2224 – 7201M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

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Mathematics:
Applications and interpretation

Higher level

Paper 1

20 pages

International Baccalaureate®
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1. (a) **EITHER**

$N = 96$
$PV = \mp 100000$
$FV = \pm 150000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

OR

$N = 8$
$PV = \mp 100000$
$FV = \pm 150000$
$P/Y = 1$
$C/Y = 12$ ***(M1)(A1)***

OR

$150000 = 100000\left(1 + \frac{I}{100 \times 12}\right)^{12 \times 8}$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app (at least 3 entries, not necessarily correct) or an attempt to use a compound interest formula.
Award ***A1*** for all entries correct in financial app or correct substitution in compound interest formula.

THEN

$I = 5.08\ (5.07903\ldots)$ ***A1***

[3 marks]

(b) $N = 120$
$I\% = 6.1$
$PV = \mp 150000$
$PMT = \pm 1000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app (at least 3 entries, not necessarily correct). Award ***A1*** for all entries correct in financial app (condone missing -/+ sign if the correct final answer is seen).

$FV = (\$)\ 110867$ ***A1***

Note: Answer must be correct to nearest dollar to award the final ***A1***. Award ***(M1)(A1)A0*** for an unsupported final answer to a greater degree of accuracy eg. (\$) $110866.70\ldots$
Award ***M1A1A0*** for a truncated answer of 110866 if no working is shown.

[3 marks]
[Total: 6 marks]

2. (a) (i) $r = 0.995 \quad (0.994705\ldots)$ ***A2***

Note: Award ***A1*** for 0.99.

(ii) $m = 10.6t + 43.9 \quad (10.6032...t + 43.8780...)$ ***A1A1***

Note: Second ***A1*** is for the correct variables.

[4 marks]

(b) **EITHER**
$10.6032... \times 1.5$ ***(M1)***
OR
$\left(10.6032...(t+1.5) + 43.8780...\right) - \left(10.6032...(t) + 43.8780...\right)$ ***(M1)***
THEN
15.9 (marks) $(15.9048\ldots)$ ***A1***

Note: Accept 16.

[2 marks]

(c) *Accept any valid reason* ***R1***
e.g:
The students in the sample might not be of equal ability / she has not controlled for ability

She might have originally obtained close to full marks so an extra 15.9 would not be possible.

[1 mark]
[Total: 7 marks]

3. (a) (i) attempt to rearrange to isolate C ***(M1)***
e.g., subtracting 32 or dividing the equation by 1.8

$C = \frac{5}{9}(F - 32) \quad \left(C = \frac{F - 32}{1.8},\ C = 0.556F - 17.8\right)$ ***A1***

Note: If the answer is not written as an equation, award at most ***M1A0***.

(ii) $C = \left(\frac{77 - 32}{1.8} =\right) \ 25 \ (^{\circ}\text{C})$ ***A1***

[3 marks]

(b) (i) $(1.8 \times 17 + 32 =) \ 62.6 \ (^{\circ}\text{F})$ ***A1***

(ii) recognizing that the "+32" does not affect the SD ***(M1)***
$(1.8 \times 9 =) \ 16.2 \ (^{\circ}\text{F})$ ***A1***

Note: Award ***M0A0*** for $1.8 \times 9 + 32 \ (= 48.2)$.

[3 marks]
[Total 6 marks]

4. attempt to use Euler

$y_{n+1} = y_n + 0.1\log(x_n + y_n)$ ***(A1)***

$y_1 \left(= 1 + 0.1 \times \log_{10}(1)\right) = 1$ ***(A1)***

$y_2 = 1.004139...$ ***(A1)***

THEN

when $x = 2$ $\quad y(2) \approx 1.61\ (1.60536...)$ ***A1***

[Total: 4 marks]

5. (a) $y = 0.5x - 1$ ***A1A1***

Note: Award ***A1*** for $0.5x$ and ***A1*** for -1 (or equivalent equation). Award at most ***A1A0*** if answer is not presented as an equation.

