2224 –7106M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

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Mathematics: analysis and approaches

Higher level

Paper 1

31 pages

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

SECTION A

1. $\tan^{-1} 1 = 45^\circ$ or equivalent *(A1)*

attempt to equate $2x - 5^\circ$ to their reference angle *(M1)*

Note: Do not accept $2x - 5° = 1$.

$2x - 5^\circ = 45^\circ, \left(225^\circ\right)$

$x = 25^\circ, 115^\circ$ ***A1A1***

Note: Do not award the final ***A1*** if any additional solutions are seen.

[4 marks]

2. recognising a quadratic in 3^x ***(M1)***

$3\times\left(3^x\right)^2+5\times 3^x-2=0$

valid attempt to solve a quadratic equation (factorising, use of formula, completing square, or otherwise) ***(M1)***

$\left(3\times 3^x-1\right)\left(3^x+2\right)=0$ OR $3^x=\dfrac{-5\pm\sqrt{25+24}}{6}$ (or equivalent) ***(A1)***

$3^x=\dfrac{1}{3}$ (or $3^x=-2$) ***(A1)***

$x=-1$ ***A1***

Note: Award the final ***A1*** if candidate's answer includes $x=-1$ and $x=\log_3\left(-2\right)$. Award ***A0*** if other incorrect answers are given.

[5 marks]

3. (a) (i) $\left(\frac{9}{2}, \frac{3\sqrt{3}}{2}\right)$ (accept $x = \frac{9}{2}$ and $y = \frac{3\sqrt{3}}{2}$) ***A1***

(ii) **METHOD 1**

using $m = \frac{\text{change in } y}{\text{change in } x}$ with their midpoint OR gradient perpendicular to AC

OR $m = \tan 30^\circ$ ***(M1)***

$m = \frac{\sqrt{3}}{3}$ ***(A1)***

$y = \frac{\sqrt{3}}{3}x$ OR $y - \frac{3\sqrt{3}}{2} = \frac{\sqrt{3}}{3}\left(x - \frac{9}{2}\right)$ (must be written as an equation) ***A1***

METHOD 2

attempt to find the vector equation of the line using a direction and a point on the line ***(M1)***

direction vector is $\begin{pmatrix} \frac{9}{2} \\ \frac{3\sqrt{3}}{2} \end{pmatrix}$ or equivalent/parallel vectors ***(A1)***

$\boldsymbol{r} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} \frac{9}{2} \\ \frac{3\sqrt{3}}{2} \end{pmatrix}$ OR $\boldsymbol{r} = \lambda \begin{pmatrix} \frac{9}{2} \\ \frac{3\sqrt{3}}{2} \end{pmatrix}$ OR $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{9}{2} \\ \frac{3\sqrt{3}}{2} \end{pmatrix} + \lambda \begin{pmatrix} \frac{9}{2} \\ \frac{3\sqrt{3}}{2} \end{pmatrix}$ (or equivalent) ***A1***

> **Note:** Vector equation must be in the form $\boldsymbol{r} =$ or $\begin{pmatrix} x \\ y \end{pmatrix} =$.
>
> Allow equivalent parametric forms such as $x = \frac{9}{2}t$, $y = \frac{3\sqrt{3}}{2}t$.

[4 marks]

continued…

Question 3 continued.

