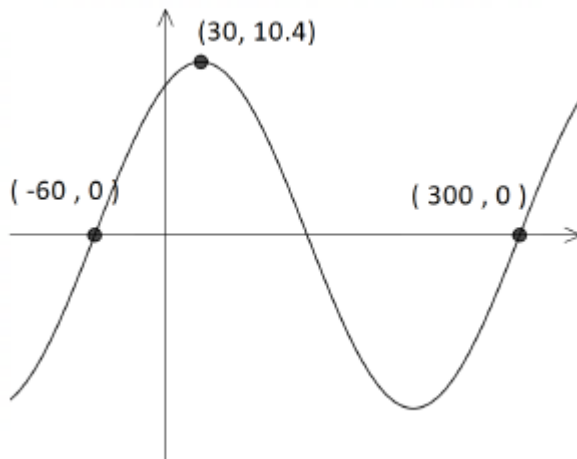
Note: Accept any value for θ that rounds to a 2sf answer of 60.

Do **not** accept a final answer for an angle in radians.

Do **not** award *A1* for answer of 60° resulting from incorrect working.

METHOD 2 Graphical approach

let $at = x$ and plot $V_1 + V_2$ curves on GDC   *(M1)*
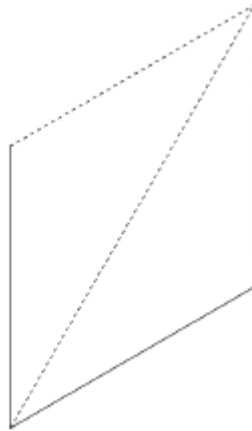

attempt to find maximum   *(M1)*

$$V = 10.\,4 \quad \textit{A1}$$

attempt to find any x-axis intercept (either -60 or 300)   *(M1)*

$\theta = 60$ (degrees) $\theta = -300$ (degrees)   *A1*

METHOD 3 Geometric approach

considering the rhombus

(M1)

$$V = \sqrt{6^2 + 6^2 - 2 \times 6 \times 6 \cos 120^\circ} \quad \textit{(M1)}$$

$$\left(= \sqrt{108} = 6\sqrt{3} \right) = 10.4 \left(10.3923\ldots \right) \quad \textit{A1}$$

$$\theta = 60 \text{ (degrees)} \quad \textit{A2}$$

Note: An answer of $\theta = -300.$ is most likely to be seen in METHOD 2, but should be condoned in METHODS 1 and 3 if seen there.
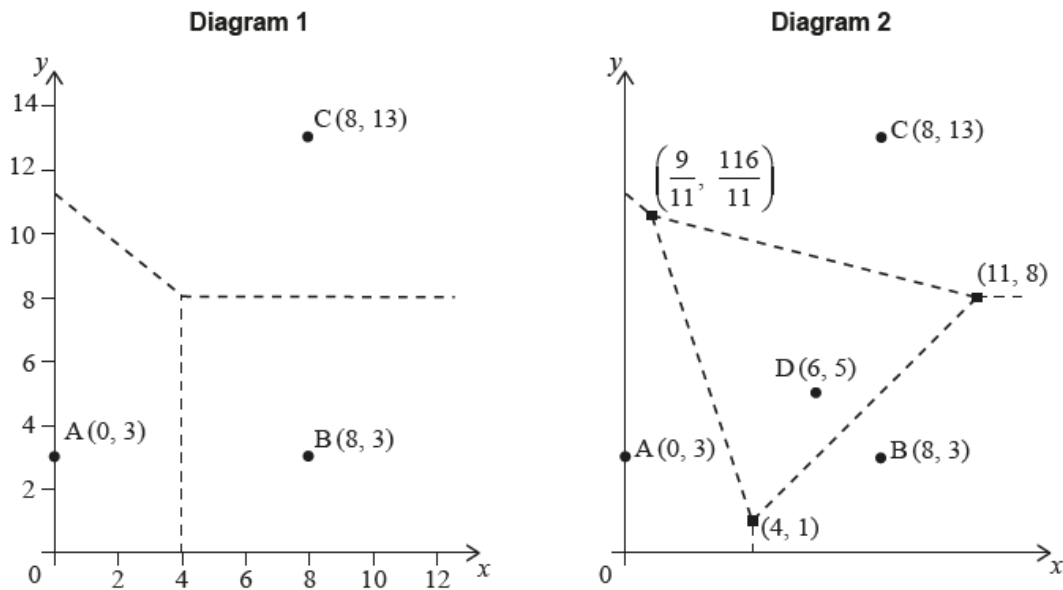
[5 marks]

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4. [Maximum mark: 8]

On the following Voronoi diagram, the coordinates of three farmhouses are

$A(0,\ 3), B(8,\ 3)$ and $C(8,\ 13)$, where distances are measured in

kilometres. Each farmhouse owns the land that is closest to it, and their boundaries are defined by the dotted lines on **Diagram 1**.

