Mixed questions [60 marks]

1. [Maximum mark: 6] 23M.1.AHL.TZ1.13

The displacement, x (cm), of the end of a spring, at time t (seconds), is given by

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + 10x = 0.$$

At $t = 0, x = 0.75$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$.

Use Euler's method, with a step length 0.1 seconds, to estimate the value of x when $t = 0.5$. [6]

2. [Maximum mark: 17] 23M.2.AHL.TZ1.3

A large international sports tournament tests their athletes for banned substances.

They interpret a positive test result as meaning that the athlete uses banned substances.

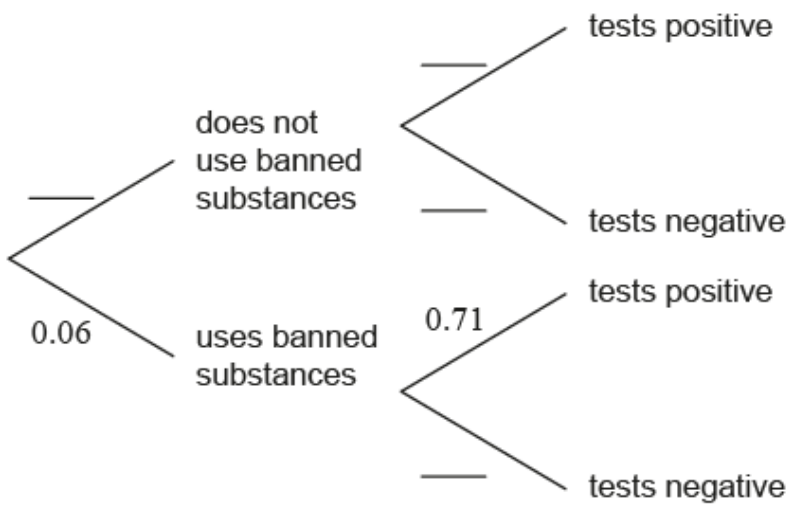
A negative result means that they do not.

The probability that an athlete uses banned substances is estimated to be 0.06.

If an athlete **uses** banned substances, the probability that they will test positive is 0.71.

If an athlete does **not use** banned substances, the probability that they will test negative is 0.98.

(a) Using the information given, complete the following tree diagram.



[2]

(b.i) Determine the probability that a randomly selected athlete does not use banned substances and tests negative. [2]

(b.ii) If two athletes are selected at random, calculate the probability that both athletes do not use banned substances and both test negative. [2]

(c.i) Calculate the probability that a randomly selected athlete will receive an **incorrect** test result. [3]

(c.ii) A random sample of 1300 athletes at the tournament are selected for testing. Calculate the expected number of athletes in the sample that will receive an incorrect test result. [2]

Team X are competing in the tournament. There are 20 athletes in this team. It is known that none of the athletes in Team X use banned substances.

(d) Calculate the probability that none of the athletes in Team X will test positive. [4]

(e) Determine the probability that more than 2 athletes in Team X will test positive. [2]

3. [Maximum mark: 5] 23M.1.AHL.TZ1.12

Two AC (alternating current) electrical sources with the same frequencies are combined. The voltages from these sources can be expressed as $V_1 = 6 \sin (at + 30^\circ)$ and $V_2 = 6 \sin (at + 90^\circ)$.

The combined total voltage can be expressed in the form $V_1 + V_2 = V \sin (at + \theta^\circ)$.

Determine the value of V and the value of θ. [5]

4. [Maximum mark: 8] 23N.1.SL.TZ1.4

On the following Voronoi diagram, the coordinates of three farmhouses are $\mathrm{A}(0,\ 3), \mathrm{B}(8,\ 3)$ and $\mathrm{C}(8,\ 13)$, where distances are measured in kilometres. Each farmhouse owns the land that is closest to it, and their boundaries are defined by the dotted lines on **Diagram 1**.

Diagram 1

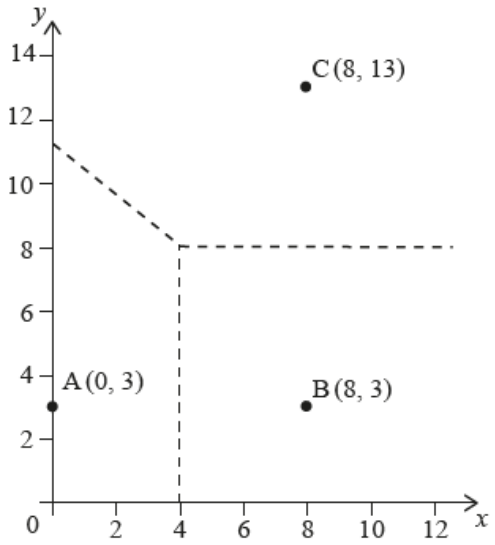
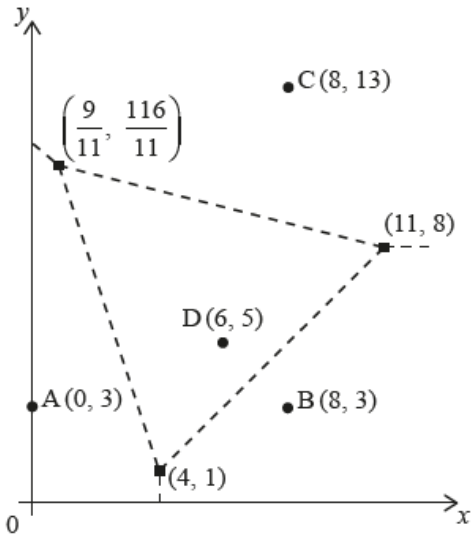


Diagram 2



To provide water to the farms it is decided to construct a well at the point where the boundaries meet on **Diagram 1**.