(b) substituting $x = 6$ into their equation **(M1)**

so at B $y = 2\sqrt{3}$ **(A1)**

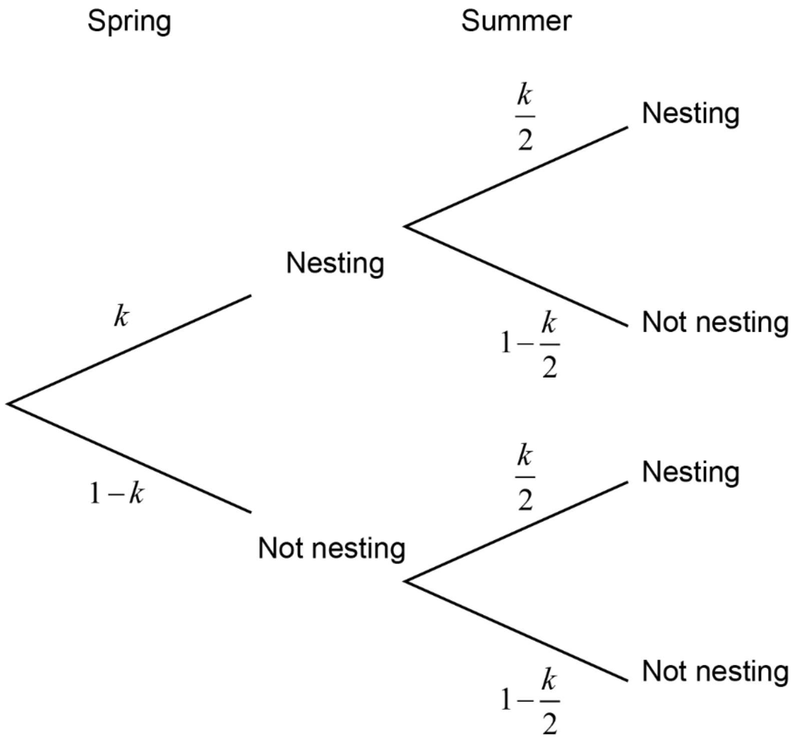
area of triangle $\mathrm{OAB} = \frac{1}{2} \times 6 \times 2\sqrt{3} = 6\sqrt{3}$

area of quadrilateral $\mathrm{OABC} = 12\sqrt{3}$ ***A1***

[3 marks]

Total [7 marks]

4. (a)



$1-k$ for Spring **_A1_**

$1-\dfrac{k}{2}$ for both Summers **_A1_**

[2 marks]

continued…

Question 4 continued.

(b) (i) multiplying the two correct branches ***(A1)***

$$(1-k)\left(1-\frac{k}{2}\right)$$

attempt to expand and equate to $\frac{5}{9}$ ***(M1)***

$$1-k-\frac{k}{2}+\frac{k^2}{2}=\frac{5}{9}$$

$18-18k-9k+9k^2=10$ OR $\frac{k^2}{2}-\frac{3k}{2}+\frac{4}{9}=0$ OR $\frac{9k^2}{2}-\frac{27k}{2}+4=0$ ***A1***

$9k^2-27k+8=0$ ***AG***

(ii) ($k=\frac{1}{3}$ is the only valid solution as) $\frac{8}{3}>1$ ***R1***

Note: Accept any valid reasoning indicating that any probability cannot be greater than 1 and/or probability cannot be less than 0.

[4 marks]

Total [6 marks]

5. (a) $y=\frac{2}{3}$ (must be written as equation with $y=$) ***A1***

[1 mark]

(b) (i) 2 ***A1***

(ii) **EITHER**

$$\frac{2(x+3)}{3(x+2)}=mx+1$$

attempt to expand to obtain a quadratic equation ***(M1)***

$2x+6=3mx^2+6mx+3x+6$

$3mx^2+(6m+1)x=0$ OR $3mx^2+6mx+x=0$ ***A1***

recognition that discriminant $\Delta=0$ for one solution ***(M1)***

$(6m+1)^2=0$

continued…

Question 5 continued.

OR

$$\frac{2(x+3)}{3(x+2)} = mx+1$$

attempt to expand to obtain a quadratic equation **(M1)**

$$2x+6 = 3mx^2+6mx+3x+6$$

$3mx^2+(6m+1)x=0$ OR $3mx^2+6mx+x=0$ **A1**

attempt to solve their quadratic for x and equating their solutions **(M1)**

$$x(3mx+6m+1)=0$$

$x=0$ OR $x=-\dfrac{6m+1}{3m}(=0)$

$$-\frac{6m+1}{3m}=0$$

OR

attempt to find $f'(x)$ using the quotient rule **(M1)**

$f'(x)=\dfrac{2}{3}\left(\dfrac{(x+2)-(x+3)}{(x+2)^2}\right)=\left(\dfrac{-2}{3(x+2)^2}\right)$ OR $\dfrac{2(3x+6)-3(2x+6)}{(3x+6)^2}$ or equivalent **A1**

recognition that m is the derivative of $f(x)$ at $x=0$ **(M1)**

THEN

$\Rightarrow m=-\dfrac{1}{6}$ **A1**

continued…

Question 5 continued.

