Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 2

29 January 2025

Zone A morning | **Zone B** morning | **Zone C** morning

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[83 marks].**

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 15]

The company Fred Express delivers packages. From past experience, the time taken, T, to deliver a package follows a normal distribution with mean 64 hours and standard deviation 12 hours.

(a) State $\mathrm{P}(T < 64)$. [1]

(b) Find $\mathrm{P}(44 < T < 64)$. [2]

30% of packages are delivered in less than k hours.

(c) (i) Sketch a diagram of this normal distribution, shading the region that represents $\mathrm{P}(T < k)$.

(ii) Find the value of k. [4]

For quality control, the manager randomly selects five outgoing packages. These selections are independent.

(d) Find the probability that exactly two of these packages are delivered in less than k hours. [3]

Fred Express charges a fixed amount of $\$4.50$ for any package weighing $1\,\mathrm{kg}$ or less. Heavier packages are charged an additional fee of $\$2.00$ per kg. This fee is applied for any weight **in excess of** $1\,\mathrm{kg}$. For example, a $1.5\,\mathrm{kg}$ package is charged an additional $\$1.00$.

(e) Write down an expression for the amount charged to deliver a package of weight $x\,\mathrm{kg}$, where $x > 1$. [2]

(f) Find the amount Fred Express charges for a $5.3\,\mathrm{kg}$ package. [1]

Meiling is charged $\$7.20$ for the delivery of a package.

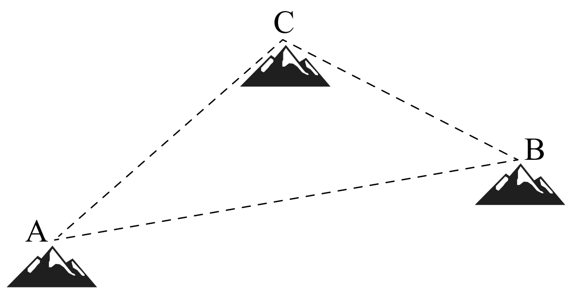
(g) Find the weight of Meiling's package. [2]

2. [Maximum mark: 12]

A national park contains three mountains whose summits are at points A, B and C.

According to a coordinate system, the position of A is $(0, 0, 2.8)$ and the position of B is $(7.2, 5.1, 2.4)$. All the values are in kilometres.

diagram not to scale



(a) (i) Find the vector $\vec{AB}$.

(ii) Hence find AB, the distance between A and B. [3]

The vector $\vec{AC}$ is parallel to the vector $\begin{pmatrix} 1.1 \\ 8.4 \\ 0.2 \end{pmatrix}$.

(b) Find the angle between $\begin{pmatrix} 1.1 \\ 8.4 \\ 0.2 \end{pmatrix}$ and $\vec{AB}$. [5]

The angle between $\vec{BA}$ and $\vec{BC}$ is 55.2°.

(c) Use the sine rule to find AC. [4]

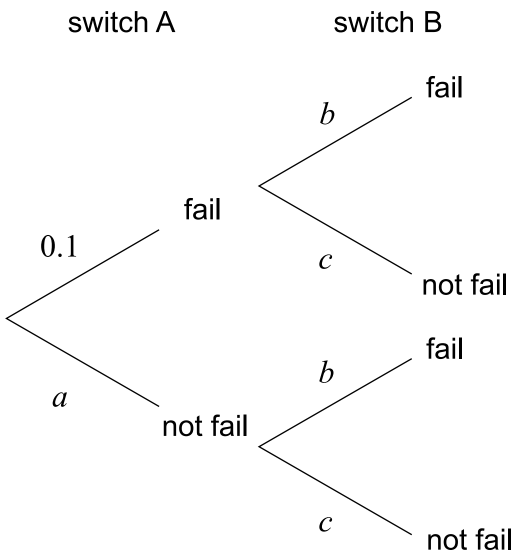
4. [Maximum mark: 12]

A type of generator will only function if a particular switch is working. The generator has a main switch, A, and a ‘back up’ switch, B.

The manufacturer claims the probability of switch A failing within one month of being fitted is 0.1 and the probability of the cheaper switch B failing within one month is 0.3. Whether or not a switch fails is independent of the state of the other switch.