(a) Write down the coordinates of this point. [1]

(b) Find the equation of the perpendicular bisector of $[\mathrm{AC}]$. [3]

An additional farmhouse $\mathrm{D}(6,\ 5)$ is built on the land. The Voronoi diagram has been redrawn to show the new boundaries. The coordinates of the vertices of these boundaries are indicated on **Diagram 2**.

A wind turbine is to be built at one of the vertices.

(c) The wind turbine should be as far from the nearest farmhouses as possible.

(c.i) By calculating appropriate distances, find the location of the wind turbine. [3]

(c.ii) Hence, write down the distance of the wind turbine to the nearest farmhouse. [1]

5. [Maximum mark: 18] SPM.2.AHL.TZ0.3

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14 % **compounded quarterly**. No deposit required and repayments are made each quarter.

(a.i) Find the repayment made each quarter. [3]

(a.ii) Find the total amount paid for the car. [2]

(a.iii) Find the interest paid on the loan. [2]

Finance option B:

A 6 year loan at a nominal annual interest rate of r % **compounded monthly**. Terms of the loan require a 10 % deposit and monthly repayments of €250.

(b.i) Find the amount to be borrowed for this option. [2]

(b.ii) Find the annual interest rate, r. [3]

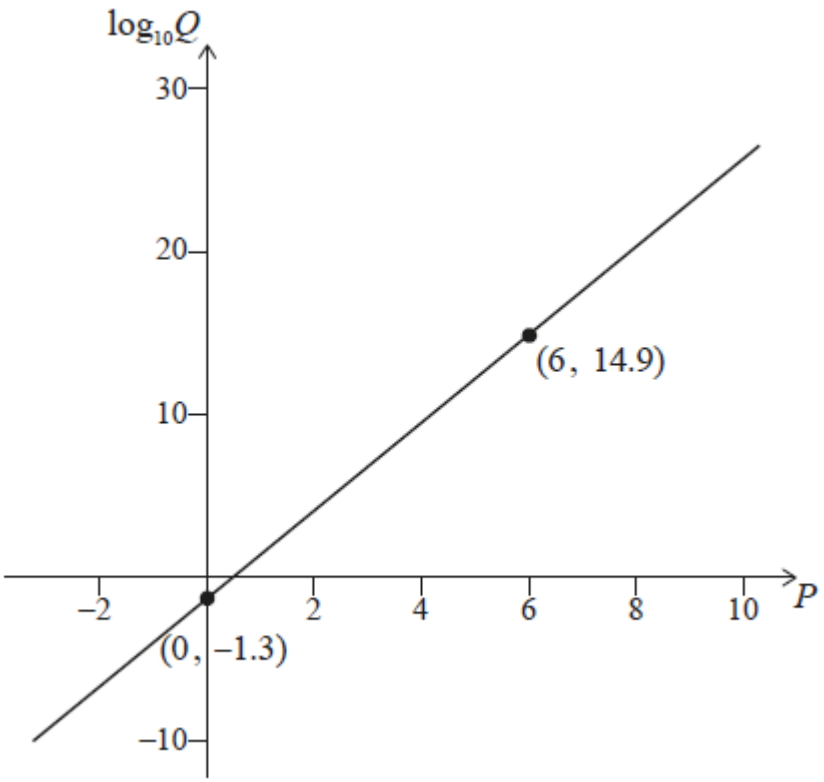
(c) State which option Bryan should choose. Justify your answer. [2]

(d) Bryan chooses option B. The car dealership invests the money Bryan pays as soon as they receive it.

If they invest it in an account paying 0.4 % interest per month and inflation is 0.1 % per month, calculate the real amount of money the car dealership has received by the end of the 6 year period. [4]

6. [Maximum mark: 6] 22N.1.AHL.TZ0.13

Gen is investigating the relationship between two sets of data, labelled P and Q, that she collected. She created a scatter plot with P on the x-axis and $\log_{10} Q$ on the y-axis. Gen noticed that the points had a strong linear correlation, so she drew a line of best fit, as shown in the diagram. The line passes through the points $(0,\ -1.3)$ and $(6,\ 14.9)$.



(a) Find an equation for Q in terms of P. [3]

Gen also investigates the relationship between the same data, Q, and some new data, R. She believes that the data can be modelled by $Q = a \ln R + b$ and she decides to create a scatter plot to verify her belief.

(b) State what expression Gen should plot on each axis to verify her belief. [1]

The scatter plot has a linear relationship and Gen finds $a = 4.3$ and $b = 12.1$.

(c) Find an equation for P in terms of R. [2]