## ■ Mixed Practice

1 **a** For the function $f(x) = 3x - 1$, find the inverse function, $f^{-1}(x)$.
   b Verify that $(f \circ f^{-1})(x) = x$ for all x.

2 A function is defined by $h(x) = \sqrt{5 - x}$ for $x \leqslant a$.
   a State the largest possible value of a.
   b Find $h^{-1}(3)$.

3 Given that $f(x) = e^{3x}$ evaluate $f^{-1}(4)$.

4 The table shows some values of the function $f(x)$.

x	0	1	2	3	4
$f(x)$	3	4	0	1	2

   a Find $(f \circ f)(2)$.
   b Find $f^{-1}(4)$.

5 Let $f(x) = x + 3$ and $g(x) = e^{2x}$. Solve the equation $(g \circ f)(x) = 1$.

6 Given that $g(x) = 3\ln(x - 2)$,
   a Find the largest possible domain for g.
   b Using the domain for part **a**, find an expression for $g^{-1}(x)$ and state its range.

7 Given the functions $f(x) = 3x + 1$ and $g(x) = x^3$, find $(f \circ g)^{-1}(x)$.

8 Let $f(x) = 2x + 3$ and $g(x) = x^3$.
   a Find $(f \circ g)(x)$.
   b Solve the equation $(f \circ g)(x) = 0$.

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9 The following diagram shows the graph of $y = f(x)$, for $-4 \leqslant x \leqslant 5$.
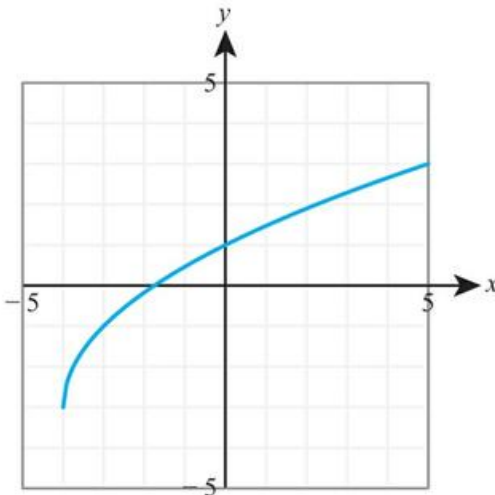
   a Write down the value of
      i $f(-3)$;
      ii $f^{-1}(1)$.
   b Find the domain of f^{-1}.
   c Sketch the graph of f^{-1}.

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10 For the function $f(x) = x - 2$ and $g(x) = \dfrac{1}{x-1}$ $(x \neq 1)$,

    a Find $(f \circ g)(x)$ and state its domain.

    b Verify that $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$ for all x.

11 Given that $f(x) = \dfrac{3x-1}{x+4}$ for $x \neq -4$, find an expression for $f^{-1}(x)$.

12 Let the function $g(x) = x + e^x$ be defined for all real numbers x.

    a By sketching a graph, or otherwise, show that g has an inverse function.

    b Solve the equation $g^{-1}(x) = 2$.

13 Let $f(x) = \sqrt{x}$ $(x \geqslant 0)$ and $g(x) = 9^x$ $(x \in \mathbb{R})$.

    a Evaluate $(g \circ f)\left(\dfrac{1}{4}\right)$.

    b Solve the equation $(f^{-1} \circ g)(x) = \dfrac{1}{3}$.

14 Let $h(x) = \ln(x - 2)$ for $x > 2$, and $g(x) = e^x$ for $x \in \mathbb{R}$.

    a Find $h^{-1}(x)$ and state its range.

    b Find $(g \circ h)(x)$, giving your answer in the form $ax + b$, where $a, b \in \mathbb{Z}$.

15 Function f is defined by $f(x) = (2x + 1)^2$ for $x \leqslant a$.

    a By using a graph, or otherwise, find the largest value of a for which f has an inverse function.

    b Find an expression for $f^{-1}(x)$.

16 Let $f(x) = x^2 + 3$, $x \geqslant 1$ and $g(x) = 12 - x$.

    a Evaluate $f(5)$.

    b Find and simplify an expression for $gf(x)$.

    c **i** State the geometric relationship between the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

        ii Find an expression for $f^{-1}(x)$.

        iii Find the range of $f^{-1}(x)$.

17 Let $f(x) = 3x + 1$, $x \in \mathbb{R}$ and $g(x) = \dfrac{x+4}{x-1}$, $x \neq 1$.

    a Find and simplify

        i $f(T)$

        ii $fg(x)$

        iii $ff(x)$

    b State the range of f. Hence, explain why $gf(x)$ does not exist.

    c **i** Show that $g^{-1}(x) = g(x)$ for all $x \neq 1$.

        ii State the range of $g^{-1}(x)$.