If both switches fail, the generator needs to shut down to replace the switches. Both switches are replaced after a month of use (whether they have failed or not) or whenever the generator needs to be shut down.

The following tree diagram shows the probabilities of a switch failing within one month of them both being replaced, assuming the manufacturer’s claim is correct.



(a) Write down the values of

(i) a.

(ii) b.

(iii) c. [2]

(b) Hence find the probability that the generator needs to shut down within one month of the switches being replaced. [1]

(This question continues on the following page)

(Question 4 continued)

The owner of the generator is suspicious of the switch manufacturer's claims, so they look back through the past 200 occasions when the switches were replaced. The records show whether no switches, one switch or two switches had failed.

The data the owner collected are shown in the following table.

No switch fails	One switch fails	Two switches fail
118	72	10

(c) Perform a χ^2 goodness of fit test at the 5% significance level to test whether the manufacturer's claims are correct using the following hypotheses.

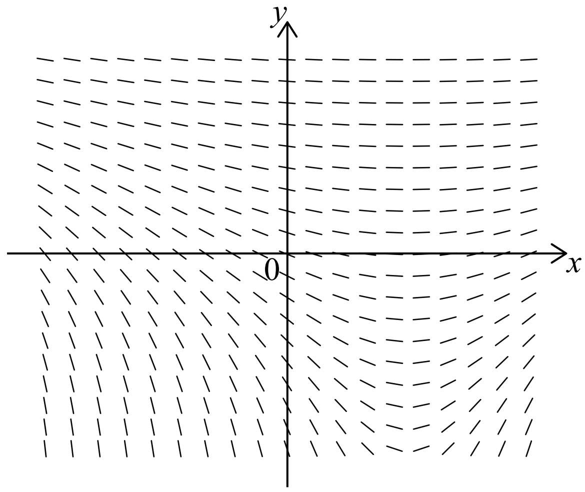
H_0: The manufacturer's claims are correct.
H_1: The manufacturer's claims are not both correct. [9]

5. [Maximum mark: 13]

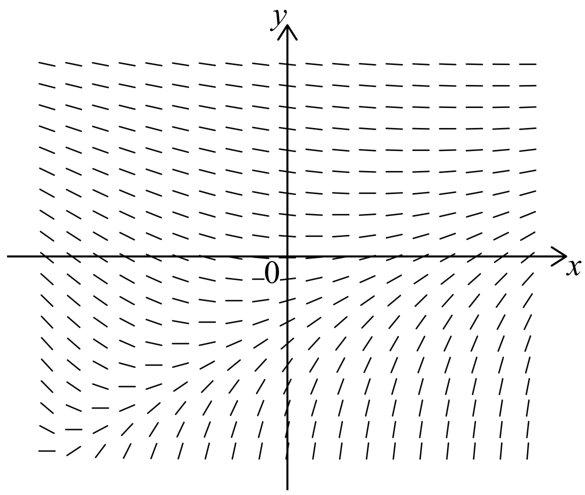
Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{\mathrm{e}^{2y}}$.

(a) Identify which of the following diagrams, **A**, **B** or **C**, represents the slope field for the differential equation. Give a reason for your answer. [2]

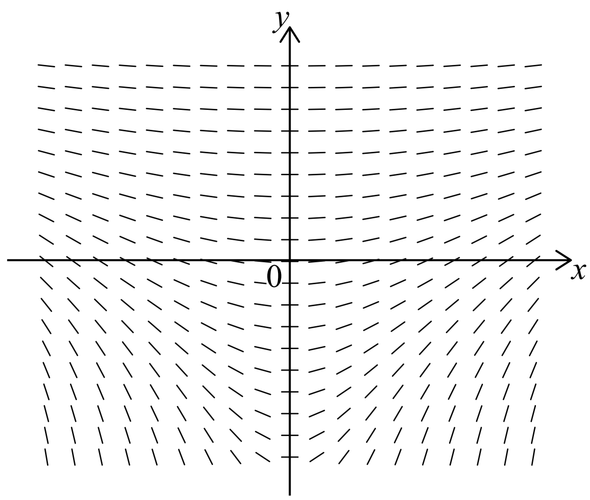
A.



