Trigonometric modelling

In this presentation we will look at applications of trigonometric functions to modelling periodic behaviour.

In this presentation we will look at applications of trigonometric functions to modelling periodic behaviour.

Before proceeding with this presentation you should look at Example 5 and Example 6 in 17D Core HL (blue book).

In this presentation we will look at applications of trigonometric functions to modelling periodic behaviour.

Before proceeding with this presentation you should look at Example 5 and Example 6 in 17D Core HL (blue book). In the presentation we will discuss in detail solutions to questions 1 and 9 from exercise set 17D and then the use of technology as in chapter 17E.

In this presentation we will look at applications of trigonometric functions to modelling periodic behaviour.

Before proceeding with this presentation you should look at Example 5 and Example 6 in 17D Core HL (blue book). In the presentation we will discuss in detail solutions to questions 1 and 9 from exercise set 17D and then the use of technology as in chapter 17E.

If you understand graphs of trig functions and their transformations, this should all be very easy.

Exercise 17D question 1

We have a function:

$$T(t) = 6\sin\left(\frac{\pi}{12}t\right) + 26$$

Exercise 17D question 1

We have a function:

$$T(t) = 6\sin\left(\frac{\pi}{12}t\right) + 26$$

First the obvious: horizontal axis is time t measured in hours (after midday); vertical axis is temperature T measured in degrees Celsius.

Exercise 17D question 1

We have a function:

$$T(t) = 6\sin\left(\frac{\pi}{12}t\right) + 26$$

First the obvious: horizontal axis is time t measured in hours (after midday); vertical axis is temperature T measured in degrees Celsius.

Now we start with the graph.

Exercise 17D question 1

We have a function:

$$T(t) = 6\sin\left(\frac{\pi}{12}t\right) + 26$$

First the obvious: horizontal axis is time t measured in hours (after midday); vertical axis is temperature T measured in degrees Celsius.

Now we start with the graph. The principle axis is $T = 26$.

Exercise 17D question 1

We have a function:

$$T(t) = 6\sin\left(\frac{\pi}{12}t\right) + 26$$

First the obvious: horizontal axis is time t measured in hours (after midday); vertical axis is temperature T measured in degrees Celsius.

Now we start with the graph. The principle axis is $T = 26$. The amplitude is 6 (and the graph is not reflected), so the maximum temperature is 32, the minimum is 20.

Exercise 17D question 1

We have a function:

$$T(t) = 6\sin\left(\frac{\pi}{12}t\right) + 26$$

First the obvious: horizontal axis is time t measured in hours (after midday); vertical axis is temperature T measured in degrees Celsius.

Now we start with the graph. The principle axis is $T = 26$. The amplitude is 6 (and the graph is not reflected), so the maximum temperature is 32, the minimum is 20. The period is $\frac{2\pi}{\pi/12} = 24$.

Exercise 17D question 1

We have a function:

$$T(t) = 6\sin\left(\frac{\pi}{12}t\right) + 26$$

First the obvious: horizontal axis is time t measured in hours (after midday); vertical axis is temperature T measured in degrees Celsius.

Now we start with the graph. The principle axis is $T = 26$. The amplitude is 6 (and the graph is not reflected), so the maximum temperature is 32, the minimum is 20. The period is $\frac{2\pi}{\pi/12} = 24$. There is no horizontal shift.

Exercise 17D question 1

We have a function:

$$T(t) = 6\sin\left(\frac{\pi}{12}t\right) + 26$$

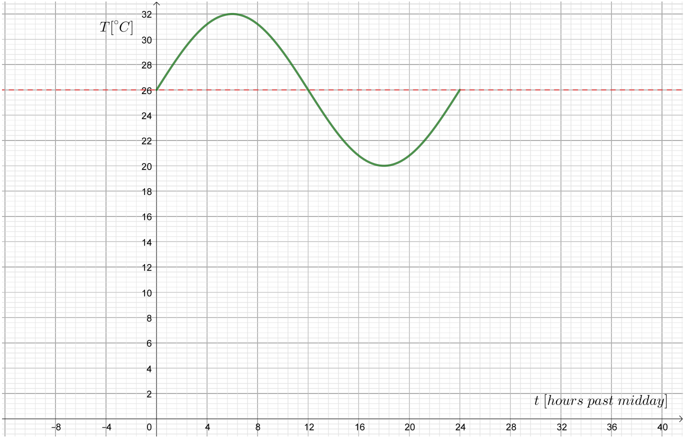
First the obvious: horizontal axis is time t measured in hours (after midday); vertical axis is temperature T measured in degrees Celsius.

