2224 – 7207M

Markscheme

May 2024

Mathematics: applications and interpretation

Higher level

Paper 2

16 pages

1. (a) $\text{BC} = 20\ (\text{m})$ ***A1***

[1 mark]

(b) use of Pythagoras ***(M1)***

$\text{AB} = \sqrt{12^2 + 4^2}$

$= 12.6\ (\text{m})\quad (12.6491\ldots,\ \sqrt{160})$ ***A1***

[2 marks]

(c) **METHOD 1 – finding angle ABC**

correct use of a trig ratio to find $\text{A}\hat{\text{B}}\text{C}$ (or finding the bearing of B from A) ***(A1)***

e.g. $\tan\left(\text{A}\hat{\text{B}}\text{C}\right) = \dfrac{12}{4}$, $\cos \text{A}\hat{\text{B}}\text{C} = \dfrac{20^2 + 12.649^2 - 20^2}{2 \times 20 \times 12.649}$, $\cos \text{A}\hat{\text{B}}\text{C} = \dfrac{6.3245}{20}$

$\text{A}\hat{\text{B}}\text{C} = 71.6\ (71.5650\ldots)$ ***(A1)***

Note: Angle $\text{A}\hat{\text{B}}\text{C}$ can be 71.5 or 72.2 depending on their working out. Bearings should be given in degrees.

$180 + 71.5650\ldots = 252°\quad (251.565\ldots)$ ***A1***

Note: The final ***A1*** can be awarded for 180 plus their 71.6. If radians used, award ***A1A1*** for $1.24904\ldots$ or $4.39063\ldots$ seen, and then ***A0*** for the radian answer.

METHOD 2 – finding angle that AB makes with the horizontal (angle *H*)

correct use of a trig ratio to find H, the angle AB makes with horizontal ***(A1)***

e.g. $\tan \hat{H} = \dfrac{4}{12}$, $\cos \hat{H} = \dfrac{12^2 + 12.649^2 - 4^2}{2 \times 12 \times 12.649}$

$\hat{H} = 18.4\ (18.4349...)$ ***(A1)***

Note: Accept 18.5 $(18.5078\ldots)$ from use of 3sf answer from part (b). Bearings should be given in degrees.

$270 - 18.4348... = 252°\quad (251.565\ldots)$ ***A1***

Note: The final ***A1*** can be awarded for 270 minus their 18.4. If radians used, award ***A1A1*** for $0.321750\ldots$ or $4.39063\ldots$ seen, and then ***A0*** for the radian answer.

[3 marks]

(d) (i) $-\frac{4}{3}$ $\left(-\frac{16}{12}\right)$ ***A1***

(ii) $(6, 8)$ ***A1A1***

Note: Award ***A1A0*** if parentheses are missing.

(iii) gradient of (their) perp line = $\frac{3}{4}$ ***(M1)***

equation of perpendicular bisector of AC ***(A1)***

e.g. $(y-8)=\frac{3}{4}(x-6)$ **OR** $y=\frac{3}{4}x+3.5$

EITHER

equation of perpendicular bisector of BC is $y=10$ ***(A1)***

OR

equation of perpendicular bisector of AB is $y=-3x+36$ ***(A1)***

Note: The ***A1*** is for either equation of perpendicular bisector of BC or AB.

point of intersection $\left(8\frac{2}{3}, 10\right)$ **OR** $(8.67, 10)$ $[(8.666\ldots, 10)]$ ***(M1)A1***

Note: Award ***M1*** for an attempt to equate their perpendicular bisectors
Award the final ***A1*** for the correct coordinate pair – parentheses omitted or not.

[8 marks]
[Total: 14 marks]

2. (a) heights, 0, 4, 1.75, 3 and 3.75 seen ***(A2)***

> **Note:** Award ***A1A0*** if **two** of 1.75, 3 or 3.75 are seen.

attempt to use trapezoidal rule formula for their heights ***(M1)***

$$\frac{1}{2}\times1\times\{0+4+2(1.75+3+3.75)\}$$ ***(A1)***

> **Note:** Award ***(M1)(A1)*** for correctly expressing this as 3 trapezoids and a triangle. The "$\times1$" need not be seen.

