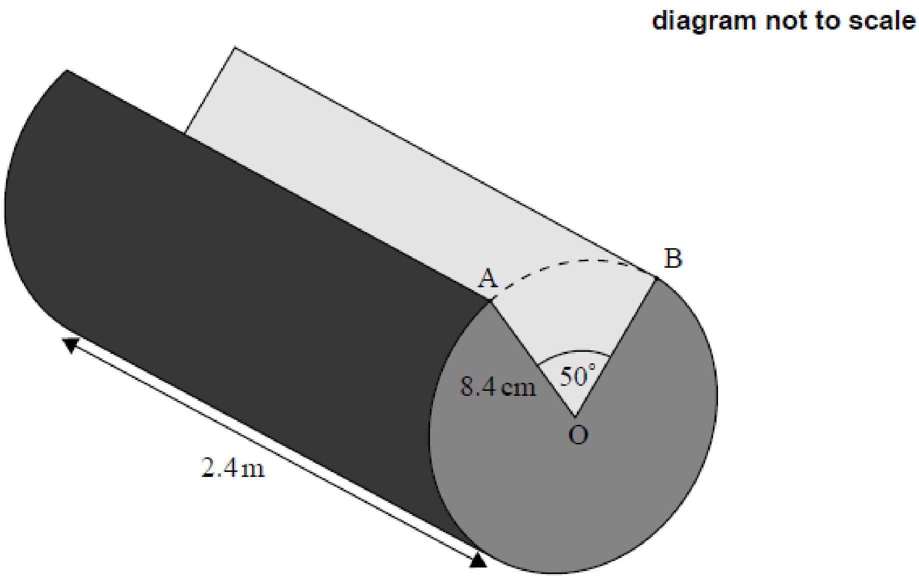
Review Set 1 [91 marks]

1. [Maximum mark: 4] SPM.1.SL.TZ0.11

Helen is building a cabin using cylindrical logs of length 2.4 m and radius 8.4 cm. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram.



Find the volume of this log. [4]

Markscheme

volume $= 240\left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664\ldots\right)$ ***M1M1M1***

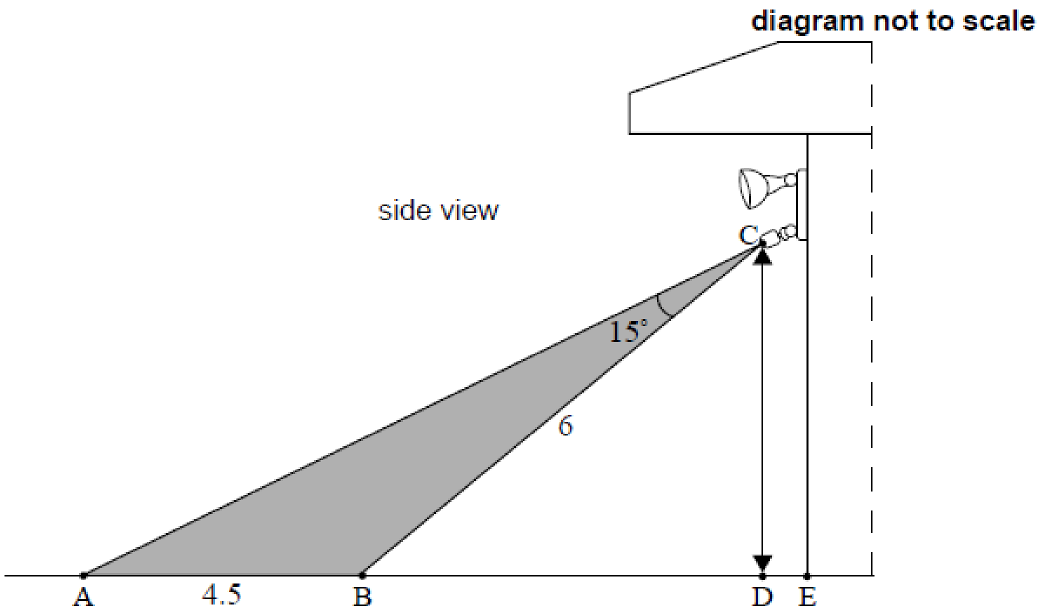
Note: Award ***M1*** 240 × area, award ***M1*** for correctly substituting area sector formula, award ***M1*** for subtraction of their area of the sector from area of circle.

= 45800 (= 45811.96071) ***A1***

[4 marks]

2. [Maximum mark: 8] SPM.1.SL.TZ0.14

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is 4.5 m and the distance from B to C is 6 m. Angle AĈB is 15°.



(a) Find CÂB. [3]

Markscheme

$$\frac{\sin \hat{CAB}}{6} = \frac{\sin 15^\circ}{4.5}$$ ***(M1)(A1)***

CÂB = 20.2° (20.187415…) ***A1***

Note: Award ***(M1)*** for substituted sine rule formula and award ***(A1)*** for correct substitutions.