(iii)

Note: In this part, FT may be awarded only for values of m between -1 and 0.

$-\frac{1}{6} < m < 0$ ***A2***

Note: Award ***A1*** for only $m > -\frac{1}{6}$. Award ***A1*** for only $m < 0$.

[7 marks]

Total [8 marks]

6. (a) $2.5(\%)$ ***A1***

[1 mark]

(b) $\mathrm{P}(\text{weight of cooking apples} > 140) = 0.5\left(50\%\right)$ (seen anywhere) ***(A1)***

recognition of conditional probability in context ***(M1)***

$$\mathrm{P}\left(\text{eating apple}|\text{weight} > 140\right) = \frac{\mathrm{P}(\text{eating apple and weight of eating apple} > 140)}{\mathrm{P}(\text{weight of apple} > 140)}$$

OR

$$\frac{\mathrm{P}(\text{eating apple } \cap \text{ weight of eating apple} > 140)}{\mathrm{P}(\text{eating apple} \cap \text{weight of eating apple} > 140) + \mathrm{P}(\text{cooking apple } \cap \text{ weight of cooking apple} > 140)}$$

OR

$$\frac{\mathrm{P}\left(\text{eating apple}\right)\mathrm{P}(\text{weight of eating apple} > 140)}{\mathrm{P}\left(\text{eating apple}\right)\mathrm{P}\left(\text{weight of eating apple} > 140\right) + \mathrm{P}\left(\text{cooking apple}\right)P(\text{weight of cooking apple} > 140)}$$

$$= \frac{0.8\times 0.025}{0.8\times 0.025 + 0.2\times 0.5}\left(= \frac{80\times 2.5}{80\times 2.5 + 20\times 50}\right)$$ (***A1***)

$$= \frac{200}{1200}\left(= \frac{1}{6}\right)$$ ***A1***

Note: Accept any equivalent exact answer written as a fraction.

[4 marks]

Total [5 marks]

7. (a) $\alpha+\beta+\gamma=\frac{7}{2}$ ***A1***

[1 mark]

(b) $p-3\mathrm{i}$ is also a root (seen anywhere) ***A1***

recognition of 5 roots and attempt to sum these roots ***(M1)***

$p+3\mathrm{i}+p-3\mathrm{i}+\frac{7}{2}$

$p+3\mathrm{i}+p-3\mathrm{i}+\frac{7}{2}=\frac{11}{2}$ ***A1***

$p=1$ ***AG***

[3 marks]

(c) (i) attempt to find product of 5 roots and equate to ± 10 ***(M1)***

$(1+3\mathrm{i})(1-3\mathrm{i})\frac{1}{2}\alpha\beta=10$

$\alpha\beta=2$ ***A1***

(ii) $\alpha=1$ and $\beta=2$ ***A1***

[3 marks]

Total [7 marks]

8.

