# Mathematics: applications and interpretation
# Higher level
# Paper 1

28 January 2025

**Zone A** afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

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1 hour 30 minutes

**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[85 marks].**

2224 – 7206

19 pages

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1.** [Maximum mark: 9]

Imani invests $\$3000$ in a bank that pays a nominal annual interest rate of $1.25\%$ compounded monthly.

(a) Calculate the amount of money Imani will have in the bank at the end of $6$ years. Give your answer correct to two decimal places. [3]

(b) Calculate the number of months it takes until Imani has at least $\$3550$ in the bank. [2]

Imani uses the $\$3550$ as a partial payment for a used car costing $\$22\,000$. For the remainder she takes out a loan from a bank.

(c) Write down the amount of money that Imani takes out as a loan. [1]

The loan is for $8$ years and the nominal annual interest rate is $12.6\%$ compounded monthly. Imani will pay the loan in fixed monthly instalments at the end of each month.

(d) Calculate the amount, correct to the nearest dollar, that Imani will have to pay the bank each month. [3]

**(This question continues on the following page)**

**(Question 1 continued)**

**2.** [Maximum mark: 6]

Jerry makes handcrafted chocolates. On average, $1$ in $25$ of the chocolates that Jerry makes is flawed. Whether or not a chocolate is flawed is independent of all other chocolates.

(a) In a batch of $20$ chocolates, chosen at random, find the probability that

(i) two are flawed.

(ii) more than two are flawed. [4]

Jerry sells the perfect chocolates for $50$ pesos each and the flawed ones for $15$ pesos each.

(b) Calculate the expected number of pesos Jerry makes from selling a batch of $20$ randomly selected chocolates. [2]

**4.** [Maximum mark: 7]

Gustav plays a game in which he first tosses an unbiased coin and then rolls an unbiased six-sided die.

If the coin shows tails, the score on the die is Gustav's final number of points.

If the coin shows heads, one is added to the score on the die for Gustav's final number of points.

(a) Find the probability that Gustav's final number of points is $7$. [2]

(b) Complete the following table. [3]

| **Final number of points** | $1$ | $2$ | $3$ | $4$ | $5$ | $6$ | $7$ |
|---|---|---|---|---|---|---|---|
| **Probability** | | | | | | | |

(c) Calculate the expected value of Gustav's final number of points. [2]

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**6.** [Maximum mark: 6]

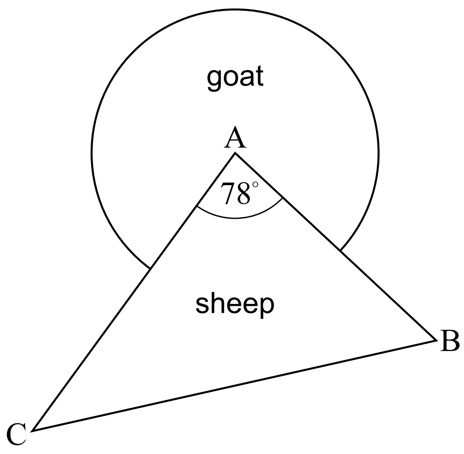
A sheep is in a field in the shape of a triangle, ABC.

$AC = 21$ metres, $AB = 15$ metres and $C\hat{A}B = 78^\circ$.

A goat is in an adjacent field in the shape of a sector of a circle with centre, A, and radius 8 metres.

The fields are shown in the diagram.

**diagram not to scale**




Determine which animal, the sheep or the goat, is in the field with the larger area, and state how many extra square metres are in this larger field.

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**7.** [Maximum mark: 4]

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = xy - 1$, given that $y = 2$ when $x = 1$.

Use Euler's method with step size $0.1$ to find the approximate value of $y$ when $x = 1.5$.

**9.** [Maximum mark: 6]

The number of traffic accidents at a road junction is modelled by a Poisson distribution with a mean of $0.76$ accidents per week.

(a) Under this model, calculate the probability that

(i) there are at least $2$ accidents in a particular week.