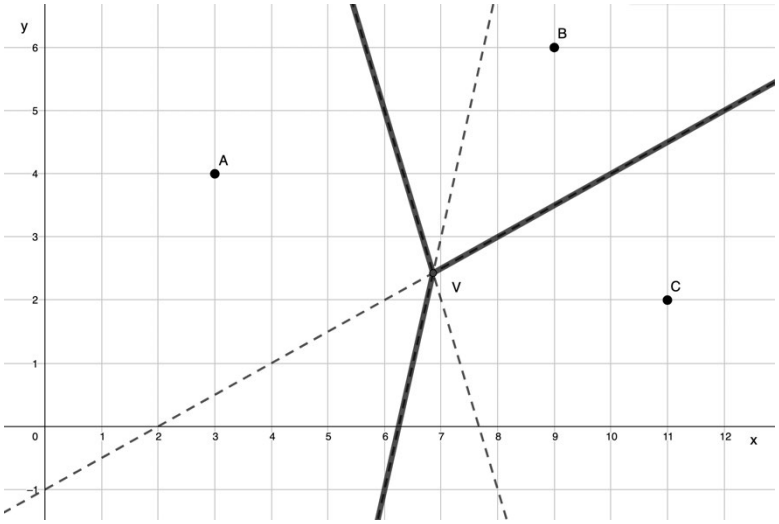
[2 marks]

(b) $(6.857,\ 2.429)$ ***A1A1***

Note: If both answers are not correct to 4 sig figs, award at most ***A1A0***.
Accept $x = 6.857$, $y = 2.429$.
Award ***A1A0*** for $\left(\frac{48}{7}, \frac{17}{7}\right)$. Award ***A0A1*** for $(2.429, 6.857)$.

[2 marks]

(c)



A2

Note: Award marks as shown in the table below. Condone edges that do not extend to the sides of the graph or beyond the x-axis.

Correct edges	Incorrect edges	Marks
3	0	***A2***
3	1	***A1A0***
3	2 or more	***A0A0***
2	0	***A1A0***
2	1	***A1A0***
2	2 or more	***A0A0***
1	0	***A1A0***
1	1 or more	***A0A0***

[2 marks]
[Total 6 marks]

6. (a) attempt to substitute into geometric sequence formula for twelfth term **OR** at least three correct terms of the sequence ***(M1)***

$u_{12} = 40 \times 1.1^{12-1}$ **OR** $40, 44, 48.4, \ldots$

$114 \quad (114.124\ldots)$ ***A1***

[2 marks]

(b) (i) attempt to substitute into geometric series formula **OR** a sum of at least the first three terms ***(M1)***

$S_{12} = \dfrac{40\left(1.1^{12}-1\right)}{1.1-1}$ **OR** $\sum_{1}^{12}\left(40 \times 1.1^{n-1}\right)$ **OR** $40+44+48.4+\ldots$

Note: Award ***M1*** for $u_1 = 40$ and $r = 1.1$ seen as part of a geometric series formula, or ***M1*** for sigma notation and their u_n formula (condone missing limits), or ***M1*** for the sum of at least the **correct** first three terms of the sequence.

$S_{12} = 855 \quad (855.371\ldots)$ ***A1***

(ii) finding $S_{24} = 3539.89...$ or attempt to find the sum between u_{13} and u_{24} ***(M1)***

Note: Award ***M1*** for $S_{24} = 3539.89\ldots$ **or** sigma notation that includes correct limits and their u_n formula **or** a substituted geometric series formula that includes $125.537\ldots$ and $n = 12$ **or** a list of terms that includes at least the 13th term and the 24th term.

$3539.89\ldots - 855.371\ldots$ **OR** $\sum_{13}^{24}\left(40 \times 1.1^{n-1}\right)$ **OR**

$\left(S_{13 \text{ to } 24} =\right)\dfrac{125.537\ldots\left(1.1^{12}-1\right)}{1.1-1}$ **OR** $125.537 + \ldots + 358.172\ldots$ ***(A1)***

Note: Accept a calculation using $u_{13} = 125$ or 126.

$2680 \quad \left(2684.52\ldots, 2684, 2685\right)$ ***A1***

Note: For $u_{13} = 125$, the sum is $2673.03\ldots$ and for $u_{13} = 126$, the sum is $2694.41\ldots$

[5 marks]

[Total: 7 marks]

7. (a) (i) $m = ah^3$ ***(M1)***

$64 = a \times 0.8^3 \ \ (\Rightarrow a = 125)$ ***(A1)***

$m = 125h^3$ ***A1***

Note: The final answer must be written as m in terms of h to award the final ***A1***.

