2224 – 7203M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2024

Mathematics:
Applications and interpretation

Higher level

Paper 3

14 pages

1. (a) (i) $141 - 88$ ***A1***
$= 53$ ***AG***
[1 mark]

(ii) $(120 - 53 - 54 =)\ 13$ ***A1***
[1 mark]

(b) Restricting the size of the sample space to 150 $(54 + 96\ )$ ***(M1)***
$= 0.36\ \left(\frac{54}{150}, \frac{9}{25}\right)$ ***A1***
[2 marks]

(c) (i) H_0: The development of cracks and the technique used are independent
H_1: The development of cracks and the technique used are not independent
A1

Note: Condone equivalent statements such as 'not dependent' but do not accept "uncorrelated" or "not related" in place of "independent".

[1 mark]

(ii) (p-value$=$) $0.0170\ \ (0.0169864\ldots)$ ***(M1)A1***
[2 marks]

(iii) $0.0170 < 0.05$ ***R1***

hence there is sufficient evidence to reject the null hypothesis that the development of cracks and the technique used are independent. ***A1***

Note: Do not award ***R0A1***.

[2 marks]

(d) $\frac{96 + 56}{240}\ \left(= \frac{152}{240}\right)$ ***A1***
$\frac{19}{30}$ ***AG***
[1 mark]

continued…

Question 1 continued.

(e) **EITHER**

the probability of each component developing cracks is independent of all the other components in the sample. ***R1***

Note: Do not accept the word "independence" on its own. Appropriate context must be seen.

OR

the development of cracks can be partitioned into two clear groups ***R1***

Note: Do not accept 'the samples are representative of the population'.

[1 mark]

(f) 67 seen ***(A1)***

EITHER

attempt to find a probability ≤ 67 (condone strict inequality for ***(M1)***) ***(M1)***

$(\mathrm{P}(X \leq 67) =)\ 0.0549\ \ (0.0549093\ldots)$ ***A1***

Note: Award ***(A1)(M1)A0*** for an unsupported $p = 0.0372$, from use of strict inequality.

$0.0549 > 0.05$ ***R1***

OR

attempt to find the critical region ***(M1)***

critical region is $X \leq 66$ ***A1***

$66 < 67$ or '67 is not in the critical region' ***R1***

THEN

EITHER
do not reject the null hypothesis (as there is insufficient evidence that the new technique reduces the number of cracks). ***A1***

OR
do not accept the alternative hypothesis (as there is insufficient evidence that the new technique reduces the number of cracks). ***A1***

continued…

Question 1 continued.

Note: Do not award ***R0A1***.
Only follow through within part (f) for final ***R1A1*** if the ***(M1)*** has been awarded.
Do not condone "accept the null hypothesis".

[5 marks]

(g) (i) the test for a proportion is directional and so considers whether the new treatment reduces the number of components developing cracks. ***R1***

[1 mark]

(ii) **EITHER**

there could be variation in the value of p chosen for the null hypothesis / the value of p from the sample might not be a representative of the current technique ***R1***

OR
the test in (f) does not treat minor and major cracks as different attributes / the test in (c) does treat minor and major cracks as different attributes ***R1***

OR
the test in (f) has to make an additional assumption (for example 'independence') ***R1***

[1 mark]

continued…

Question 1 continued.

(h) **EITHER**

let μ_1 be the mean length of time before cracks appear with the new technique and μ_2 be the mean length with the current technique

H_0: $\mu_1 = \mu_2$ ***A1***

H_1: $\mu_1 > \mu_2$ ***A1***

Note: Award ***A1A0*** for correct hypotheses in which the two population means are not clearly defined (e.g. unsupported μ_1 and μ_2).

OR

H_0: the POPULATION mean length of time before cracks appear is the same for both groups ***A1***

H_1:the new technique increases the POPULATION mean length of time before cracks appear. ***A1***

OR

H_0: the mean length of time before cracks appear in ALL components made with the new technique is the same as for ALL components made with the current technique. ***A1***

H_1: the mean length of time before cracks appear in ALL components made with the new technique is greater than the mean for ALL components made with the current technique. ***A1***

Note: Award ***A1A0*** if "population" (or equivalent, such as "all") is omitted from an otherwise correct answer.

THEN

recognition of the need to use of a two-sample test ***(M1)***

p-value $= 0.0162$ $(0.0162328\ldots)$ ***A2***

Note: If not pooled, answer is $0.0164368\ldots$ award ***(M1)A2***.

$0.0162 < 0.05$ ***R1***

reject the null hypothesis (**OR** accept the alternative hypothesis) ***A1***
(there is sufficient evidence to that the new technique increases the mean length of time before the cracks appear)

Note: Do not award ***R0A1***.

Follow through within part (h) for the last ***R1A1***, provided their p-value is between 0 and 1 inclusive.