Diagram 1   Diagram 2

To provide water to the farms it is decided to construct a well at the point where the boundaries meet on **Diagram 1**.

(a)   Write down the coordinates of this point.   [1]

> Markscheme
>
> $(4, 8)$ *A1*
>
> *[1 mark]*

(b)   Find the equation of the perpendicular bisector of $[AC]$.   [3]

> Markscheme
>
> attempt to find the gradient of AC *(M1)*
>
> $\frac{13-3}{8-0}$, $\frac{10}{8}$, $\left(\frac{5}{4}\right)$, (1.25)
>
> attempt to substitute **their** coordinates and the negative reciprocal of **their** gradient into the equation of a straight line   *(M1)*

$$y - 8 = -\tfrac{4}{5}(x - 4) \text{ OR } 8 = -\tfrac{4}{5}(4) + c \text{ OR}$$
$$c = 11.2$$

$$y - 8 = -\tfrac{4}{5}(x - 4)$$
$$(y = -0.8x + 11.2, \ 4x + 5y - 56 = 0) \qquad \textit{A1}$$

[3 marks]

An additional farmhouse $D(6, \ 5)$ is built on the land. The Voronoi diagram has been redrawn to show the new boundaries. The coordinates of the vertices of these boundaries are indicated on **Diagram 2**.

A wind turbine is to be built at one of the vertices.

(c)   The wind turbine should be as far from the nearest farmhouses as possible.

(c.i)   By calculating appropriate distances, find the location of the wind turbine.   [3]

Markscheme

attempt to find one distance from a farm to any closest vertex   *M1*

finding a correct distance from at least two distinct vertices   *A1*

$$7.58968\ldots, \ 4.472135\ldots \left(\sqrt{20}\right), \ 5.830951\ldots \left(\sqrt{34}\right)$$

$\left(\tfrac{9}{11}, \tfrac{166}{11}\right)$ (is furthest)   *A1*

[3 marks]

(c.ii)   Hence, write down the distance of the wind turbine to the nearest farmhouse.   [1]

$7.59 \, (\text{km}) \, (= 7.58968\ldots)$ **A1**

[1 mark]

5.   [Maximum mark: 18]   SPM.2.AHL.TZ0.3

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14 % **compounded quarterly**. No deposit required and repayments are made each quarter.

(a.i)   Find the repayment made each quarter.   [3]

Markscheme

N = 24
I % = 14
PV = −14000
FV = 0
P/Y = 4
C/Y = 4   *(M1)(A1)*

Note: Award *M1* for an attempt to use a financial app in their technology, award *A1* for all entries correct. Accept PV = 14000.

(€)871.82   *A1*

[3 marks]

(a.ii) Find the total amount paid for the car. [2]

(a.iii) Find the interest paid on the loan. [2]

Finance option B:

A 6 year loan at a nominal annual interest rate of r % **compounded monthly**. Terms of the loan require a 10 % deposit and monthly repayments of €250.

(b.i) Find the amount to be borrowed for this option. [2]

(b.ii) Find the annual interest rate, r. [3]

N = 72

PV = 12600

PMT = −250

FV = 0

P/Y = 12

C/Y = 12   *(M1)(A1)*

Note: Award *M1* for an attempt to use a financial app in their technology, award *A1* for all entries correct. Accept PV = −12600 provided PMT = 250.

12.56(%)   *A1*

[3 marks]

(c)   State which option Bryan should choose. Justify your answer.   [2]

EITHER

Bryan should choose Option A   *A1*

no deposit is required   *R1*

Note: Award *R1* for stating that no deposit is required. Award *A1* for the correct choice from that fact. Do not award *R0A1*.