(ii) $m = (125(0.75)^3 =)\ 52.7\ (\text{kg})\ \ (52.7343...)$ ***A1***

[4 marks]

(b) **EITHER (finding the height of the lion)**

$(220 = 125h^3)$

$h = \sqrt[3]{1.76}\ \ (1.20736...)$ ***(A1)***

$E = k_1h^2$ (seen anywhere) ***(A1)***

$k = \dfrac{\left(\sqrt[3]{1.76}\right)^2}{0.8^2}$ ***(M1)***

$= 2.28\ \ (2.27769\ldots)$ ***A1***

OR (finding a formula for E in terms of m)

$\left(m = 125h^3\right)$

$E = k_1h^2$ ***(A1)***

$E = k_2m^{\frac{2}{3}}$ ***(A1)***

$k = \dfrac{220^{\frac{2}{3}}}{64^{\frac{2}{3}}}$ ***(M1)***

$= 2.28\ \ (2.27769\ldots)$ ***A1***

[4 marks]

[Total 8 marks]

8. (a) B and C ***A1***

[1 mark]

(b) correct intervals seen ($x \leq 5$ (or $x < 5$) **AND** $x \geq 5$ (or $x > 5$)) ***A1***

Note: The case of $x = 5$ must be included for this ***A1*** to be awarded.

attempt to add edges to $33 + x$ ***(M1)***

(If $x < 5$ (or $x \leq 5$) then repeat BC and) length is $33 + 2x$ ***A1***

(If $x > 5$ (or $x \geq 5$) then repeat AB and AC and) length is $(33 + x + 5 =) 38 + x$

A1

Note: If the intervals are not explicit, award at most ***A0(M1)A1A1***.

[4 marks]

[Total 5 marks]

9. (a) attempt to integrate by substitution or inspection ***(M1)***

$4\ln|2x+3|+c$ **OR** $4\ln|x+1.5|+c$ ***A1A1***

Note: Award ***M1*** for $\ln(2x+3)$ or $\ln(x+1.5)$, ***A1*** for the 4 and ***A1*** for c. The ***A*** marks can only be awarded if the ***M*** mark is awarded. Condone absence of modulus signs.

[3 marks]

(b) recognizing that area is $\left[4\ln(2x+3)\right]_0^6$ ***(M1)***

$= 4\ln(15) - 4\ln(3)$ ***(A1)***

use of log laws for their expression ***(M1)***

$= 4\ln(5)$ $(= 2\ln(25) = 1\ln(625))$ ***A1***

Note: Award ***(M1)A0M0A0*** for an unsupported final answer of $6.43775\ldots$
Award at most ***(M1)A1FTM0A0*** if their answer from part (a) does not include ln.

[4 marks]

[Total 7 marks]

10. (recognition that OB is a radius)

(radius =) $\sqrt{5^2+8^2}$ $(=\sqrt{89})$ ***(A1)***

EITHER (finding angle BOQ)

correct calculation for finding $\mathrm{B\hat{O}A}$ ***(A1)***

$\mathrm{B\hat{O}A} = \arctan\left(\frac{8}{5}\right)$ **OR** $\tan \mathrm{B\hat{O}A} = \frac{8}{5}$

expressing $\mathrm{B\hat{O}Q}$ as $90 + \mathrm{B\hat{O}A}$ ***(M1)***

$\mathrm{B\hat{O}Q} = 90 + \arctan\left(\frac{8}{5}\right)$ **OR** $\mathrm{B\hat{O}Q} = \frac{\pi}{2} + \arctan\left(\frac{8}{5}\right)$

$\left(\mathrm{B\hat{O}Q} =\right) 147.994°\ldots$ **OR** $2.58299\ldots$

substituting *their* radius and angle BOQ correctly into arc length formula ***(M1)***

(arc BQ =) $\frac{90 + \arctan\left(\frac{8}{5}\right)}{360} \times 2\pi\left(\sqrt{5^2+8^2}\right)$ **OR** $\left(\frac{\pi}{2} + \arctan\left(\frac{8}{5}\right)\right) \times \left(\sqrt{5^2+8^2}\right)$

24.4 (m) $(24.3679\ldots)$ ***A1***

OR (finding angle BOP)

correct calculation for finding angle $\mathrm{B\hat{O}P}$ ***(A1)***

$\mathrm{B\hat{O}P} = \arctan\left(\frac{5}{8}\right)$ **OR** $\tan \mathrm{B\hat{O}P} = \frac{5}{8}$

substituting *their* radius and $\mathrm{B\hat{O}P}$ correctly into arc length formula ***(M1)***

(arc BP =) $\frac{\arctan\left(\frac{5}{8}\right)}{360} \times 2\pi\left(\sqrt{5^2+8^2}\right)$

subtracting *their* arc BP from arc PQ ***(M1)***

(arc BQ =) $\pi\sqrt{5^2+8^2} - \frac{\arctan\left(\frac{5}{8}\right)}{360} \times 2\pi\left(\sqrt{5^2+8^2}\right)$

24.4 (m) $(24.3679\ldots)$ ***A1***

[Total: 5 marks]

11. (a) $\begin{pmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{pmatrix}$ ***A1A1***

Note: Award ***A1*** for selecting the correct matrix, ***A1*** for substituting 2α into a rotation matrix
Award ***A1A0*** for clockwise rotation. These marks can be awarded independently.
Condone the use of a different symbol.