(ii) there will be exactly $3$ accidents in a particular $4$-week period. [4]

The local traffic authority wishes to determine the probability that, in an $8$-week period, fewer than $2$ accidents occur in a week on exactly $5$ occasions. It assumes that the weekly occurrence of accidents is independent of the week in which these occur.

(b) State the appropriate model that the traffic authority should use to determine this probability. [2]

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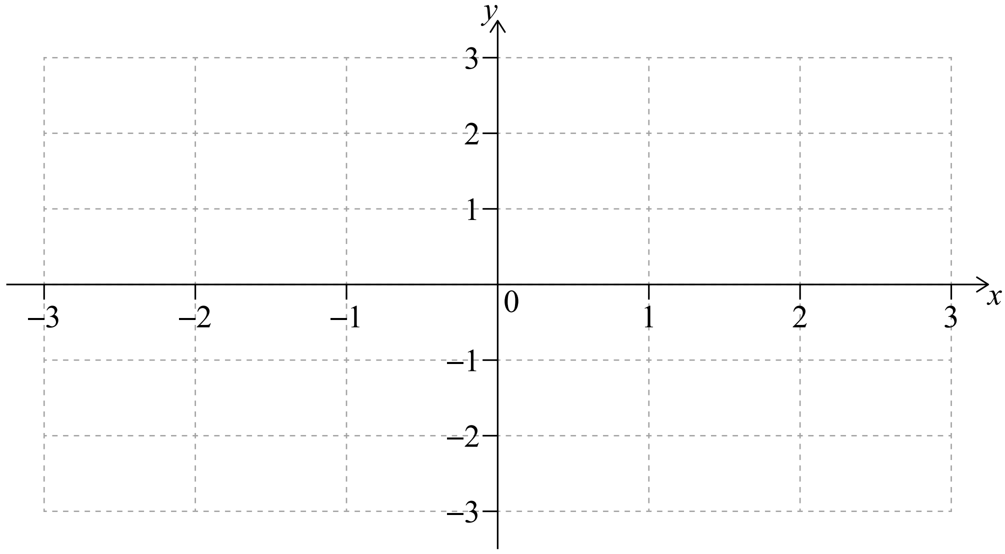
**10.** [Maximum mark: 9]

Consider the function $f(x) = x\sqrt{3 - x^2}$ , $-\sqrt{3} \le x \le \sqrt{3}$ .

(a) Sketch the graph of $y = f(x)$ on the following pair of axes. [2]




The area between the graph of $y = f(x)$ and the $x$-axis is rotated through $360^\circ$ about the $x$-axis.

(b) (i) Write down an integral that represents this volume.

(ii) Calculate the value of this integral. [4]

The graph of the function $f$ is transformed, to give the graph of the function $g$, in the following way:

- It is first stretched by scale factor $2$, parallel to the $x$-axis with the $y$-axis invariant.
- It is then stretched by scale factor $0.5$, parallel to the $y$-axis with the $x$-axis invariant.

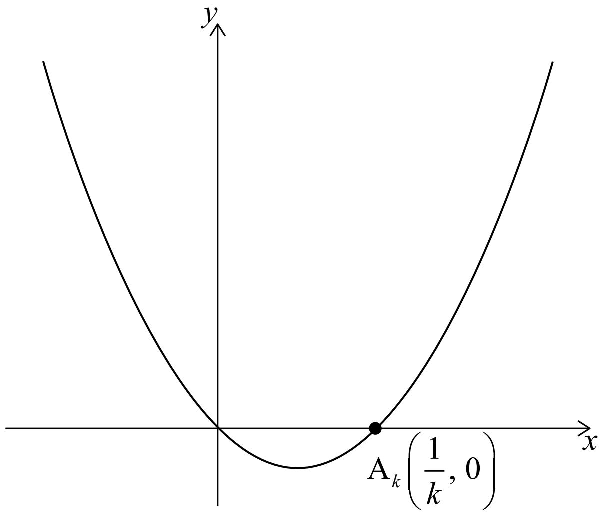
(c) Find the volume obtained when the area between the graph of $y = g(x)$ and the $x$-axis is rotated through $360^\circ$ about the $x$-axis. [3]

**(This question continues on the following page)**

**(Question 10 continued)**

**11.** [Maximum mark: 9]

The diagram shows the curve with equation $y_k = kx^2 - x$, $k > 0$, which intersects the $x$-axis at the origin and at the point $A_k\left(\frac{1}{k}, 0\right)$.