[3 marks]

(b) Point B on the ground is 5 m from point E at the entrance to Ollie’s house. He is 1.8 m tall and is standing at point D, below the sensor. He walks towards point B.

Find the distance Ollie is **from the entrance to his house** when he first activates the sensor. [5]

Markscheme

$\hat{CBD} = 20.2 + 15 = 35.2°$ ***A1***

(let X be the point on BD where Ollie activates the sensor)

$\tan 35.18741\ldots° = \frac{1.8}{BX}$ ***(M1)***

Note: Award ***A1*** for their correct angle $\hat{CBD}$. Award ***M1*** for correctly substituted trigonometric formula.

$BX = 2.55285\ldots$ ***A1***

$5 - 2.55285\ldots$ ***(M1)***

= 2.45 (m) (2.44714…) ***A1***

[5 marks]

3. [Maximum mark: 12] SPM.2.SL.TZ0.5

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed, $s \text{ m s}^{-1}$, and braking distance, d m, of a truck were recorded. This information is summarized in the following table.

Speed, s m s^{-1}	0	6	10
Braking distance, d m	0	12	60

This information was used to create Model A, where d is a function of s, $s \geq 0$.

Model A: $d\left(s\right)=ps^2+qs$, where $p,q\in\mathbb{Z}$

At a speed of 6 m s^{-1}, Model A can be represented by the equation $6p+q=2$.

(a.i) Write down a second equation to represent Model A, when the speed is 10 m s^{-1}. [2]

Markscheme

$p(10)^2+q\left(10\right)=60$ ***M1***

$10p+q=6\left(100p+10q=60\right)$ ***A1***

[2 marks]

(a.ii) Find the values of p and q. [2]

Markscheme

$p=1,q=-4$ ***A1A1***

Note: If p and q are both incorrect then award ***M1A0*** for an attempt to solve simultaneous equations.

[2 marks]

(a.iii) Find the coordinates of the vertex of the graph of $y=d\left(s\right)$. [2]

Markscheme

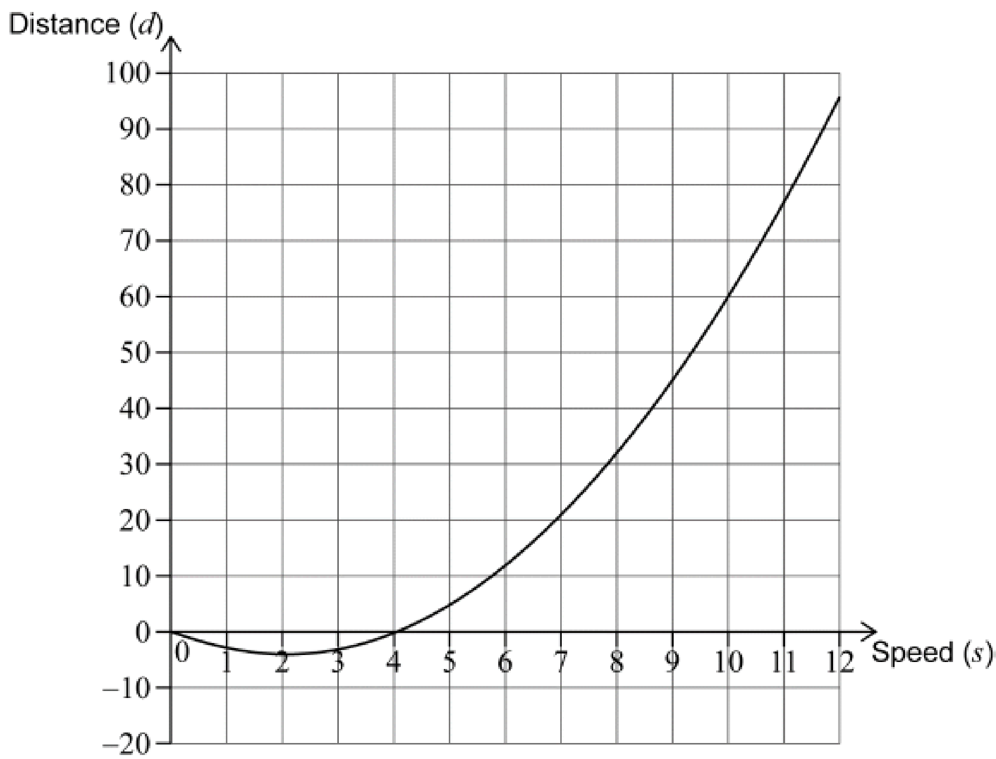
(2, −4) ***A1A1***

Note: Award ***A1*** for each correct coordinate.
Award ***A0A1*** if parentheses are missing.