Note: To award full marks limit notation $\lim\limits_{x \to 0}$ must be seen at least once in their working. If no limit notation is seen but otherwise all correct, do not award the final ***A1.***

$$\lim_{x \to 0} \frac{4\sec^4 x \tan x + 2\sin x \cos x}{4x^3 - 2x}$$ ***A1A1***

Note: Award ***A1*** for numerator and ***A1*** for denominator.

$$= \lim_{x \to 0} \frac{16\sec^4 x \tan^2 x + 4\sec^6 x - 2\sin^2 x + 2\cos^2 x}{12x^2 - 2}$$ ***M1A1A1***

Note: Award ***M1 for second use of*** l'Hôpital's rule providing their expression is in indeterminate form as $x \to 0$ and providing there is **no third attempt** at using l'Hôpital's Rule.

$$= -3$$ ***A1***

[6 marks]

9. (a) ${}^{n}C_{3}$ ***A1***

[1 mark]

(b) **EITHER**

finding the number of ways to assign the students with the two students apart

number of ways to assign two students ${}^{2}C_{1}$ (seen anywhere) ***(A1)***

number of ways to assign others ${}^{n-2}C_{2}$ to have one group of 3 (seen anywhere) ***(A1)***

number of ways = ${}^{2}C_{1} \times {}^{n-2}C_{2}$

attempt to set up an equation involving either half of their answer to part (a) and their number of ways or their answer to part (a) is twice their number of ways ***M1***

$\frac{1}{2}\,{}^{n}C_{3} = {}^{2}C_{1} \times {}^{n-2}C_{2}$ OR ${}^{n}C_{3} = 2 \times {}^{2}C_{1} \times {}^{n-2}C_{2}$

valid attempt to eliminate all factorials from their equation ***(M1)***

$\frac{n(n-1)(n-2)}{3 \times 2} = 2 \times 2 \times \frac{(n-2)(n-3)}{2}$ or equivalent with no factorials

$n(n-1) = 12(n-3)$

continued…

Question 9 continued.

OR

finding the number of ways to assign the students with the two students together

number of ways to assign two students and one other to the first group ${}^{n-2}C_1$ (seen anywhere) ***(A1)***

number of ways to assign three other students the first group ${}^{n-2}C_3$ (seen anywhere) ***(A1)***

number of ways $= {}^{n-2}C_1 + {}^{n-2}C_3$

attempt to set up an equation involving either half of their answer to part (a) and their number of ways or their answer to part (a) is twice their number of ways ***M1***

$\frac{1}{2}\,{}^{n}C_3 = {}^{n-2}C_1 + {}^{n-2}C_3$ OR ${}^{n}C_3 = 2\left({}^{n-2}C_1 + {}^{n-2}C_3\right)$

valid attempt to eliminate all factorials from their equation ***(M1)***

$$\frac{n(n-1)(n-2)}{3\times 2} = 2\times(n-2) + 2\times\frac{(n-2)(n-3)(n-4)}{3\times 2}$$

$n(n-1) = 12 + 2(n-3)(n-4)$

THEN

$n^2 - 13n + 36 = 0$ ***A1***

$(n-9)(n-4) = 0$

$n = 9$ ***A1***

Note: Do not award the final ***A1*** if additional values of n are given.

[6 marks]

Total [7 marks]

SECTION B

10. (a) attempt to find a difference ***(M1)***

$d = p - a, 2d = q - a, d = q - p$ OR $p = a + d, q = a + 2d, q = p + d$

correct equation ***A1***

$p - a = q - p$ OR $q - a = 2(p - a)$ OR $p = \frac{a+q}{2}$ (or equivalent)

$2p - q = a$ ***AG***

[2 marks]

(b) attempt to find a ratio ***(M1)***

$r = \frac{s}{a}, r^2 = \frac{t}{a}, \quad r = \frac{t}{s}$ OR $s = ar, t = ar^2, t = sr$

correct equation ***A1***

$\left(\frac{s}{a}\right)^2 = \frac{t}{a}$ OR $\frac{s}{a} = \frac{t}{s}$ (or equivalent)

$s^2 = at$ ***AG***

[2 marks]

continued…

Question 10 continued

(c) **EITHER**

$2p-1=s^2$ (or equivalent) ***A1***

$\left(s^2>0\right) \Rightarrow\ 2p-1>0$ OR $s=\sqrt{2p-1} \Rightarrow 2p-1>0$ OR $p=\frac{s^2+1}{2}$ (and $s^2>0$) ***R1***

OR

$2p-1=a$ and $s^2=a$ ***A1***

($s^2>0$, so) $a>0\ \Rightarrow\ 2p-1>0$ OR $p=\frac{a+1}{2}$ and $a>0$ ***R1***

$\Rightarrow p>\frac{1}{2}$ ***AG***

Note: Do not award ***A0R1***.

