l'Hopital [46 marks]

1. [Maximum mark: 5] EXN.1.AHL.TZ0.6

Use l'Hôpital's rule to determine the value of $\lim_{x \to 0} \left(\frac{2x \cos\left(x^2\right)}{5 \tan x} \right)$. [5]

2. [Maximum mark: 6] 24M.1.AHL.TZ2.8

Use l'Hôpital's rule to find $\lim_{x \to 0} \frac{\sec^4 x - \cos^2 x}{x^4 - x^2}$. [6]

3. [Maximum mark: 8] 22M.2.AHL.TZ2.7

Consider $\lim_{x \to 0} \frac{\arctan(\cos x) - k}{x^2}$, where $k \in \mathbb{R}$.

(a) Show that a finite limit only exists for $k = \frac{\pi}{4}$. [2]

(b) Using l'Hôpital's rule, show algebraically that the value of the limit is $-\frac{1}{4}$. [6]

4. [Maximum mark: 5] 21M.1.AHL.TZ1.8

Use l'Hôpital's rule to find $\lim_{x \to 0} \left(\frac{\arctan 2x}{\tan 3x} \right)$. [5]

5. [Maximum mark: 6] 20N.1.AHL.TZ0.F_1

Use l'Hôpital's rule to determine the value of

$\lim_{x \to 0} \frac{2 \sin x - \sin 2x}{x^3}$. [6]

6. [Maximum mark: 7] 20N.3.AHL.TZ0.Hca_1

Use l'Hôpital's rule to find

$$\lim_{x \to 1} \frac{\cos\left(x^2 - 1\right) - 1}{e^{x-1} - x}.$$

[7]

7. [Maximum mark: 9] 19M.3.AHL.TZ0.Hca_4

Using L'Hôpital's rule, find $\lim_{x \to 0} \left(\frac{\tan 3x - 3\tan x}{\sin 3x - 3\sin x} \right)$.

[9]