[2 marks]

(a.iiii) Using the values in the table and your answer to part (b), sketch the graph of $y = d\left(s\right)$ for $0 \le s \le 10$ and $-10 \le d \le 60$, clearly showing the vertex. [3]

Markscheme



A3

Note: Award ***A1*** for smooth quadratic curve on labelled axes and within correct window.
Award ***A1*** for the curve passing through (0, 0) and (10, 60). Award ***A1*** for the curve passing through their vertex. Follow through from part (b).

[3 marks]

(a.iiiii) Hence, identify why Model A may not be appropriate at lower speeds. [1]

Markscheme

the graph indicates there are negative stopping distances (for low speeds) ***R1***

Note: Award ***R1*** for identifying that a feature of their graph results in negative stopping distances (vertex, range of stopping distances…).

[1 mark]

Additional data was used to create Model B, **a revised model** for the braking distance of a truck.

Model B: $d\left(s\right) = 0.95s^2 - 3.92s$

(a.iiiii) Use Model B to calculate an estimate for the braking distance at a speed of $20\ \mathrm{m\ s}^{-1}$. [2]

Markscheme

$0.95 \times 20^2 - 3.92 \times 20$ ***(M1)***

$= 302\ (\mathrm{m})\ \ (301.6\ldots)$ ***A1***

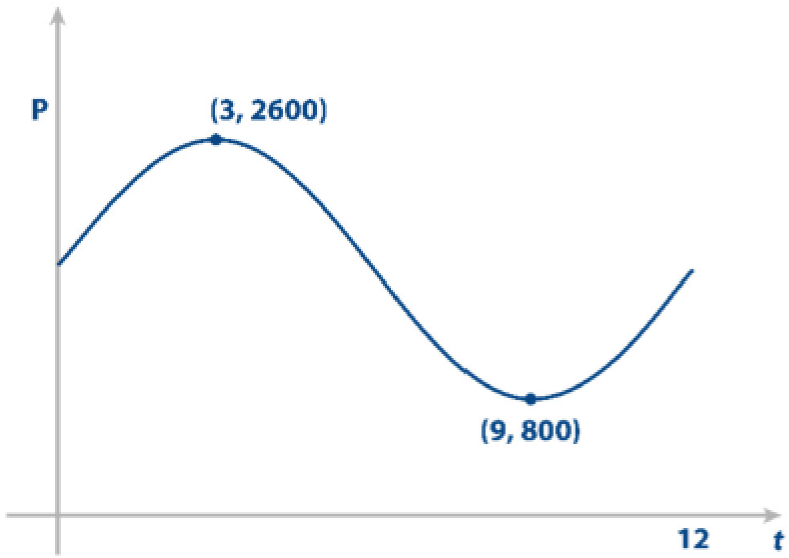
[2 marks]

4. [Maximum mark: 7] EXN.1.SL.TZ0.6

The size of the population (P) of migrating birds in a particular town can be approximately modelled by the equation $P = a\sin(bt) + c,\ \ a,\ b,\ c \in \mathbb{R}^+$, where t is measured in months from the time of the initial measurements.

In a 12 month period the maximum population is 2600 and occurs when $t = 3$ and the minimum population is 800 and occurs when $t = 9$.

This information is shown on the graph below.



(a.i) Find the value of a.

[2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

$\frac{2600-800}{2} = 900$ **(M1)A1**

[2 marks]

(a.ii) Find the value of b.

[2]

Markscheme

$\frac{360}{12} = 30$ **(M1)A1**

Note: Accept $\frac{2\pi}{12} = 0.524\ (0.523598\ldots)$.

[2 marks]

(a.iii) Find the value of c. [1]

Markscheme

$\frac{2600+800}{2} = 1700$ **A1**

[1 mark]

(b) Find the value of t at which the population first reaches 2200. [2]

Markscheme

Solve $900\ \sin(30t) + 1700 = 2200$ **(M1)**

$t = 1.12\ (1.12496\ldots)$ **A1**

[2 marks]

5. [Maximum mark: 9] EXN.1.SL.TZ0.11

A farmer owns a triangular field ABC. The length of side $[\text{AB}]$ is 85 m and side $[\text{AC}]$ is 110 m. The angle between these two sides is 55°.

(a) Find the area of the field. [3]

Markscheme

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Area $= \frac{1}{2} \times 110 \times 85 \times \sin 55^\circ$ **(M1)(A1)**

$= 3830\ (3829.53\ldots)\ \text{m}^2$ **A1**

Note: units must be given for the final **A1** to be awarded.

[3 marks]

(b) The farmer would like to divide the field into two equal parts by constructing a straight fence from A to a point D on $[\text{BC}]$.