Now we start with the graph. The principle axis is $T = 26$. The amplitude is 6 (and the graph is not reflected), so the maximum temperature is 32, the minimum is 20. The period is $\frac{2\pi}{\pi/12} = 24$. There is no horizontal shift.

We want to sketch the graph for $0 \leq t \leq 24$, so for one full period.

Exercise 17D question 1 (a)

We get the following graph:



Exercise 17D question 1 (b)

We want the temperature at midnight (so for $t = 12$) and at 2 pm. (so for $t = 2$).

Exercise 17D question 1 (b)

We want the temperature at midnight (so for $t = 12$) and at 2 pm. (so for $t = 2$).

i. at midnight: $T(12) = 6\sin(\frac{\pi}{12} \times 12) + 26 = 6\sin\pi + 26 = 26^\circ C$.

Exercise 17D question 1 (b)

We want the temperature at midnight (so for $t = 12$) and at 2 pm. (so for $t = 2$).

i. at midnight: $T(12) = 6\sin(\frac{\pi}{12} \times 12) + 26 = 6\sin\pi + 26 = 26^\circ C$. We could've also found this temperature from the graph.

Exercise 17D question 1 (b)

We want the temperature at midnight (so for $t = 12$) and at 2 pm. (so for $t = 2$).

i. at midnight: $T(12) = 6\sin(\frac{\pi}{12} \times 12) + 26 = 6\sin\pi + 26 = 26^\circ C$. We could've also found this temperature from the graph.

ii. at 2 pm.: $T(2) = 6\sin(\frac{\pi}{12} \times 2) + 26 = 6\sin\frac{\pi}{6} + 26 = 29^\circ C$.

Exercise 17D question 1 (c)

We want the maximum temperature and the time it occurs. This is fairly easy to do using the graph, but let's do this algebraically.

Exercise 17D question 1 (c)

We want the maximum temperature and the time it occurs. This is fairly easy to do using the graph, but let's do this algebraically.

The maximum of $\sin x$ is 1, so the maximum of $6 \sin x + 26$ is 32.

Exercise 17D question 1 (c)

We want the maximum temperature and the time it occurs. This is fairly easy to do using the graph, but let's do this algebraically.

The maximum of $\sin x$ is 1, so the maximum of $6 \sin x + 26$ is 32. The maximum for the first positive argument occurs when $x = \frac{\pi}{2}$. So the maximum of $6 \sin(\frac{\pi}{12} t) + 26$ will occur when

$$\frac{\pi}{12} t = \frac{\pi}{2}$$

this gives $t = 6$.

Exercise 17D question 1 (c)

We want the maximum temperature and the time it occurs. This is fairly easy to do using the graph, but let's do this algebraically.

The maximum of $\sin x$ is 1, so the maximum of $6 \sin x + 26$ is 32. The maximum for the first positive argument occurs when $x = \frac{\pi}{2}$. So the maximum of $6 \sin(\frac{\pi}{12} t) + 26$ will occur when

$$\frac{\pi}{12} t = \frac{\pi}{2}$$

this gives $t = 6$. So the maximum temperature of $32^\circ C$ occurs at 6 pm.

Exercise 17D question 9 (a)

We want the function $h(t)$, where h is the height in cm of the tip of the hour hand relative to the centre of the clock and t is time in hours after midnight.

Exercise 17D question 9 (a)

We want the function $h(t)$, where h is the height in *cm* of the tip of the hour hand relative to the centre of the clock and t is time in hours after midnight.

The maximum height is 6 *cm* and occurs at time $t = 0$ and then every 12 hours.

Exercise 17D question 9 (a)

We want the function $h(t)$, where h is the height in cm of the tip of the hour hand relative to the centre of the clock and t is time in hours after midnight.