[7 marks]

continued…

Question 1 continued.

(i) **EITHER**

(though statistically significant) the new technique only seems to increase the time before cracks appear by 1 hour out of 250, so it is not a significant increase
(i.e. the effect size is small) ***R1***

OR

the minimum time (not mean time) before cracks appear should be considered given the context / An appropriate confidence interval should be considered, and not simply the mean. ***R1***

Note: If a not significant p-value was seen in part (h), do not award ***R1*** for an answer of "the result is not significant" in part (i).

[1 mark]

[Total 26 marks]

2 (a) (i) $(34,\ 4.1)$ ***A1A1***

[2 marks]

(ii) recognizing the sequence is arithmetic, with a common difference of 0.5 and 1.0 ***(M1)***

$\left(30+(n-1),\ 2.1+(n-1)0.5\right)$ ***A1A1***

$\left(=\left(29+n,\ 1.6+0.5n\right)\right)$

[3 marks]

(b) (i) evidence of integration of the acceleration vector **OR** use of $\boldsymbol{v}=\boldsymbol{u}+\boldsymbol{a}t$ ***(M1)***

$\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix}=\begin{pmatrix}c_1\\ -9.8t+c_2\end{pmatrix}$

$\left(\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix}=\right)\begin{pmatrix}29.4\cos\theta\\ 29.4\sin\theta-9.8t\end{pmatrix}$ ***A1A1***

Note: The first ***A1*** is for $\dot{x}$ and the second is for $\dot{y}$.

[3 marks]

(ii) $\theta=90^\circ$ ***(A1)***

$29.4-9.8t=0$ ***M1A1***

Note: Award ***M1*** for setting their $\dot{y}$ to zero (may still include θ), ***A1*** for correct equation, leading to given result.
If they substitute $t=3$ award at most ***(A1)M0A0.***

maximum point when $t=\dfrac{29.4}{9.8}$

$=3$ (seconds) ***AG***

[3 marks]

(c) correct substitution OR use of correct graph ***(M1)***

maximum height is $29.4\times3-4.9\times3^2$

$=44.1$ (m) ***A1***

[2 marks]

continued…

Question 2 continued.

(d) (i) $29.4\sin\theta t - 4.9t^2 = 0$ ***M1***

$t = 6\sin\theta$ (or $t = 0$) ***A1***

$x = 29.4\cos\theta \times 6\sin\theta$ ***A1***

$= 176.4\cos\theta\sin\theta$ ***AG***

[3 marks]

(ii) valid method to find maximum (e.g. sketch graph, find derivative) ***(M1)***

maximum value of x is 88.2 (m) ***A1***

Note: Award ***(M1)A0*** for an unsupported answer of "45" or $\frac{\pi}{4}$ $(0.785398...)$.

[2 marks]

(e) (i) $(c =)$ 44.1 ***A1***

Note: follow through from 2c

[1 mark]

(ii) **EITHER**

$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow 2a \times 0 + b = 0$ ***(M1)***

OR

vertex is at $x = -\frac{b}{2a} = 0$ ***(M1)***

THEN

$\Rightarrow b = 0$ ***A1***

[2 marks]

(iii) point $(88.2, 0)$ used ***(A1)***

$0 = a \times 88.2^2 + 44.1$ ***(M1)***

$\Rightarrow a = -\frac{44.1}{88.2^2}$

$= -\frac{5}{882} = -0.0056689...$ ***A1***

[3 marks]

Note: The method for finding the parameters might appear in a different part, for example use of $(88.2, 0)$ might appear in part (ii), or the parts might not be clearly numbered. Accept any correct working wherever it is seen, and ascribe the marks to the correct part.

continued…

Question 2 continued.

(f) use of their arithmetic sequence from (a)(ii) with $n = 40$ ***(M1)***
coordinates of seat $A_{40} = (69, 21.6)$ ***(A1)***

EITHER

substitution of their 69 into their $y = -\frac{5}{882}x^2 + 44.1$ from part (e) ***(M1)***

$y = -\frac{5}{882} \times 69^2 + 44.1$

$= 17.1\ (17.1102...)$ ***A1***

$21.6 > 17.1$ ***R1***

OR

substitution of their $y=21.6$ into their $y = -\frac{5}{882}x^2 + 44.1$ from part (e) ***(M1)***

$21.6 = -\frac{5}{882}x^2 + 44.1$

$x = 63$ ***A1***

$69 > 63$ ***R1***

Note: Accept equivalent justification in words, provided both values are seen.
Award ***R1*** for correct reasoning only if ***M1*** has been awarded and their seat coordinates lie outside their equation for the curve.

so the T-shirt cannot reach seat A_{40} ***AG***

[5 marks]

[Total 29 marks]