Find BD. Fully justify any assumptions you make. [6]

Markscheme

$\text{BC}^2 = 110^2 + 85^2 - 2 \times 110 \times 85 \times \cos 55^\circ$ **(M1)A1**

$\text{BC} = 92.7\ (92.7314\ldots)\ (\text{m})$ **A1**

METHOD 1

Because the height and area of each triangle are equal they must have the same length base **R1**

D must be placed half-way along BC **A1**

$\mathrm{BD} = \frac{92.731\ldots}{2} \approx 46.4\ (\mathrm{m})$ **A1**

Note: the final two marks are dependent on the **R1** being awarded.

METHOD 2

Let $\mathrm{C\widehat{B}A} = \theta^\circ$

$\frac{\sin \theta}{110} = \frac{\sin 55^\circ}{92.731\ldots}$ **M1**

$\Rightarrow \theta = 76.3^\circ\ (76.3354\ldots)$

Use of area formula

$\frac{1}{2} \times 85 \times \mathrm{BD} \times \sin(76.33\ldots^\circ) = \frac{3829.53\ldots}{2}$ **A1**

$\mathrm{BD} = 46.4\ (46.365\ldots)\ (\mathrm{m})$ **A1**

[6 marks]

6. [Maximum mark: 13] EXM.2.SL.TZ0.3

Urvashi wants to model the height of a moving object. She collects the following data showing the height, h metres, of the object at time t seconds.

t (seconds)	2	5	7
h (metres)	34	38	24

She believes the height can be modeled by a quadratic function, $h(t) = at^2 + bt + c$, where $a,\ b,\ c \in \mathbb{R}$.

(a) Show that $4a + 2b + c = 34$. [1]

Markscheme

$t = 2,\ h = 34 \Rightarrow\ 34 = a2^2 + 2b + c$ ***M1***

$\Rightarrow\ 34 = 4a + 2b + c$ ***AG***

[1 mark]

(b) Write down two more equations for a, b and c. [3]

Markscheme

attempt to substitute either (5, 38) or (7, 24) ***M1***

$25a + 5b + c = 38$ ***A1***

$49a + 7b + c = 24$ ***A1***

[3 marks]

(c) Solve this system of three equations to find the value of a, b and c. [4]

Markscheme

$a = -\frac{5}{3},\ b = 13,\ c = \frac{44}{3}$ ***M1A1A1A1***

[3 marks]

Hence find

(d.i) when the height of the object is zero. [3]

Markscheme

$-\frac{5}{3}t^2 + 13t + \frac{44}{3} = 0$ ***M1***

$t = 8.8$ seconds ***M1A1***

[3 marks]

(d.ii) the maximum height of the object. [2]

Markscheme

attempt to find maximum height, e.g. sketch of graph ***M1***

$h = 40.0$ metres ***A1***

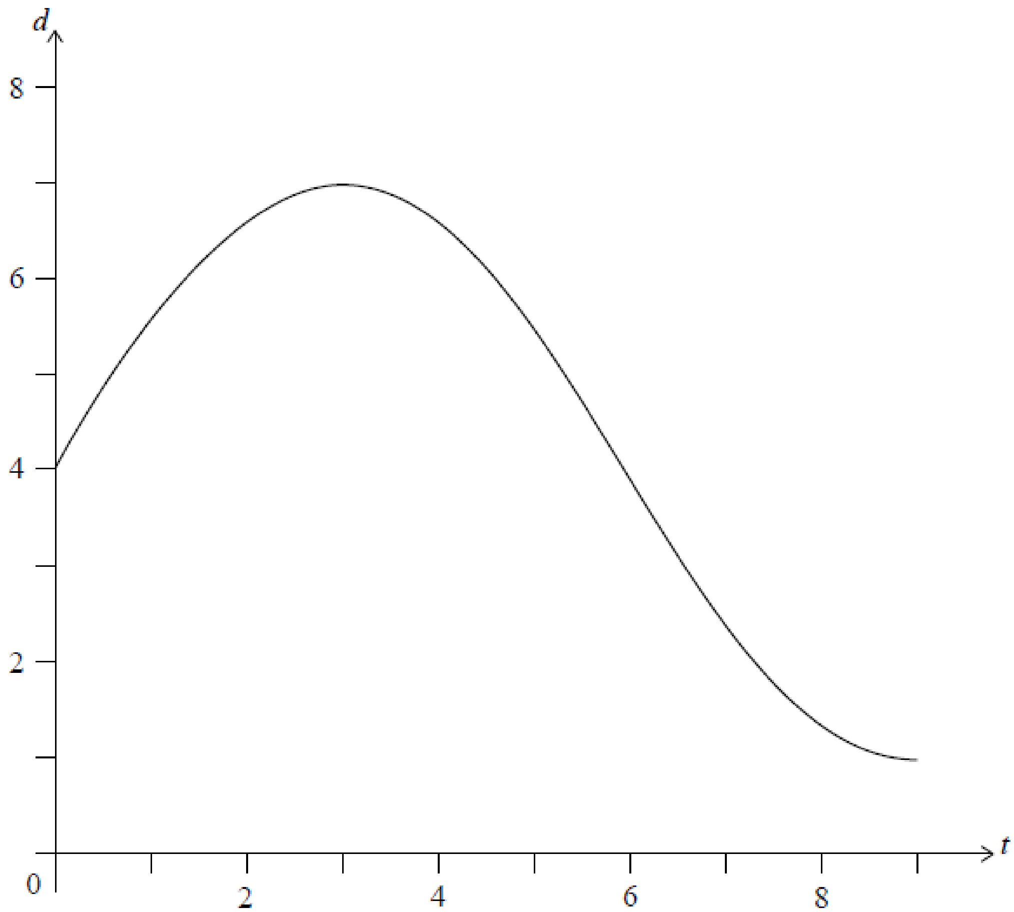
[2 marks]