The maximum height is 6 cm and occurs at time $t = 0$ and then every 12 hours. The minimum height is -6 cm and occurs at time $t = 6$ and again every 12 hours.

Exercise 17D question 9 (a)

We want the function $h(t)$, where h is the height in cm of the tip of the hour hand relative to the centre of the clock and t is time in hours after midnight.

The maximum height is 6 cm and occurs at time $t = 0$ and then every 12 hours. The minimum height is -6 cm and occurs at time $t = 6$ and again every 12 hours. This means that the principle axis is $h = 0$, the amplitude is 6 (with no reflection) and the period is 12 (which gives $b = \frac{\pi}{6}$).

Exercise 17D question 9 (a)

We want the function $h(t)$, where h is the height in *cm* of the tip of the hour hand relative to the centre of the clock and t is time in hours after midnight.

The maximum height is 6 *cm* and occurs at time $t = 0$ and then every 12 hours. The minimum height is -6 *cm* and occurs at time $t = 6$ and again every 12 hours. This means that the principle axis is $h = 0$, the amplitude is 6 (with no reflection) and the period is 12 (which gives $b = \frac{\pi}{6}$). If we choose to use cosine function, then there's no horizontal shift, we start at the top and go down.

Exercise 17D question 9 (a)

We want the function $h(t)$, where h is the height in cm of the tip of the hour hand relative to the centre of the clock and t is time in hours after midnight.

The maximum height is 6 cm and occurs at time $t = 0$ and then every 12 hours. The minimum height is -6 cm and occurs at time $t = 6$ and again every 12 hours. This means that the principle axis is $h = 0$, the amplitude is 6 (with no reflection) and the period is 12 (which gives $b = \frac{\pi}{6}$). If we choose to use cosine function, then there's no horizontal shift, we start at the top and go down. We get:

$$h(t) = 6\cos\left(\frac{\pi}{6} \times t\right)$$

Exercise 17D question 9 (b)

We want the function $d(t)$, where h is the horizontal displacement in cm of the tip of the minute hand relative to the centre of the clock and t is time in hours after midnight.

Exercise 17D question 9 (b)

We want the function $d(t)$, where h is the horizontal displacement in cm of the tip of the minute hand relative to the centre of the clock and t is time in hours after midnight.

The maximum horizontal displacement is 12 cm and occurs at time $t = \frac{1}{4}$ and then every hour.

Exercise 17D question 9 (b)

We want the function $d(t)$, where h is the horizontal displacement in cm of the tip of the minute hand relative to the centre of the clock and t is time in hours after midnight.

The maximum horizontal displacement is 12 cm and occurs at time $t = \frac{1}{4}$ and then every hour. The minimum is -12 and occurs at time $t = \frac{3}{4}$ and again every hour.

Exercise 17D question 9 (b)

We want the function $d(t)$, where h is the horizontal displacement in cm of the tip of the minute hand relative to the centre of the clock and t is time in hours after midnight.

The maximum horizontal displacement is 12 cm and occurs at time $t = \frac{1}{4}$ and then every hour. The minimum is -12 and occurs at time $t = \frac{3}{4}$ and again every hour. This means that the principle axis is $d = 0$, the amplitude is 12 (with no reflection) and the period is 1 (which gives $b = 2\pi$).

Exercise 17D question 9 (b)

We want the function $d(t)$, where h is the horizontal displacement in cm of the tip of the minute hand relative to the centre of the clock and t is time in hours after midnight.

The maximum horizontal displacement is 12 cm and occurs at time $t = \frac{1}{4}$ and then every hour. The minimum is -12 and occurs at time $t = \frac{3}{4}$ and again every hour. This means that the principle axis is $d = 0$, the amplitude is 12 (with no reflection) and the period is 1 (which gives $b = 2\pi$). If we choose to use sine function, then there's no horizontal shift, we start at 0 and go up.

Exercise 17D question 9 (b)

We want the function $d(t)$, where h is the horizontal displacement in cm of the tip of the minute hand relative to the centre of the clock and t is time in hours after midnight.