$=10.5\ (\mathrm{m}^2)$ ***A1***

[5 marks]

(b) $-\frac{1}{12}x^3+x^2+c$ ***A1A1A1***

[3 marks]

(c) $\int_0^4\left(-\frac{1}{4}x^2+2x\right)\mathrm{d}x+1\times4+\frac{1}{2}\times7\times4$ ***(A1)(M1)(A1)***

> **Note:** Award ***A1*** for correct area of rectangle **OR** triangle, ***M1*** for substituting correct limits into given integral (may be seen in part (b)), and ***A1*** for entire expression correct.

$=10.6666\ldots + 4 + 14$

$= 28\frac{2}{3}\ (\mathrm{m}^2)\ \left(\frac{86}{3}\right)$ ***A1***

> **Note:** The answer must be **exact** for the ***A1*** to be awarded. For an answer of 28.7 or 28.66 award ***(A1)(M1)(A1)A0***.

[4 marks]

(d) (Total area using part (a) =) 28.5 ***(A1)***

Percentage error $=\left|\frac{28.5-28.6666\ldots}{28.6666\ldots}\right|\times100$ ***(M1)***

> **Note:** if their trapezoid value is incorrect but is used correctly in the percentage error formula, award at most ***A0M1A0***. If it is clear from the answer that ×100 has been used, then condone the omission and award the ***M*** mark.

$=0.581\ (\%)\ (0.581395\ldots)$ ***A1***

(accept 0.697 from use of 28.7)

[3 marks]
[Total: 15 marks]

3. (a) (i) correct approach to find missing length ***(A1)***

$\sqrt{4^2 - 1^2} \quad (= \sqrt{15})$

attempt to find cross-section ***(M1)***

e.g. use of area of trapezoid formula or rectangle+triangle or rectangle – triangle

use of volume of prism formula ***(M1)***

(their cross-section multiplied by 3)

$$3\left[\frac{1}{2}(10+11)\left(\sqrt{4^2-1^2}\right)\right]$$

$= 122\,(\text{m}^3) \quad (121.998\ldots)$ ***A1***

(ii) correct approach to find missing height ***(A1)***

$\sqrt{4^2 - 3.2^2} \quad (= 2.4)$

attempt to find volume ***(M1)***

(multiplication by 3.2 and 3 seen)

$$3\left[\frac{1}{2}\left(10+10+\sqrt{4^2-3.2^2}\right)(3.2)\right]$$

$= 108\,(\text{m}^3) \quad (107.52...)$ ***A1***

(iii) correct approach to find missing lengths ***(A1)***

$\sin\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{\pi}{3}\right)$ **OR** $\sin\left(\frac{\pi}{3}\right)$ and Pythagoras etc seen in work

$$3\left[\frac{1}{2}\left(10+10+4\cos\left(\frac{\pi}{3}\right)\right)4\sin\left(\frac{\pi}{3}\right)\right]$$

$= 114\,(\text{m}^3) \quad (114.315\ldots)$ ***A1***

[9 marks]

(b) $V = 3\left[\frac{1}{2}(10+10+4\cos(\theta))4\sin(\theta)\right]$ ***A1***

all correct intermediate working leading to given answer ***A1***

e.g. $V = 6\sin(\theta)(20+4\cos(\theta))$

$V = 24\sin(\theta)(5+\cos(\theta))$ ***AG***

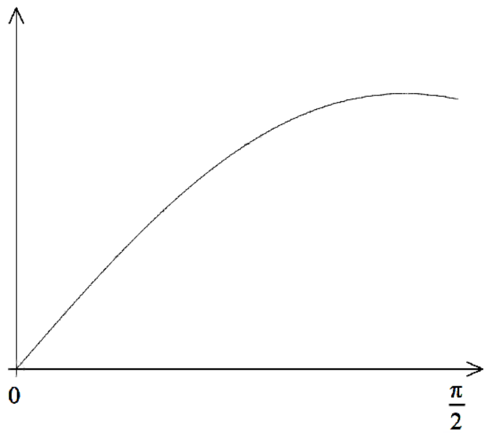
Note: The ***AG*** line must be seen for the final ***A1*** to be awarded.