18 Let $f(x) = \sqrt{x} - 5$, for $x \geqslant 5$.

    a Find $f^{-1}(2)$.

    b Let g be a function such that g^{-1} exists for all real numbers. Given that $g(30) = 3$, find $(f \circ g^{-1})(3)$.

19 Let $f(x) = 3x - 2$ and $g(x) = \dfrac{5}{3x}$, for $x \neq 0$.

    a Find $f^{-1}(x)$.

    b Show that $(g \circ f^{-1})(x) = \dfrac{5}{x+2}$.

    Let $h(x) = \dfrac{5}{x+2}$, for $x \geqslant 0$. The graph of h has a horizontal asymptote at $y = 0$.

c **i** Find the y-intercept of the graph of h.

  ii Hence, sketch the graph of h.

d For the graph of h^{-1},

  i write down the x-intercept;

  ii write down the equation of the vertical asymptote.

e Given that $h^{-1}(a) = 3$, find the value of a.

20 For the functions $f(x) = e^{2x}$ and $g(x) = \ln(x - 2)$, verify that $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$.

21 Let $f(x) = \dfrac{1}{1 + \sqrt{x}}$ and $g(x) = x + 7$. Solve $f^{-1}(g^{-1}(x)) = 9$.

22 Let $f(x) = 2 + x - x^3$ for $x \geq a$.

a Find the smallest value of a so that f has an inverse function.

b State the geometric relationship between the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

c Find the exact solution of the equation $f^{-1}(x) = f(x)$.

23 Consider the functions given below.

$$f(x) = 2x + 3$$

$$g(x) = \frac{1}{x}, \; x \neq 0$$

a **i** Find $(g \circ f)(x)$ and write down the domain of the function.

  ii Find $(f \circ g)(x)$ and write down the domain of the function.

b Find the coordinates of the point where the graph of $y = f(x)$ and the graph of $y = (g^{-1} \circ f \circ g)(x)$ intersect.

5 a $f^{-1}(x) = \frac{1}{3}(2^x - 1)$ b $f^{-1}(x) = \frac{1}{4}(3^x + 1)$ b $f^{-1}(x) = 2\ln(x+1), \; x > -1$

6 a $f^{-1}(x) = x^2 + 2$ b $f^{-1}(x) = x^2 - 3$

22 a $\sqrt{\dfrac{4x+1}{x-1}}$

7 a $f^{-1}(x) = \sqrt[3]{x} - 2$ b $f^{-1}(x) = \sqrt[3]{x} + 3$

8 a $f^{-1}(x) = \sqrt[3]{x+2}$ b $f^{-1}(x) = \sqrt[3]{x-5}$

   b $f^{-1}(x) > 2$

9 a $f^{-1}(x) = \dfrac{2x+3}{x-1}$ b $f^{-1}(x) = \dfrac{3x+1}{x-1}$

23 $\dfrac{1}{7}$

10 a $f^{-1}(x) = \dfrac{2x+1}{3x-2}$ b $f^{-1}(x) = \dfrac{3x-1}{2x-3}$

24 a

11 a $x \geqslant 2$ b $x \geqslant -5$

12 a $x \leqslant -1$ b $x \leqslant 3$

13 a $x \geqslant 2$ b $x \geqslant 0$

14 a $-1 \leqslant x \leqslant 1$ b $-2 \leqslant x < 2$

15 a $x \leqslant -1$ b $x \leqslant 1$

16 a $x \geqslant -2$ b $x \geqslant 3$

17 a $x < -2$ b $x < -1$

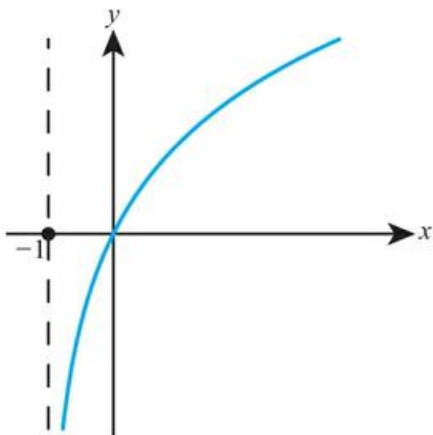**18** a $\dfrac{5}{3}$ b $\dfrac{3x-4}{x}$ domain $x \neq 0$
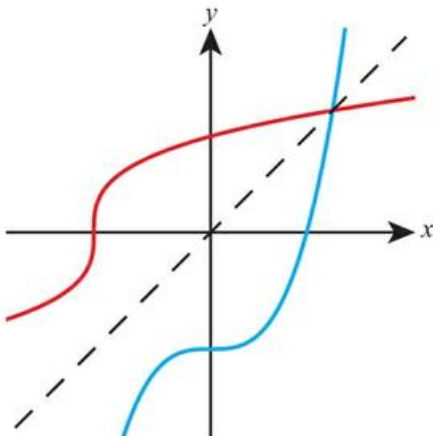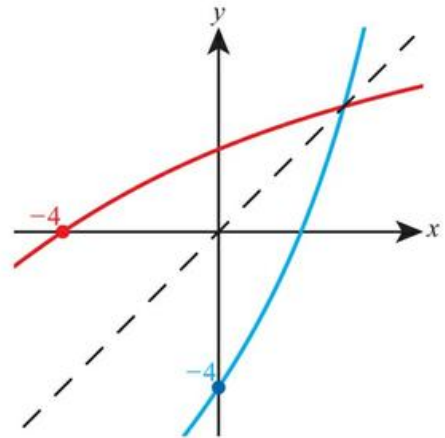
   b $2\ln 5$