The maximum horizontal displacement is 12 cm and occurs at time $t = \frac{1}{4}$ and then every hour. The minimum is -12 and occurs at time $t = \frac{3}{4}$ and again every hour. This means that the principle axis is $d = 0$, the amplitude is 12 (with no reflection) and the period is 1 (which gives $b = 2\pi$). If we choose to use sine function, then there's no horizontal shift, we start at 0 and go up. We get:

$$d(t) = 12\sin(2\pi t)$$

Now we move on to finding trigonometric models based on real-life data.

Now we move on to finding trigonometric models based on real-life data. Before you proceed go through example 7 in section 17E Core HL.

Trigonometric model

Consider the following data for the average number of hours of daylight each month in Warsaw.

Trigonometric model

Consider the following data for the average number of hours of daylight each month in Warsaw.

Sunrise and sunset by month (Warsaw)

Month	Sunrise	Sunset	Hours of daylight
Januar	07:36 am	03:54 pm	8:18 hours
Februar	06:50 am	04:49 pm	9:59 hours
März	05:47 am	05:42 pm	11:55 hours
April	05:35 am	07:36 pm	14:00 hours
Mai	04:38 am	08:26 pm	15:49 hours
Juni	04:11 am	09:01 pm	16:50 hours
Juli	04:31 am	08:52 pm	16:21 hours
August	05:18 am	08:02 pm	14:44 hours
September	06:09 am	06:52 pm	12:43 hours
Oktober	07:00 am	05:43 pm	10:43 hours
November	06:55 am	03:45 pm	8:50 hours
Dezember	07:37 am	03:25 pm	7:49 hours

Trigonometric model

We will try to model the hours of daylight using *sine* function.

Trigonometric model

We will try to model the hours of daylight using *sine* function. We will let January correspond to 1 on the horizontal axis, February 2 and so on. The vertical axis will represent hours of daylight.

Trigonometric model

We will try to model the hours of daylight using *sine* function. We will let January correspond to 1 on the horizontal axis, February 2 and so on. The vertical axis will represent hours of daylight. We will denote the horizontal axis with m (month) and the vertical axis with D (daylight).

Trigonometric model

We will try to model the hours of daylight using *sine* function. We will let January correspond to 1 on the horizontal axis, February 2 and so on. The vertical axis will represent hours of daylight. We will denote the horizontal axis with m (month) and the vertical axis with D (daylight). We want a function $D(m)$.

Trigonometric model

We will try to model the hours of daylight using *sine* function. We will let January correspond to 1 on the horizontal axis, February 2 and so on. The vertical axis will represent hours of daylight. We will denote the horizontal axis with m (month) and the vertical axis with D (daylight). We want a function $D(m)$.

The maximum $D_{max} = 16.8(3)$, the minimum $D_{min} = 7.81(6)$. This gives the principle axis $D = 12.325$ and the amplitude of $4.508(3)$.

Trigonometric model

We will try to model the hours of daylight using *sine* function. We will let January correspond to 1 on the horizontal axis, February 2 and so on. The vertical axis will represent hours of daylight. We will denote the horizontal axis with m (month) and the vertical axis with D (daylight). We want a function $D(m)$.

The maximum $D_{max} = 16.8(3)$, the minimum $D_{min} = 7.81(6)$. This gives the principle axis $D = 12.325$ and the amplitude of $4.508(3)$. The period is of course 12, so we get $b = \frac{\pi}{6}$.

Trigonometric model

We will try to model the hours of daylight using *sine* function. We will let January correspond to 1 on the horizontal axis, February 2 and so on. The vertical axis will represent hours of daylight. We will denote the horizontal axis with m (month) and the vertical axis with D (daylight). We want a function $D(m)$.

The maximum $D_{max} = 16.8(3)$, the minimum $D_{min} = 7.81(6)$. This gives the principle axis $D = 12.325$ and the amplitude of $4.508(3)$. The period is of course 12, so we get $b = \frac{\pi}{6}$. Finally we need to figure out the horizontal shift. We will use the maximum value to establish it's value. The maximum of $\sin x$ occurs at $x = \frac{\pi}{2}$, in our case maximum occurs for $m = 6$ (June) so we solve:

$$\frac{\pi}{6}(6 - c) = \frac{\pi}{2}$$

which gives $c = 3$.

