Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 2

2 May 2024

Zone A morning | **Zone B** morning | **Zone C** morning

2 hours

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

2224 – 7207

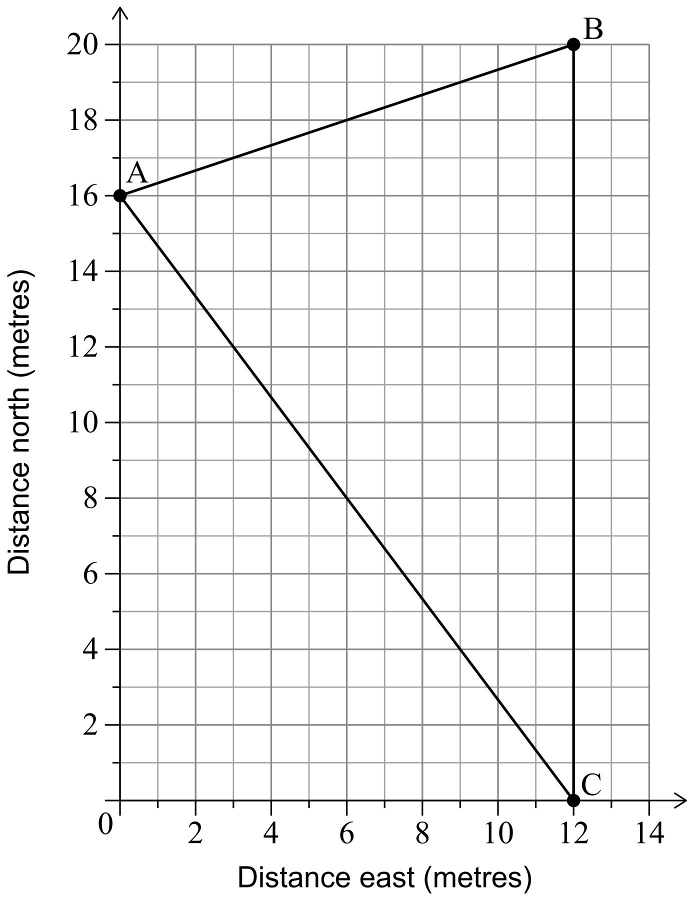
13 pages

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 14]

Mai is at an amusement park. A map of part of the amusement park is represented on the following coordinate axes.

Mai's favourite three attractions are positioned at A(0, 16), B(12, 20) and C(12, 0). All measurements are in metres.



(a) Write down the distance between B and C. [1]

(b) Calculate the distance between A and B. [2]

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(Question 1 continued)

Mai is standing at the attraction at B and wants to walk directly to the attraction at A.

(c) Calculate the bearing of A from B. [3]

A drinking fountain is to be installed at a point that is an equal distance from each of the attractions at A, B and C.

(d) (i) Write down the gradient of [AC].

(ii) Write down the mid-point of [AC].

(iii) Hence calculate the coordinates of the drinking fountain. [8]

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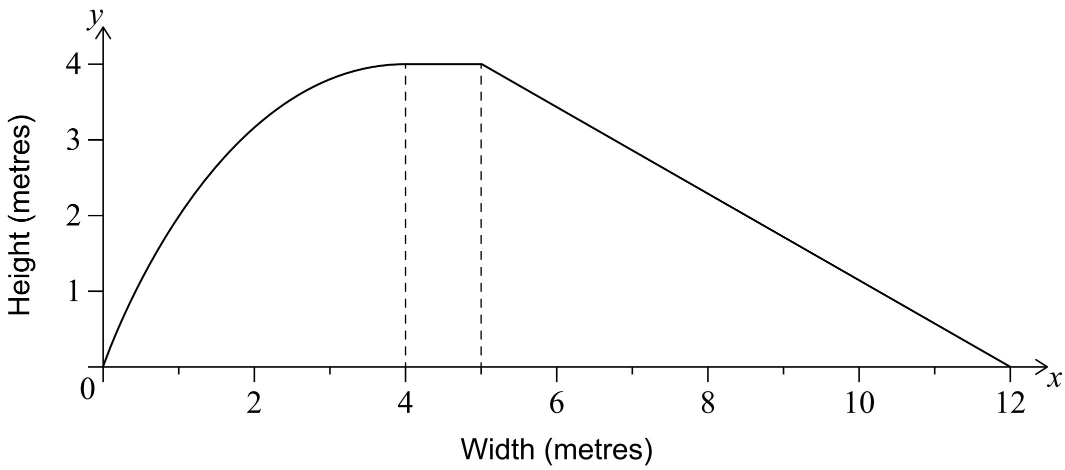
2. [Maximum mark: 15]

The following diagram shows a model of the side view of a water slide. All lengths are measured in metres.