B.



C.



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(Question 5 continued)

It is given that, for a particular solution, $x = 0$ and $y = 0$.

(b) Find an expression for y, in terms of x, for this solution. [7]

(c) Find $\frac{dy}{dx}$, in terms of x, by differentiating your answer from part (b). [2]

(d) Hence verify that your answer to part (b) is a solution to $\frac{dy}{dx} = \frac{x}{e^{2y}}$. [2]

6. [Maximum mark: 13]

Taylor is playing a computer game in which they shoot at spaceships and battleships. The number of spaceships they hit per minute can be modelled by a Poisson distribution with mean 4.2. The number of battleships they hit per minute can be modelled by a Poisson distribution with a mean of 2.3. Any single hit occurs independently of all others.

(a) Find the probability Taylor hits

(i) at most 10 spaceships in 2 minutes.

(ii) a total of more than 10 spaceships and battleships in one minute. [5]

Every spaceship that is hit earns Taylor 3 points and every battleship 5 points. Let T be the total points earned in one minute.

(b) Find

(i) $\mathrm{E}(T)$.

(ii) $\mathrm{Var}(T)$. [3]

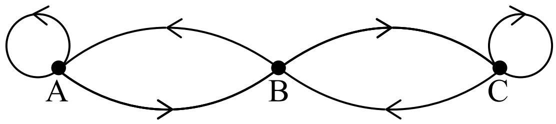
(c) State one reason why the distribution of T cannot be Poisson. [1]

Taylor intends to play the game for one hour.

(d) Use the central limit theorem to find the probability that Taylor's mean score per minute is greater than 25. [4]

8. [Maximum mark: 18]

(a) Write down the adjacency matrix for the directed graph shown below. [2]



(b) Find the total number of walks of length 5 from A to B. [3]

A bird sits on one of three posts, labelled A, B and C, with B between A and C. When the bird moves, it will either fly to an adjacent post or return to the same post according to the following pattern.

- If it is on B, it will fly to A or C, each with a probability of 0.5.
- If it is on A or C, it will return to the same post with a probability of 0.5 or fly to B with a probability of 0.5.

The possible flights of the bird can be represented by the graph in part (a).

(c) (i) Every possible sequence of 5 flights by the bird has the same probability of occurring. State this probability.

(ii) Use your answer to part (b) to find the probability that if the bird was initially on post A, it will be on post B after 5 flights. [3]

A second bird often joins the first on the posts. Their flights now follow the pattern given below.

- The birds will never sit on the same post.
- They will always fly from the posts at the same time.
- If they are on adjacent posts, the bird on post B will **always** fly to the vacant end post. The other bird will fly to post B with a probability of 0.4 or return to the same post with a probability of 0.6.
- If they are each on one of the end posts, they will fly to post B with a probability of 0.5 or return to the same post with a probability of 0.5. However, if they both try to fly to post B at the same time, they will see the other one doing so and both will immediately return to the post they were previously on.

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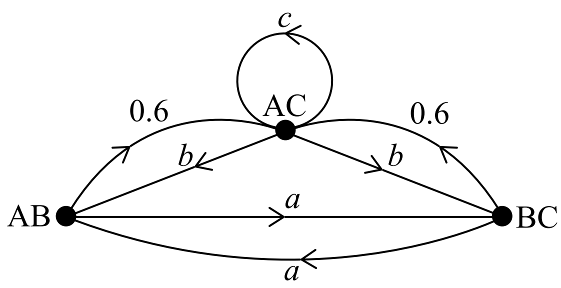
(Question 8 continued)

The possible flights of the two birds can be represented by the following transition diagram, where the three vertices represent which posts are **occupied**.

diagram not to scale



(d) Write down the value of

(i) a.

(ii) b.

(iii) c. [3]

(e) Given that the birds are initially on posts A and B, find the probability they will be on posts B and C after 5 flights. [4]

The birds continue this pattern of flights for a long period.

(f) Given that the time between flights is always the same, find the post which is sat on least and the proportion of the time it is free. [3]