Trigonometric model

We have the following equation:

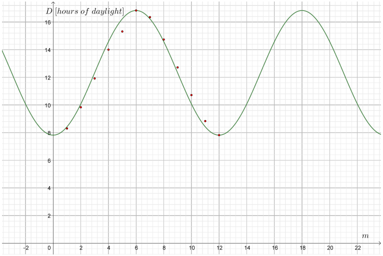
$$D(m) = 4.5083 \sin\left(\frac{\pi}{6}(m-3)\right) + 12.325$$

Trigonometric model

We have the following equation:

$$D(m) = 4.5083 \sin\left(\frac{\pi}{6}(m-3)\right) + 12.325$$

Which gives the following graph (together with all the points):

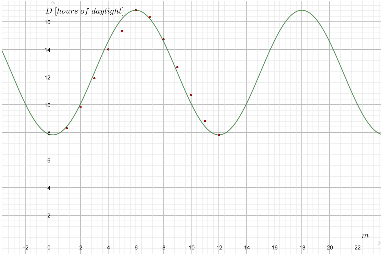


Trigonometric model

We have the following equation:

$$D(m) = 4.5083 \sin\left(\frac{\pi}{6}(m-3)\right) + 12.325$$

Which gives the following graph (together with all the points):



Looks like we could've done a slightly better job with horizontal shift by choosing a different point.

Trigonometric model

We will now use GDC to find the model.

Trigonometric model

We will now use GDC to find the model.

- Go to stats and enter two lists of values L1 is m, L2 is d.

Trigonometric model

We will now use GDC to find the model.

- Go to stats and enter two lists of values L1 is m, L2 is d.
- Press CALC $\rightarrow$ SET and take a look at 2Var statistics (now we have two variables).

Trigonometric model

We will now use GDC to find the model.

- Go to stats and enter two lists of values L1 is m, L2 is d.
- Press CALC $\rightarrow$ SET and take a look at 2Var statistics (now we have two variables). XList should be your L1 and YList L2. If it's ok, then press exit.

Trigonometric model

We will now use GDC to find the model.

- Go to stats and enter two lists of values L1 is m, L2 is d.
- Press CALC $\to$ SET and take a look at 2Var statistics (now we have two variables). XList should be your L1 and YList L2. If it's ok, then press exit.
- Press REG (or if you are back at the beginning CALC $\to$ REG), then press F6 to go to other options and finally choose Sin (F4) and voilá

Trigonometric model

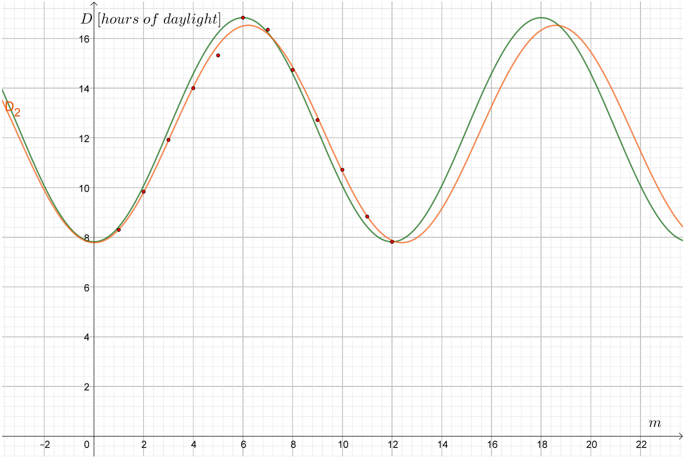
We will now use GDC to find the model.

- Go to stats and enter two lists of values L1 is m, L2 is d.
- Press CALC $\rightarrow$ SET and take a look at 2Var statistics (now we have two variables). XList should be your L1 and YList L2. If it's ok, then press exit.
- Press REG (or if you are back at the beginning CALC $\rightarrow$ REG), then press F6 to go to other options and finally choose Sin (F4) and voilá

The function we got is:

$$D(m) = 4.3711\sin(0.5075m - 1.5814) + 12.1548$$

Graphs of both functions:



Admittedly the GDC has done a slightly better job.

In case of any questions you can contact me on MS Teams or via Librus.