OR

Bryan should choose Option B   *A1*

cost of Option A (6923.69) > cost of Option B ($72 \times 250 - 12600 = 5400$) **R1**

Note: Award **R1** for a correct comparison of costs. Award **A1** for the correct choice from that comparison. Do not award **R0A1**.

[2 marks]

(d)   Bryan chooses option B. The car dealership invests the money Bryan pays as soon as they receive it.

Если they invest it in an account paying 0.4 % interest per month and inflation is 0.1 % per month, calculate the real amount of money the car dealership has received by the end of the 6 year period.   [4]

Markscheme

real interest rate is $0.4 - 0.1 = 0.3\%$ **(M1)**

value of other payments $250 + 250 \times 1.003 + \ldots + 250 \times 1.003^{71}$

use of sum of geometric sequence formula or financial app on a GDC **(M1)**

$= 20\,058.43$

value of deposit at the end of 6 years

$1400 \times (1.003)^{72} = 1736.98$ **(A1)**

Total value is (€) 21 795.41   **A1**

Note: Both **M** marks can awarded for a correct use of the GDC's financial app:

N = 72 (6×12)
I % = 3.6 (0.3×12)
PV = 0

PMT = −250

FV =

P/Y = 12

C/Y = 12

OR

N = 72 (6 × 12)

I % = 0.3

PV = 0

PMT = −250

FV =

P/Y = 1

C/Y = 1

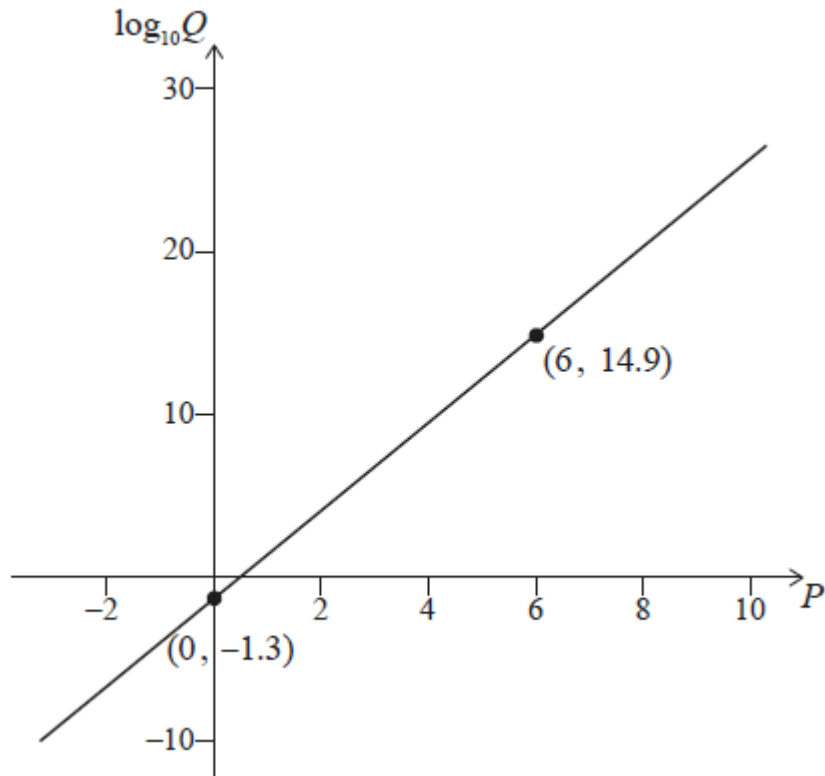*[4 marks]*

6.   [Maximum mark: 6]   22N.1.AHL.TZ0.13

Gen is investigating the relationship between two sets of data, labelled P and Q, that she collected. She created a scatter plot with P on the x-axis and $\log_{10} Q$ on the y-axis. Gen noticed that the points had a strong linear correlation, so she drew a line of best fit, as shown in the diagram. The line passes through the points $(0, \ -1.3)$ and $(6, \ 14.9)$.

(a)　　Find an equation for Q in terms of P.　　　　　　　　　　　　　　　　[3]

Gen also investigates the relationship between the same data, Q, and some new data, R. She believes that the data can be modelled by $Q = a \ln R + b$ and she decides to create a scatter plot to verify her belief.

(b)   State what expression Gen should plot on each axis to verify her
       belief.   [1]

The scatter plot has a linear relationship and Gen finds $a = 4.3$ and
$b = 12.1$.

(c)   Find an equation for P in terms of R.   [2]