The curved edge of the slide is modelled by

$$f(x) = -\frac{1}{4}x^2 + 2x \text{ for } 0 \le x \le 4.$$

The remainder of the slide is modelled by

$$g(x) = \begin{cases} 4, \text{ for } 4 \le x \le 5 \\ \frac{48}{7} - \frac{4x}{7}, \text{ for } 5 \le x \le 12 \end{cases}$$

(a) Use the trapezoidal rule with an interval width of 1 to calculate the approximate area under the model of the slide in the interval $0 \le x \le 4$. [5]

(b) Find $\int \left(-\frac{1}{4}x^2 + 2x\right) \mathrm{d}x$. [3]

(c) Calculate the exact area under the entire model of the slide, for $0 \le x \le 12$. [4]

(d) Find the percentage error in the **total** area under the entire model of the slide when using the approximate value from part (a). [3]

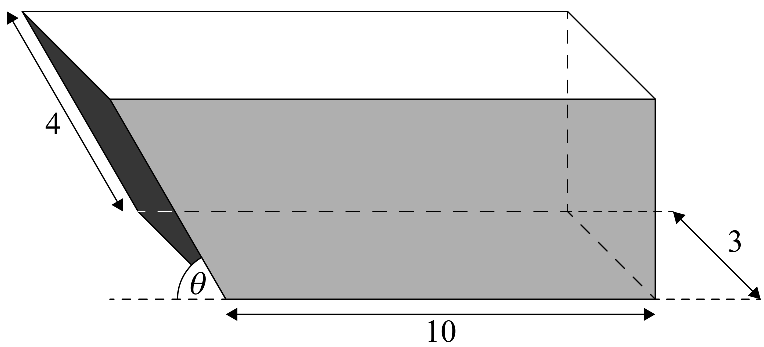
3. [Maximum mark: 22]

A skip is a container used to carry garbage away from a construction site. For safety reasons the garbage must not extend beyond the top of the skip. The maximum volume of garbage to be removed is therefore equal to the volume of the skip.

A particular design of skip can be modelled as a prism with a trapezoidal cross section. For the skip to be transported, it must have a rectangular base of length $10\,\text{m}$ and width $3\,\text{m}$. The length of the sloping edge is fixed at $4\,\text{m}$, and makes an angle of θ with the horizontal.

The following diagram shows such a skip.

diagram not to scale



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(Question 3 continued)

(a) Find the volume of this skip,

(i) if the length of the top edge of the skip is $11\,\mathrm{m}$.

(ii) if the height of the skip is $3.2\,\mathrm{m}$.

(iii) if θ is $\frac{\pi}{3}$. [9]

(b) Show that the volume, $V\,\mathrm{m}^3$, of the skip is given by

$$24\sin(\theta)(5+\cos(\theta)).$$ [2]

(c) Explain, in context, why $\theta \neq 0$. [1]

(d) (i) Sketch the graph of $V = 24\sin(\theta)\big(5+\cos(\theta)\big)$, $0<\theta<\frac{\pi}{2}$.

(ii) Find the maximum volume of the skip and the value of θ for which this maximum volume occurs. [4]

(e) Show, by differentiation, that the maximum volume occurs at a value of θ that satisfies the equation $2\cos^2\theta + 5\cos\theta - 1 = 0$. [6]

4. [Maximum mark: 12]

A hygiene inspector lives in Town A and must visit restaurants in five towns $(\mathrm{B}-\mathrm{F})$, before returning to A. The inspector must not repeat any of the towns. The distances, in km, between the six towns are shown in the table.

	A	**B**	**C**	**D**	**E**	**F**
A		31	28	26	22	23
B	31		25	20	27	25
C	28	25		19	22	24
D	26	20	19		21	22
E	22	27	22	21		24
F	23	25	24	22	24	

(a) Starting at A, use the nearest neighbour algorithm to find an upper bound for the length of the journey the inspector must take. State the order in which the towns are to be visited. [4]

(b) By deleting A, use Prim's algorithm starting at B to find a lower bound for the length of the inspector's journey. [5]

(c) By considering the minimum spanning tree found in part (b), determine whether the journey given by this lower bound is an achievable solution. [3]

5. [Maximum mark: 16]

The drivers of a delivery company can park their vans overnight either at its headquarters or at home.

Urvashi is a driver for the company. If Urvashi has parked her van overnight at headquarters on a given day, the probability that she parks her van at headquarters on the following day is 0.88. If Urvashi has parked her van overnight at her home on a given day, the probability that she parks her van at home on the following day is 0.92.

(a) Write down a transition matrix, $\boldsymbol{T}$, that shows the movement of Urvashi's van between headquarters and home. [2]

On Monday **morning** she collected her van from headquarters where it was parked overnight.