7. [Maximum mark: 6] 24M.1.SL.TZ1.7

The following graph shows the depth of water, d metres, in a river at t hours after $12:00$.

At $15:00$, the depth of water reaches 7 m, its highest level. At $21:00$, the depth of water drops to 1 m, its lowest level.

The depth can be modelled by the function $d(t) = a\sin(bt) + 4$.



(a) Find the value of a. [1]

Markscheme

$a = 3$ ***A1***

[1 mark]

(b) Find the value of b. [2]

Markscheme

period$= 12$ ***(A1)***

$\left(\frac{360}{b} = 12 \quad \textbf{OR} \quad \frac{2\pi}{b} = 12\right)$

$b = 30$ $\quad b = \frac{\pi}{6}$ ***A1***

[2 marks]

(c) Find the first time after $12:00$ when the depth is equal to $3\,\mathrm{m}$. Give your answer to the nearest minute. [3]

Markscheme

equating their expression to 3 ***(M1)***

$3 = 3\sin(30t) + 4$ **OR** $3 = 3\sin\left(\frac{\pi}{6}t\right) + 4$

$t = 6.64904\ldots$ ***(A1)***

$6.39\ (\mathrm{pm})\ (18:39)$ ***A1***

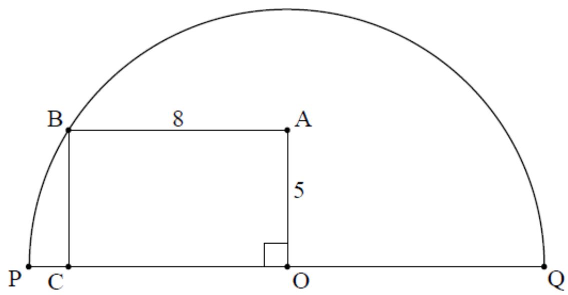
Note: Follow through within the part for the final ***A1***; this mark is awarded for expressing **their** intermediate answer (seen) as a time correct to the nearest minute.

[3 marks]

8. [Maximum mark: 5] 24M.1.SL.TZ1.11

The following diagram shows a semicircle with centre O and diameter PQ. A rectangle OABC is also shown, such that $\mathrm{AB} = 8$ and $\mathrm{OA} = 5$.

diagram not to scale



[5]

Find the length of the arc BQ.

Markscheme

(recognition that OB is a radius)

(radius=) $\sqrt{5^2+8^2}\left(=\sqrt{89}\right)$ ***(A1)***

EITHER (finding angle BOQ)

correct calculation for finding $\mathrm{B\hat{O}A}$ ***(A1)***

$\mathrm{B\hat{O}A}=\arctan\left(\frac{8}{5}\right)$ **OR** $\tan \mathrm{B\hat{O}A}=\frac{8}{5}$

expressing $\mathrm{B\hat{O}Q}$ as $90+\mathrm{B\hat{O}A}$ ***(M1)***

$\mathrm{B\hat{O}Q}=90+\arctan\left(\frac{8}{5}\right)$ **OR** $\mathrm{B\hat{O}Q}=\frac{\pi}{2}+\arctan\left(\frac{8}{5}\right)$

$\left(\mathrm{B\hat{O}Q}=\right)147.994^{\circ}\ldots$ **OR** $2.58299\ldots$

substituting *their* radius and angle $\mathrm{B\hat{O}Q}$ correctly into arc length formula ***(M1)***

$(\text{arc BQ}=)\ \frac{90+\arctan\left(\frac{8}{5}\right)}{360}\times 2\pi\left(\sqrt{5^2+8^2}\right)$ **OR** $\left(\frac{\pi}{2}+\arctan\left(\frac{8}{5}\right)\right)\times\left(\sqrt{5^2+8^2}\right)$