[2 marks]

(b) $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$

$= \begin{pmatrix} \cos^2\alpha - \sin^2\alpha & -2\cos\alpha\sin\alpha \\ 2\cos\alpha\sin\alpha & \cos^2\alpha - \sin^2\alpha \end{pmatrix}$ ***M1A1***

Note: Award ***M1*** for an attempt to multiply matrices, e.g. at least one entry correct.

[2 marks]

(c) (i) (Because matrix multiplication represents the composition of transformations)
two rotations of α are equivalent to a rotation of 2α ***R1***
so the two matrices are equal (so each of the entries are also equal) ***R1***
$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$ ***AG***

(ii) $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ ***A1***

replacing $\cos^2\alpha$ with $1 - \sin^2\alpha$ ***M1***
$= 1 - \sin^2\alpha - \sin^2\alpha$

$= 1 - 2\sin^2\alpha$ ***AG***

[4 marks]
[Total 8 marks]

12. (a) $40 = 100 - 100 \times 2^{-t}$ ***(A1)***

0.737 (hours) $(0.736965\ldots)$ ***A1***

Note: Accept 44.2 minutes.

[2 marks]

(b) values of P are $50, 75, 87.5, 93.75$ ***(A1)***

$SS_{res} = (50-48)^2 + (75-74)^2 + (87.5-86)^2 + (93.75-91)^2$ ***(M1)(A1)***

$= 14.8 \ (14.8125)$ ***A1***

[4 marks]

(c) (i) The sum of the square residuals is smaller so it is a better fit ***R1***

(ii) *Accept a valid argument in favour of model P or against the quadratic model.* ***R1***

e.g. any one of the following:
quadratic has no asymptote
quadratic will begin to go down
quadratic will become negative
quadratic might not go through $(0, 0)$
model P is the manufacturer's model

[2 marks]
[Total 8 marks]

13. (a) (i) $\left(|z_1| = \sqrt{4^2 + 5^2} =\right) \ 6.40 \ (6.40312..., \sqrt{41})$ ***A1***

(ii) $\left(\arg(z_1) =\right) \ 0.896 \ \left(0.896055..., 51.3401...^\circ, \arctan\left(\frac{5}{4}\right)\right)$ ***A1***

[2 marks]

(b) angle in triangle is $2 - 0.896055\ldots$ **OR** $114.591^\circ - 51.3401...^\circ$ ***(A1)***
use of area of triangle formula ***(M1)***

$\frac{1}{2} \times 6.40312... \times 3 \times \sin(2 - 0.896055...)$ ***(A1)***

$8.58 \ (8.57688\ldots)$ ***A1***

Note: Accept methods that use Cartesian form or vector product.

[4 marks]
[Total 6 marks]

14. (a) $x_B = 4\sqrt{2(t-3)} \qquad (t \geq 3)$ ***A1A1***

Note: Award ***A1*** for multiplying by 2 and ***A1*** for $t-3$. Award ***A1A0*** for $4\sqrt{2t-3}$.

[2 marks]

(b) equating their x_B to $4\sqrt{t}$ ***(M1)***

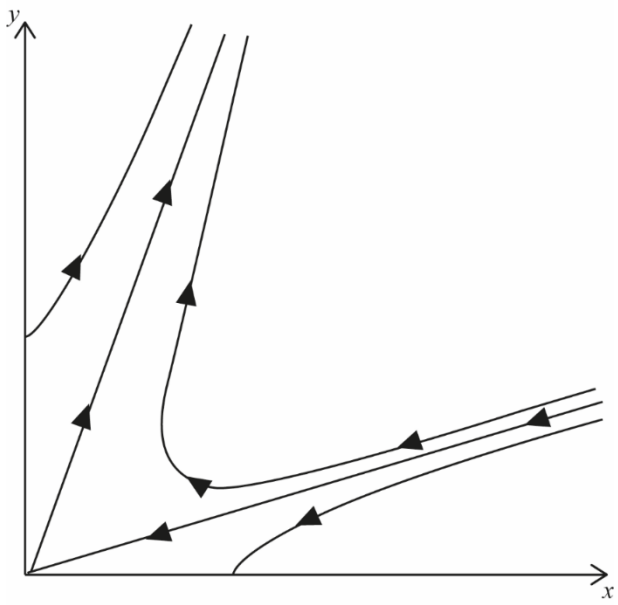
$4\sqrt{2(t-3)} = 4\sqrt{t}$

$t = 6$ (seconds) ***A1***

Note: Do not ***FT*** from part (a) to part (b).