[2 marks]

(c) *accept any reasoning along the lines:* “skip would have zero volume” or “if the angle is zero, then the contents would fall out” ***R1***

[1 mark]

(d) (i)



A1A1

> **Note:** Award ***A1*** for the correct shape and ***A1*** for the graph on the correct, labelled, domain. Condone omission of θ / V labels (or x/y).

(ii) $\theta = 1.38 \;\; (1.38356...) \;\; (79.3^\circ \;\; (79.2723...^\circ))$ ***A1***

$V_{\text{max}} = 122 \;\; (122.292...)$ ***A1***

> **Note:** Award ***A0A1*** if values are reversed and ***A0A0*** for a coordinate pair.

[4 marks]

(e) recognizing that derivative is equal to zero (seen at any stage) ***M1***

$\frac{dV}{d\theta} = 0$ (accept $\frac{dy}{dx} = 0$)

(from graph, turning point is a global maximum)

use of product rule ***M1***

$\left(\frac{dV}{d\theta} = \right) 24\cos(\theta)\left(5 + \cos(\theta)\right) + 24\sin(\theta)\left(-\sin(\theta)\right)$ ***A1***

$= 120\cos(\theta) + 24\cos^2(\theta) - 24\sin^2(\theta) \;\; (= 0)$ (or equivalent) ***A1***

substituting $1 - \cos^2(\theta)$ for $\sin^2(\theta)$ ***M1***

e.g $120\cos(\theta) + 24\cos^2(\theta) - 24(1 - \cos^2(\theta)) \;\; (= 0)$

correct intermediate steps leading to given answer ***A1***

$2\cos^2(\theta) + 5\cos(\theta) - 1 = 0$ ***AG***

[6 marks]
[Total: 22 marks]

4. (a) AEDCFBA ***A1A1***

Note: Award ***A1*** for AE at start, ***A1*** for correct completed route.

attempt to find the length of their route
length $22+21+19+24+25+31$ ***(M1)***
$=142$ (km) ***A1***

Note: Award ***A1A0M1A0*** for omitted final edge and their sum.

[4 marks]

(b) attempt to form MST without vertex A ***(M1)***

Note: Exactly 4 edges that form a spanning tree are required.

BD DC DE DF **OR** 20, 19, 21, 22 seen in that order ***A1***

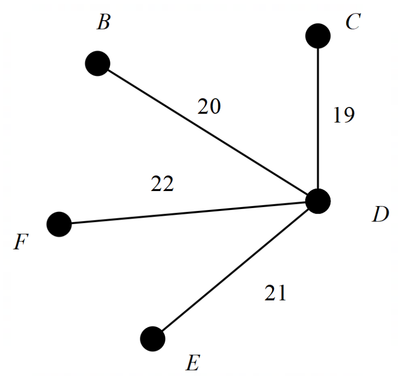
Note: Award ***M1A0*** for diagram of MST.

attempt to reconnect vertex A (one edge is sufficient) ***(M1)***
reconnecting A: AE (22) and AF (23) ***(A1)***
lower bound: $20+19+21+22+22+23$
$=127$ ***A1***

Note: If 127 seen, unsupported or without the explicit evidence of Prim's algorithm, award ***M1A0M1A1A1***.

[5 marks]

(c)



(A1)

Note: Condone the omission of the weights from their diagram. The diagram may include A with its two edges.

correct reasoning based on lack of cycle (once A is reattached) ***R1***
e.g. edges BD and CD would be repeated
this lower bound is not achievable (in this way) ***A1***

Note: Do not award ***R0A1***.

[3 marks]
[Total: 12 marks]

5. (a) $\begin{pmatrix} 0.88 & 0.08 \\ 0.12 & 0.92 \end{pmatrix}$ ***M1A1***

> **Note:** Award ***M1*** for correct values used, ***A1*** if in correct positions.
> Accept alternative consistent matrix (e.g. the transpose or diagonal elements exchanged) and follow through to eigenvectors and initial state vector.