$24.4(\mathrm{m})\ \ (24.3679\ldots)$ ***A1***

OR (finding angle BOP)

correct calculation for finding angle $\mathrm{B\hat{O}P}$ ***(A1)***

$\mathrm{B\hat{O}P} = \arctan\left(\frac{5}{8}\right)$ **OR** $\tan \mathrm{B\hat{O}P} = \frac{5}{8}$

substituting *their* radius and $\mathrm{B\hat{O}P}$ correctly into arc length formula ***(M1)***

$(\text{arc BP} =)\ \frac{\arctan\left(\frac{5}{8}\right)}{360} \times 2\pi\left(\sqrt{5^2+8^2}\right)$

subtracting *their* arc BP from arc PQ ***(M1)***

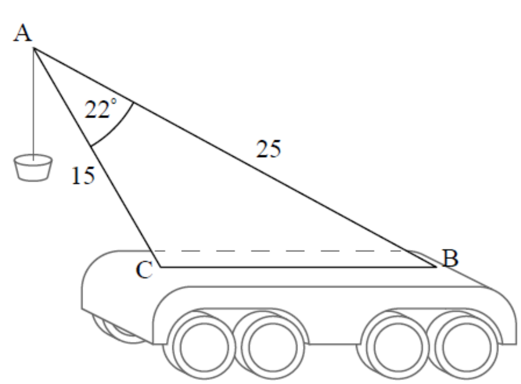
$(\text{arc BQ} =)\ \pi\sqrt{5^2+8^2} - \frac{\arctan\left(\frac{5}{8}\right)}{360} \times 2\pi\left(\sqrt{5^2+8^2}\right)$

$24.4(\mathrm{m})\ \ (24.3679\ldots)$ ***A1***

[5 marks]

9. [Maximum mark: 6] 24M.1.SL.TZ2.2

The diagram shows a toy crane.



$AB = 25$ cm, $AC = 15$ cm and $B\hat{A}C = 22°$.

(a) Calculate BC. [3]

Markscheme

attempt to substitute into cosine rule formula ***(M1)***

$(BC^2 =)\ 15^2 + 25^2 - 2 \times 15 \times 25 \times \cos(22)$ ***(A1)***

$(BC =)\ 12.4\,(\text{cm}) \quad (12.4343\ldots)$ ***A1***

[3 marks]

(b) Given that $A\hat{B}C$ is acute, calculate $A\hat{B}C$ [3]

Markscheme

selecting sine rule formula **OR** cosine rule formula ***(M1)***

$\frac{12.4343\ldots}{\sin 22} = \frac{15}{\sin A\hat{B}C}$ **OR** $\left(\cos A\hat{B}C =\right) \frac{25^2 + 12.4343\ldots^2 - 15^2}{2 \times 25 \times 12.4343\ldots}$

(A1)

Note: Award ***M1A1*** for correct cosine rule formula to find $A\hat{B}C$.

$\left(A\hat{B}C =\right)\ 26.9° \quad (26.8658\ldots°)$ ***A1***

Note: Accept $26.9461\ldots$ from use of 12.4 in the sine rule formula and $26.7267\ldots$ in the cosine rule formula.

[3 marks]

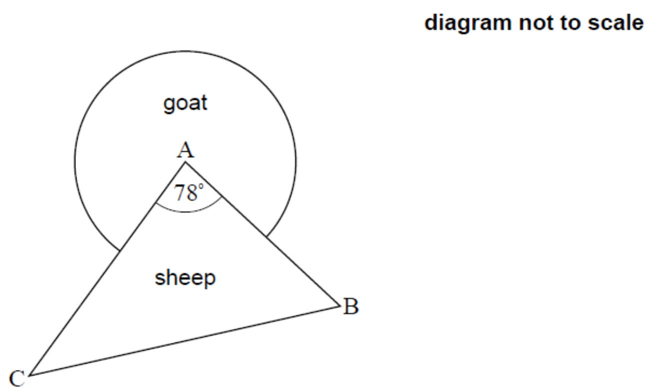
10. [Maximum mark: 6] 24M.1.SL.TZ2.11

A sheep is in a field in the shape of a triangle, ABC.

$\mathrm{AC} = 21$ metres, $\mathrm{AB} = 15$ metres and $\mathrm{C\hat{A}B} = 78^{\circ}$.

A goat is in an adjacent field in the shape of a sector of a circle with centre, A, and radius 8 metres.

The fields are shown in the diagram.