25 a 0 b $-\sqrt{x-3}$

26 a $x \leqslant 5$

   b $5 - \dfrac{1}{3}\sqrt{x}$

19 $\dfrac{1}{5}\ln\left(\dfrac{x}{3}\right)$ domain $x > 0$

27 4

20 a

Chapter 14 Mixed Practice

1 a $f^{-1}(x) = \dfrac{x+1}{3}$

2 a 5 b -4

3 0.462

4 a 3 b 1

   b $f^{-1}(x) = \sqrt[3]{5x+15}$

5 $x = -3$

21 a

6 a $x > 2$

   b $2 + e^{\frac{x}{3}}, \; g^{-1}(x) > 2$

7 $\sqrt[3]{\dfrac{x-1}{3}}$

8 a $2x^3 + 3$

   b -1.15

9 a i -1

      ii 0

   b $-3 \leqslant x \leqslant 3$

c
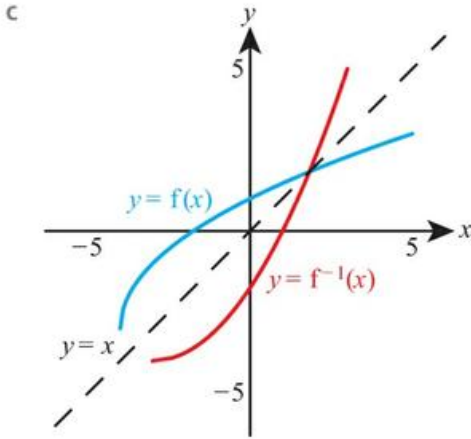
10 a $\dfrac{1}{x-1}-2, x \neq 1$

11 $f^{-1}(x) = \dfrac{4x+1}{3-x}$ domain $x \neq 3$

12 b 9.39

13 a 3   b $-\dfrac{1}{4}$

14 a $e^x + 2, y > 2$ b $x - 2$

15 a $a = -\dfrac{1}{2}$ b $\dfrac{-1-\sqrt{x}}{2}$

16 a 28   b $9 - x^2$

  c i Reflection in the line $y = x$
    ii $\sqrt{x-3}$ iii $y \geqslant 1$

17 a i 22   ii $\dfrac{4x+11}{x-1}$

    iii $9x + 4$

  b f(x) can be 1, which is not in the domain of g

  c ii $y \neq 1$
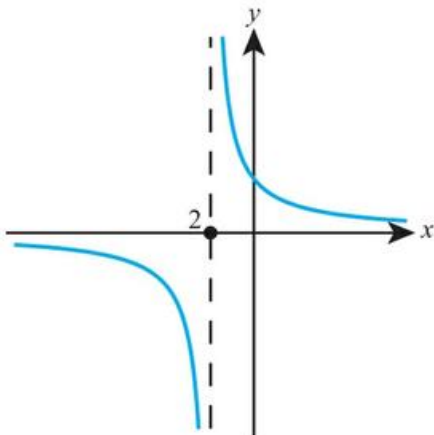
18 a 9   b 5

19 a $\dfrac{x+2}{3}$

  c i 2.5

    ii

d i 2.5   ii $x = 0$

e 1

21 $\dfrac{29}{4}$

22 a 0.577

  b Reflection in the line $y = x$

  c $\sqrt[3]{2}$

23 a i $\dfrac{1}{2x+3}, x \neq -\dfrac{3}{2}$

    ii $\dfrac{2}{x}+3, x \neq 0$

  b $(-1, 1)$

Chapter 15 Prior Knowledge

1 $3x^2 + 10x - 8$

2 $(x-10)(x+1)$

3 $2\sqrt{3}$

4 $x > \dfrac{7}{3}$

Exercise 15A

1 a i 2   ii 1   iii 3
  b i 3   ii 3   iii 1

2 a i 1   ii 1   iii 2
  b i 1   ii 2   iii 3

3 a i 2   ii 3   iii 1
  b i 2   ii 1   iii 3

4 a