[2 marks]

(b) 5 (seen) ***(A1)***

$\begin{pmatrix} 0.88 & 0.08 \\ 0.12 & 0.92 \end{pmatrix}^5 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.596608 \\ 0.403392 \end{pmatrix}$ **OR** $\begin{pmatrix} 0.88 & 0.08 \\ 0.12 & 0.92 \end{pmatrix}^5 = \begin{pmatrix} 0.596608 & 0.268928 \\ 0.403392 & 0.731072 \end{pmatrix}$

(M1)

$\text{P}(\text{Friday evening}) = 0.403 \ \ (0.403392)$ ***A1***

> **Note:** Award ***A0M1A0*** for use of 4 (and resulting probability 0.354).

[3 marks]

(c) attempt to find $\det(\boldsymbol{A} - \lambda \boldsymbol{I})$ ***(M1)***

$\begin{vmatrix} 0.88-\lambda & 0.08 \\ 0.12 & 0.92-\lambda \end{vmatrix}$ **OR** $(0.88-\lambda)(0.92-\lambda)-(0.12)(0.08)$

$\lambda^2 - 1.8\lambda + 0.8$ ***A1***

[2 marks]

(d) eigenvalues are 0.8 and 1 ***(A1)***

> **Note:** If no attempt is made to find eigenvectors, do not award ***A1*** for finding eigenvalues.

$\begin{pmatrix} 0.88 & 0.08 \\ 0.12 & 0.92 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0.8\begin{pmatrix} x \\ y \end{pmatrix}$

$0.88x + 0.08y = 0.8x$

eigenvector = eg. $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ***A1***

EITHER

$\begin{pmatrix} 0.88 & 0.08 \\ 0.12 & 0.92 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 1\begin{pmatrix} x \\ y \end{pmatrix}$ ***(M1)***

$0.88x + 0.08y = x$

$0.08y = 0.12x$

OR

eigenvalue 1 gives

$\begin{pmatrix} -0.12 & 0.08 \\ 0.12 & -0.08 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ***(M1)***

$-0.12x + 0.08y = 0$

$0.08y = 0.12x$

> **Note:** Award ***M1*** for an attempt to find the eigenvector with eigenvalue 1.

THEN

eigenvector = eg. $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ***A1***

Note: Award ***A0A1M0A0*** if only $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is seen and no eigenvalues are found.

[4 marks]

(e) $\boldsymbol{D} = \begin{pmatrix} 1 & 0 \\ 0 & 0.8 \end{pmatrix}$, $\boldsymbol{P} = \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}$ **OR** $\boldsymbol{D} = \begin{pmatrix} 0.8 & 0 \\ 0 & 1 \end{pmatrix}$, $\boldsymbol{P} = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ ***A1A1***

Note: Award ***A1*** for one of $\boldsymbol{P}$ or $\boldsymbol{D}$ correct. Do not award the second ***A1*** unless $\boldsymbol{P}$ and $\boldsymbol{D}$ are consistent.

[2 marks]

(f) **EITHER**

attempt to use $\boldsymbol{T}^n = (\boldsymbol{PDP}^{-1})^n = \boldsymbol{PD}^n\boldsymbol{P}^{-1}$ ***M1***

Note: Award ***M1*** for their $\boldsymbol{D}^n$ seen.

limit of $\boldsymbol{D}^n$ calculated ***A1***

$\begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}^{-1}$

Note: $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ must be seen to award ***A1***.

OR

attempt to expand their $\boldsymbol{PD}^n\boldsymbol{P}^{-1}$ using explicit $\boldsymbol{P}$, $\boldsymbol{P}^{-1}$ ***M1***

$(\boldsymbol{T}^n =)\ \frac{1}{5}\begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0.8^n \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 3 & -2 \end{pmatrix}$

$(\boldsymbol{T}^n =)\ \frac{1}{5}\begin{pmatrix} 2+3(0.8^n) & 2-2(0.8^n) \\ 3-3(0.8^n) & 3+2(0.8^n) \end{pmatrix}$ ***A1***

Note: Using this method, the limit of 0.8^n may be inferred and ***M1A1*** awarded.

THEN

0.6 ***A1***

Note: Multiplication by initial condition $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ may be seen at any point as part of their method. For an answer of 0.6 from incomplete methods award a maximum of ***M1A0A0***, or if no working is seen, award ***M0A0A1***.