Determine which animal, the sheep or the goat, is in the field with the larger area, and state how many extra square metres are in this larger field. [6]

Markscheme

attempt to substitute into area of triangle formula ***(M1)***

(sheep's field area =) $0.5 \times 15 \times 21 \times \sin\left(78^{\circ}\right)$

$= 54.058\ldots \left(\mathrm{m}^2\right)$ ***A1***

EITHER

(goat's field area =) $\frac{282}{360} \times \pi \times 8^2$ ***(A1)(A1)***

Note: Award ***A1*** for 282, ***A1*** for correct entries in formula (including their 282).

OR

$\pi \times 8^2 - \frac{78}{360} \times \pi \times 8^2$ ***(M1)(A1)***

Note: Award ***A1*** for minor sector area, ***M1*** for subtracting their sector area from circle area.

THEN

$= 157.498\ldots \left(\frac{752\pi}{15}\right) (\mathrm{m}^2)$ ***A1***

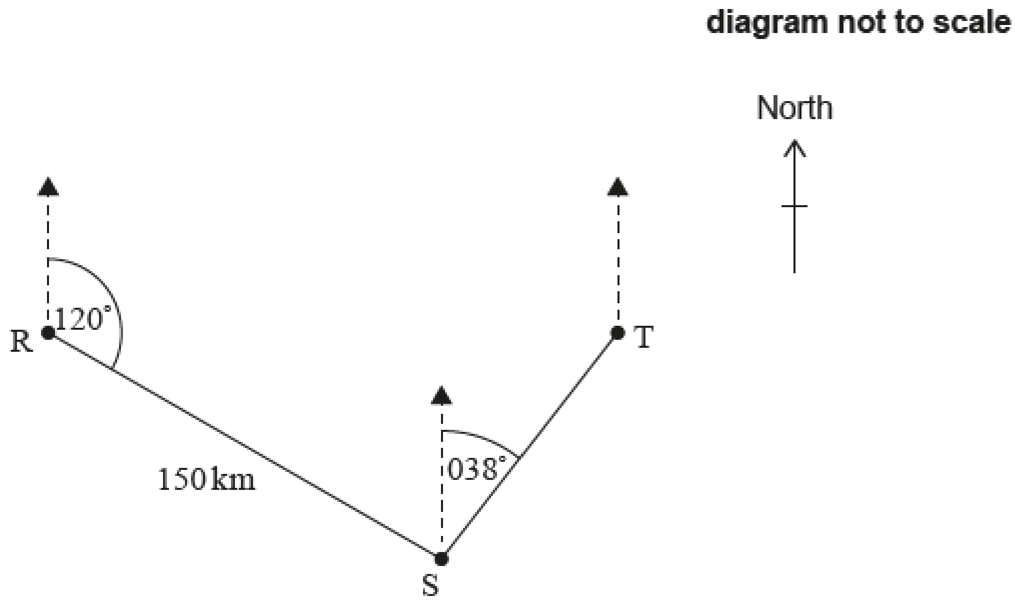
the goat has most area by $3.44\,(\mathrm{m}^2)\ (3.44026\ldots)$ ***A1***

Note: Accept $154$and 157 for the intermediate ***A1*** marks, but do NOT follow through within the question; a final answer of $3\,\mathrm{m}^2$ is awarded ***A0***.

[6 marks]

11. [Maximum mark: 6] 23N.1.SL.TZ1.5

Ron sails his boat from point R for a distance of $150\ \mathrm{km}$, on a bearing of 120°, to arrive at point S. He then sails on a bearing of 038° to point T. Ron's journey is shown in the diagram.



(a) Find $\text{R}\hat{\text{S}}\text{T}$. [2]

Markscheme

recognizing supplementary angles or acute angles in right-triangles *(M1)*

$$\left(\text{R}\hat{\text{S}}\text{T} =\right) 38^\circ + (180^\circ - 120^\circ),\ 38^\circ + (90^\circ - 30^\circ)$$

$\text{R}\hat{\text{S}}\text{T} = 98^\circ$ *A1*

[2 marks]

Point T is directly east of point R.

(b) Calculate the distance that Ron sails to return directly from point T to point R. [4]

Markscheme

$\text{R}\hat{\text{T}}\text{S} = 52^\circ$ (may be seen in part (a)) *(A1)* attempt to substitute into the sine rule (or equivalent) *(M1)*