The normal to the curve at $A_k$ intersects the curve again at point $B_k$.

(a) Show that the $x$-coordinate of $B_k$ is $-\frac{1}{k}$. [6]

Consider the case where $k = 2$.

(b) Calculate the finite area of the region between the curve with equation $y_2 = 2x^2 - x$ and the normal at $A_2$. [3]

**(This question continues on the following page)**

**(Question 11 continued)**

**12.** [Maximum mark: 7]

A duck is sitting in a duck pond at point $\mathrm{A}(7,\ 4,\ 0)$ relative to an origin O, where lengths are measured in metres and time, $t$, is measured in seconds. It takes off and flies in a straight line with vector equation

$$\boldsymbol{d} = \begin{pmatrix} 7 \\ 4 \\ 0 \end{pmatrix} + t\begin{pmatrix} 6 \\ 6 \\ 3 \end{pmatrix}.$$

(a) Find the speed of the duck through the air (in $\mathrm{m\,s^{-1}}$). [2]

A hawk hovering at position vector $\begin{pmatrix} -38 \\ 134 \\ 315 \end{pmatrix}$, relative to O, sees the duck take off and immediately dives from its position with constant velocity vector $\begin{pmatrix} 15 \\ -20 \\ -60 \end{pmatrix}$ to intercept the duck.

(b) Write down the vector equation for $\boldsymbol{h}$, that models the flight of the hawk. [1]

(c) Find the position vector at which the hawk intercepts the duck. [4]

**13.** [Maximum mark: 8]

A particle starts from rest at point O and moves in a straight line with velocity, $v$, given by

$$v = 3\sin(t)\left(1+\cos(t)\right),\ t \geq 0$$

where $v$ is measured in metres per second and time, $t$ (radians), is measured in seconds.

The particle next comes to instantaneous rest when $t = a$.

(a) Determine the value of $a$. [2]

(b) Find the maximum velocity of the particle during the interval $0 \leq t \leq a$. [2]

(c) By finding the total distance travelled between $t = 0$ and $t = a$, find the average speed of the particle during the interval $0 \leq t \leq a$. [4]

**14.** [Maximum mark: 6]

A cup of hot water is placed in a room and is left to cool for half an hour. Its temperature, measured in $^{\circ}$C, is recorded every $5$ minutes. The results are shown in the table.

| **Time (mins)** | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| **Temperature (°C)** | 59 | 52 | 46 | 40 | 35 | 29 |

Akira uses the power function $T(t) = at^{b} + 25$ to model the temperature, $T$, of the water $t$ minutes after it was placed in the room.

(a) State what the value of $25$ represents in this context. [1]

(b) Use your graphic display calculator to find the value of $a$ and of $b$. [3]

Soo Min models the temperature, $T$, of the water $t$ minutes after it was placed in the room as $T(t) = kc^{t} + 25$.

(c) Find the value of $k$ and of $c$. [1]

(d) State a reason why Soo Min's model of the temperature is a better fit for the data than Akira's model. [1]

**15.** [Maximum mark: 8]

Consider the complex number $z = -1 + \mathrm{i}$.

(a) Express $z$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $-\pi < \theta \leq \pi$. [2]

A and B are the points on the Argand diagram that represent the complex numbers $z$ and $z^2$, respectively.

A is mapped onto B by the composition of a rotation and an enlargement.

(b) (i) Describe fully this mapping of A onto B, stating the scale factor of the enlargement and the angle of rotation.

(ii) Find and simplify a matrix that maps A onto B. [5]

(c) Find the smallest positive integer, $n$, for which $z^n$ is real and positive. [1]

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