(b) Find the probability that Urvashi's van will be parked at home at the end of the week on Friday **evening**. [3]

(c) Write down the characteristic polynomial for the matrix $\boldsymbol{T}$. Give your answer in the form $\lambda^2 + b\lambda + c$. [2]

(d) Calculate eigenvectors for the matrix $\boldsymbol{T}$. [4]

(e) Write down matrices $\boldsymbol{P}$ and $\boldsymbol{D}$ such that $\boldsymbol{T} = \boldsymbol{PDP}^{-1}$, where $\boldsymbol{D}$ is a diagonal matrix. [2]

(f) Hence find the long-term probability that Urvashi's van is parked at home. [3]

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6. [Maximum mark: 14]

The kth triangle number, T_k, is defined as $T_k = \sum_{r=1}^{k} r$.

(a) (i) Calculate the value of the fifth triangle number, T_5.

(ii) Determine the formula for T_k in the form $ak^2 + bk$. [4]

(b) (i) Find the value of $T_5 + T_4$.

(ii) Find the simplest expression for $T_k + T_{k-1}$. [3]

A bag contains 15 red discs and 10 blue discs, all identical except for colour. Two discs are chosen at random from the bag without replacement.

(c) Calculate the probability that the two discs are different colours. [3]

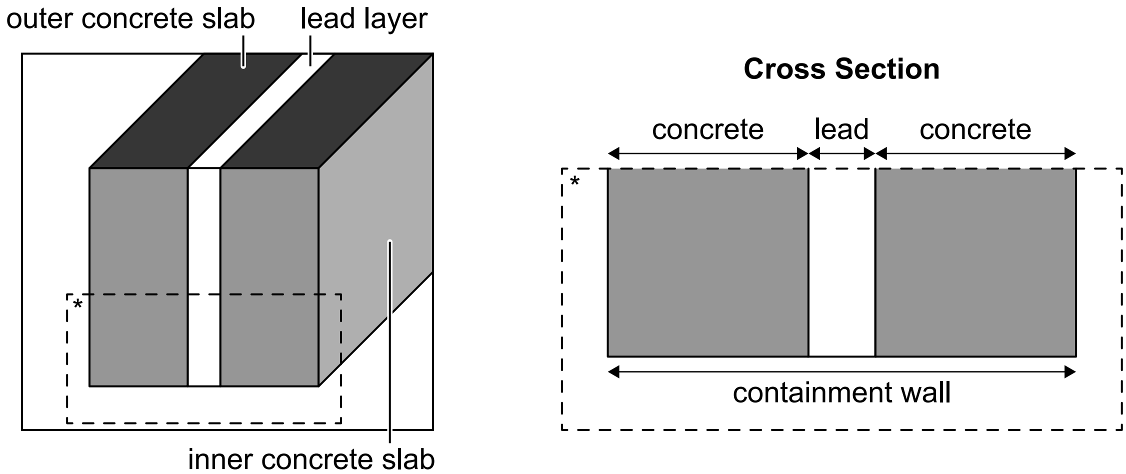
A bag contains T_k red discs and T_{k-1} blue discs, all identical except for colour. Two discs are chosen at random from the bag without replacement.

(d) Show that the probability that the two discs are different colours is independent of k. [4]

7. [Maximum mark: 17]

Containment walls to protect against radiation are constructed from two parallel concrete slabs that have a layer of lead between them as shown in the diagram.

diagram not to scale



The width of a concrete slab is modelled by a normal distribution with mean $350\,\text{mm}$ and standard deviation $10\,\text{mm}$.

(a) Find the probability that a randomly chosen concrete slab is less than $340\,\text{mm}$ in width. [2]

(b) Find the endpoints of the interval, symmetric about the mean, such that 95% of the slabs have a width that lies in this interval. [3]

Stephen assumes the lead layer is also modelled by a normal distribution, but with mean $100\,\text{mm}$ and standard deviation $5\,\text{mm}$ and is independent of the width of the slabs.

Let W be the random variable that represents the total width of the wall, measured in mm.

(c) (i) Given that the widths of any two concrete slabs are independent, calculate Stephen's value for the mean and standard deviation of W.

(ii) Hence find $\mathrm{P}(780 < W < 810)$. [7]

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(Question 7 continued)

There are concerns that the mean and standard deviation for Stephen's model of the lead layer are incorrect. However, his assumption that the model is normal and the width of the lead is independent of the width of the concrete slabs still holds.

On investigation it is found that the total width of the containment wall is normally distributed with mean $810\,\mathrm{mm}$ and standard deviation $16\,\mathrm{mm}$. The model for the width of a concrete slab does not change.

(d) Use the results for the **sum** of independent random variables to find a revised value for

(i) the mean of the width of the lead layer.

(ii) the standard deviation of the width of the lead layer. [4]

Under this revised model, 80% of the lead layers have a width less than $k\,\mathrm{mm}$.

(e) Calculate the value of k. [1]

References:

3. Andyqwe, n.d. *Dumpster truck* [image online] Available at: https://www.gettyimages.co.uk/detail/photo/dumpster-truck-royalty-free-image/157611454 [Accessed 18 April 2023] Source adapted.