$\frac{\text{RT}}{\sin 98^\circ} = \frac{150}{\sin 52^\circ}$ *(A1)*

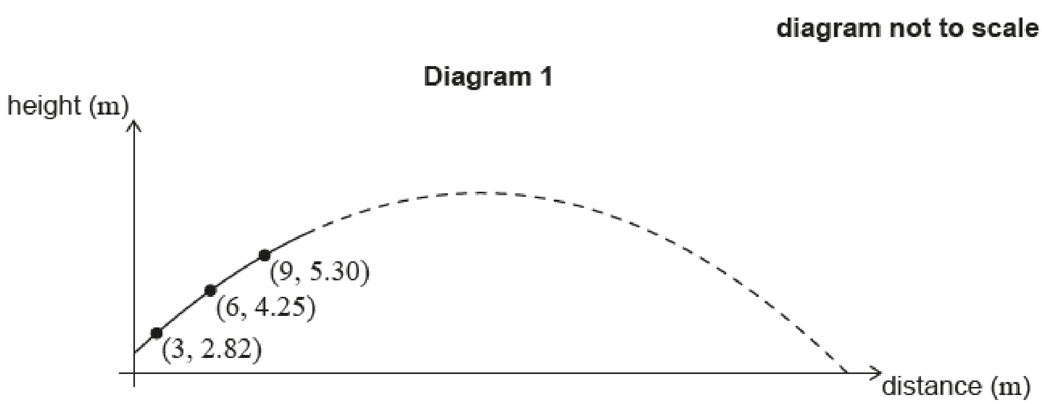
$\text{RT} = 189(\text{km}) \ \ (= 188.500\ldots)$ *A1*

[4 marks]

12. [Maximum mark: 9] 23N.1.SL.TZ1.7

An athlete on a horizontal athletic field throws a discus. The height of the discus above the field, in metres, after it is thrown can be modelled using a quadratic function of the form $f(x) = ax^2 + bx + c$, where x represents the horizontal distance, in metres, that the discus has travelled from the athlete.

A specialized camera tracks the initial path of the discus after it is thrown by the athlete. The camera records that the discus travels through the three points $(3\ ,\ 2.82)$, $(6\ ,\ 4.25)$ and $(9\ ,\ 5.30)$, as shown in **Diagram 1**.



(a) Use the coordinates $(3\ ,\ 2.82)$ to write down an equation in terms of a, b and c. [1]

Markscheme

$2.82 = a(3)^2 + b(3) + c$ **OR** $2.82 = 9a + 3b + c$ *A1*

[1 mark]

(b) Use your answer to part (a) and two similar equations to find the equation of the quadratic model for the height of the discus. [3]

Markscheme

finding other equations to solve simultaneously *(M1)*

$4.25 = a(6)^2 + b(6) + c$ **AND**
$5.30 = a(9)^2 + b(9) + c$
OR $4.25 = 36a + 6b + c$ **AND** $5.30 = 81a + 9b + c$

any one coefficient in equation correct *(A1)*

$f(x) = -0.0211x^2 + 0.667x + 1.01$ ***A1***

$(f(x) = -0.0211111\ldots x^2 + 0.666666\ldots x + 1.01)$

$(f(x) = -\frac{19}{900}x^2 + \frac{2}{3}x + \frac{101}{100})$

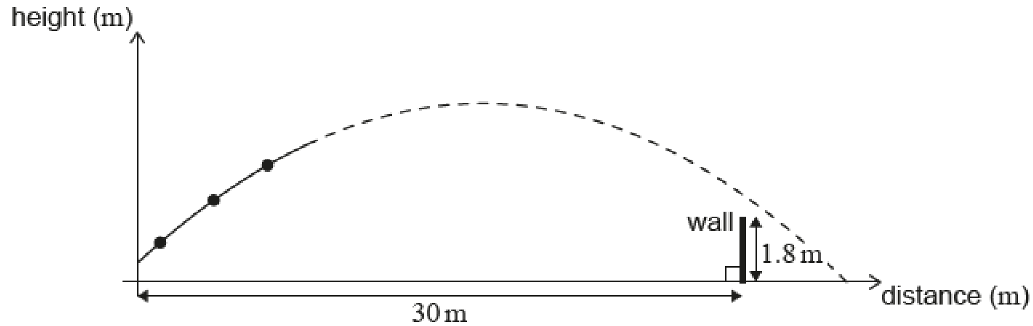
Note: Award at most ***(M1)(A1)A0*** if answer is not expressed as an equation.

[3 marks]

A 1.8-metre-high wall is 30 metres from where the athlete threw the discus, as shown in **Diagram 2**.

diagram not to scale

Diagram 2



(c) Show that the model predicts that the discus will go over the wall. [3]

Markscheme

attempt to substitute 30 into their equation *(M1)*

$(f(30) =)\ 2.01$ ***A1***

$2.01 > 1.8$ **OR** therefore the discus will go over the wall ***R1***

Note: Do not award ***A0R1***; their value must be seen to credit a correct conclusion.

[3 marks]

(d) Find the horizontal distance that the discus will travel, from the athlete until it first hits the ground, according to this model. [2]

Markscheme

setting their equation equal to zero **OR** graph with the zero indicated *(M1)*

$0 = -0.0211111\ldots x^2 + 0.666666\ldots x + 1.01\ldots$ **OR** $f(x) = 0$

$33.0\ \ (33.0275\ldots)\ (\mathrm{m})$ ***A1***

[2 marks]