[2 marks]
[Total 4 marks]

15. (a) (i)&(ii)



A1A1A1

Note: Award ***A1*** for correct directions on eigenvectors, ***A1*** for correct trajectories, ***A1*** for correct arrows on trajectories.

[3 marks]

(b) for Y not to die out $y > \frac{1}{3}x$ ***(R1)***

as $x = 252$, $y > 84$ ***(M1)***

(minimum number of new animals is) 25 ***A1***

Note: Award ***(R1)(M1)A0*** for an unsupported 24.

[3 marks]
[Total 6 marks]

16. attempt to find gradient ***(M1)***

EITHER

gradient of tangent $= -\tan 75^\circ \ (= -3.73205\ldots, -2-\sqrt{3})$ ***(A1)(A1)***

Note: Award ***A1*** for negative and ***A1*** for $\tan 75^\circ$ (or equivalent).

OR

gradient of tangent $= \tan 105^\circ \ (= -3.73205\ldots)$ ***(A2)***

THEN

$\frac{\mathrm{d}y}{\mathrm{d}x} = -5.5\sin(1.1x)$ ***(A1)***

Note: Award ***(A1)*** for a labelled sketch of the derivative function.

equating derivative to their gradient ***(M1)***
$-5.5\sin(1.1x) = -3.73205...$ **OR** line on graph

$x = 0.677993...$ ***(A1)***

Note: Award ***(A1)(M1)A0*** for an answer of $x = 38.8$, from calculator being in degrees.
Award ***A0M1A0*** if "$\frac{\mathrm{d}}{\mathrm{d}x}(5\cos(1.1x)) = -3.73205...$" seen, but leading to an incorrect x-value.

height = $= 5\cos(1.1\times 0.677993...)$ ***(M1)***
$= 3.67 \text{ (m)} \ (3.67274...)$ ***A1***

[Total 8 marks]

17. (a) $\begin{pmatrix} 0.25 & 0.25 \\ 0.1 & 0.1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ***(M1)***

Note: Accept equivalent methods including only using one line of the matrix.

$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (or any multiple) ***A1***

[2 marks]

(b) $D^n = \begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix}$ ***A1***

[1 mark]

(c) $\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}^{-1}$ ***(M1)***

EITHER

multiplying by the initial state ***(M1)***

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}^{-1}\begin{pmatrix} 7000 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2000 \\ 1000 \end{pmatrix}$ ***(A1)***

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2000 \times 0.65^n \\ 1000 \end{pmatrix}$ ***(A1)***

$\begin{pmatrix} 2000 \times 0.65^n + 5000 \\ -2000 \times 0.65^n + 2000 \end{pmatrix}$ ***(A1)***

Note: Award ***A0*** if either term in the matrix is incorrect.

OR

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}^{-1} = \frac{1}{7}\begin{pmatrix} 2 & -5 \\ 1 & 1 \end{pmatrix}$ ***A1***

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.65^n & 5 \\ -0.65^n & 2 \end{pmatrix}$ ***A1***

Note: The preceding ***A1*** marks can be awarded independently.

$\frac{1}{7}\begin{pmatrix} 5+2\times 0.65^n & 5-5\times 0.65^n \\ 2-2\times 0.65^n & 2+5\times 0.65^n \end{pmatrix}$ ***A1***

Note: Award ***A0*** if any term in the matrix is incorrect.

multiplying by the initial state ***(M1)***

$\frac{1}{7}\begin{pmatrix} 5+2\times 0.65^n & 5-5\times 0.65^n \\ 2-2\times 0.65^n & 2+5\times 0.65^n \end{pmatrix}\begin{pmatrix} 7000 \\ 0 \end{pmatrix}$

THEN

$2000-2000\times 0.65^n \quad \left(=2000\left(1-0.65^n\right)\right)$ ***A1***

Note: For the final ***A1***, follow through within the question part from the bottom-left entry of their 2x2 matrix or the bottom entry of their 2x1 matrix but only if "in terms of n".

If any mistake in the matrices is seen that DOES NOT affect the correct final answer, do not award the penultimate ***A1*** mark.

[6 marks]

[Total 9 marks]