[3 marks]
[Total: 16 marks]

6. (a) (i) 15 ***A1***

(ii) **EITHER**
attempt to use arithmetic series formula ***(M1)***
OR
attempt to set up simultaneous equations ***(M1)***
OR
attempt to use quadratic regression ***(M1)***
$(T_k =)\frac{1}{2}k^2 + \frac{1}{2}k$ ***A1A1***

> **Note:** Condone variable change (eg in quadratic regression).
> Accept $a = \frac{1}{2},\ b = \frac{1}{2}$.

[4 marks]

(b) (i) $(15 + 10 =)\ 25$ ***A1***

(ii) $\frac{k(k+1)}{2} + \frac{(k-1)((k-1)+1)}{2}$ **OR** $\frac{1}{2}k^2 + \frac{1}{2}k + \frac{1}{2}(k-1)^2 + \frac{1}{2}(k-1)$ ***(A1)***
$= k^2$ ***A1***

[3 marks]

(c) one correct product of probabilities seen: $\frac{15}{25} \times \frac{10}{24}$ **OR** $\frac{10}{25} \times \frac{15}{24}$ ***(A1)***
adding their products ***(M1)***
$\frac{15}{25} \times \frac{10}{24} + \frac{10}{25} \times \frac{15}{24}$
$= \frac{1}{2}$ ***A1***

[3 marks]

(d) attempt to add two products of probabilities involving k only ***M1***
(these may be incorrect or in terms of T_k)
$\frac{\frac{k}{2}(k+1)}{k^2} \times \frac{\frac{k}{2}(k-1)}{k^2-1} + \frac{\frac{k}{2}(k-1)}{k^2} \times \frac{\frac{k}{2}(k+1)}{k^2-1}$ ***A1***

further simplification consistent with given answer ***A1***
$= \frac{1}{2}$ ***A1***
hence independent of k ***AG***

[4 marks]
[Total: 14 marks]

7. (a) $\text{P}(X < 340)$ **OR** labelled sketch of region **OR** calc syntax with correct bounds ***(M1)***

$= 0.159 \quad (0.158656\ldots)$ ***A1***

[2 marks]

(b) recognizing endpoint occurs at either 0.975 or 0.025 ***(M1)***

$\text{P}(X < k) = 0.975$ **OR** $\text{P}(X < m) = 0.025$

$330 < X < 370 \quad (330.400... < X < 369.599...)$ ***A1A1***

[3 marks]

(c) (i) recognizing mean of W is sum of individual means within wall ***(M1)***

$W = C_1 + C_2 + L$ may be seen

$\text{E}(W) = 2\text{E}(C) + \text{E}(L)$

$= 800$ ***A1***

recognizing **variance** of W is sum of individual **variances** within wall ***(M1)***

$\text{Var}(W) = 2\text{Var}(C) + \text{Var}(L)$ **OR** 225 seen ***(A1)***

$(\text{SD}(W) =)\ 15$ ***A1***

Note: Award ***M1A0A0*** for an answer of 20.6 from using $\text{Var}(2C)$ in place of $2\text{Var}(C)$.

(ii) recognizing that W is modelled by a normal distribution ***(M1)***

$(\text{P}(780 < W < 810) =)\ 0.656 \quad (0.656296...)$ ***A1***

Note: The answer is 0.521 (0.520) from using $\text{SD} = 20.6 \quad (5\sqrt{17})$. Follow through from part (c)(i) without working seen.

[7 marks]

(d) $810 = 350 + 350 + \text{E}(L)$ (or equivalent) ***(A1)***

$(\text{E}(L) =)\ 110$ ***A1***

$\text{Var}(W) = 2\text{Var}(C) + \text{Var}(L)$ **OR** $256 = 2(100) + \text{Var}(L)$ ***(A1)***

$(\text{SD}(L) =)\ 7.48 \quad (7.48331..., \sqrt{56})$ ***A1***

[4 marks]

(e) $116 \quad (116.298)$ ***A1***

Note: Do not follow through from either a negative variance or a negative SD.

[1 mark]

[Total: 17 marks]