[2 marks]

continued…

Question 10 continued

(d) (i) $9,5,1,-3$ ***A1A1***

Note: Award ***A1*** for each of 2nd term and 4th term

(ii) $9,3,1,\frac{1}{3}$ ***A1A1***

Note: Award ***A1*** for each of 2nd term and 4th term

[4 marks]

(e) (i) attempt to find the difference between two consecutive terms ***(M1)***

$d = u_2 - u_1 = 5 + \ln 3 - 9 - \ln 9$ OR $d = u_3 - u_2 = 1 + \ln 1 - 5 - \ln 3$

$\ln 9 = 2\ln 3$ OR $\ln 1 = 0$ OR $\ln 3 - \ln 9 = \ln\frac{1}{3}\left(= \ln 3^{-1} = -\ln 3\right)$ (seen anywhere) ***(A1)***

$d = -4 - \ln 3$ ***A1***

continued…

Question 10 continued.

(ii) **METHOD 1**

attempt to substitute first term and their common difference into S_{10} ***(M1)***

$\frac{10}{2}\left(2(9+\ln 9)+9\left(-4-\ln 3\right)\right)$ OR $\frac{10}{2}\left(2(9+2\ln 3)+9\left(-4-\ln 3\right)\right)$ (or equivalent) ***A1***

$=5\left(-18-5\ln 3\right)$ (or equivalent in terms of $\ln 3$) ***A1***

$\sum_{i=1}^{10} u_i = -90-25\ln 3$ ***AG***

METHOD 2

$u_{10} = 9+\ln 9+9\left(-4-\ln 3\right)\left(=-27+\ln 9-9\ln 3\right)$

attempt to substitute first term and their u_{10} into S_{10} ***(M1)***

$\frac{10}{2}\left(2(9+\ln 9)+9\left(-4-\ln 3\right)\right)$ OR $\frac{10}{2}\left(9+\ln 9-27+\ln 9-9\ln 3\right)$ OR

$\frac{10}{2}\left(2(9+2\ln 3)+9\left(-4-\ln 3\right)\right)$ OR $\frac{10}{2}\left(9+\ln 9-27-7\ln 3\right)$ (or equivalent) ***A1***

$=5\left(-18-5\ln 3\right)$ (or equivalent in terms of $\ln 3$) ***A1***

$\sum_{i=1}^{10} u_i = -90-25\ln 3$ ***AG***

[6 marks]

Total [16 marks]

11.

(a) $2\times 2+6\times\frac{1}{2}-2\times 1=5$ ***A1***

[1 mark]

(b) $2\left(k^2-6\right)+6\left(2k+3\right)+12\left(=0\right)$ ***(A1)***

equating their scalar product of the direction normals to zero ***(M1)***

$2\left(k^2-6\right)+6\left(2k+3\right)+12=0$

$k^2-6+6k+9+6=0$ OR $\left(k+3\right)^2=0$

$k=-3$ ***A1***

attempt to substitute k, p and coordinates of A into Π_2 ***(M1)***

$q=3\times 2-3\times\frac{1}{2}-6\times 1$

$q=-\frac{3}{2}$ ***A1***

[5 marks]

continued…

Question 11 continued.

(c) attempt to equate a pair of ratios or equate vector product to zero vector ***(M1)***

$$\begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix} = \mu \begin{pmatrix} 3 \\ 9 \\ p \end{pmatrix} \Rightarrow \mu = \frac{2}{3} \text{ OR } \frac{2}{-2} = \frac{3}{p} \text{ OR } \frac{6}{-2} = \frac{9}{p} \text{ OR } 6p + 18 = 0 \text{ OR } -6 - 2p = 0$$

$p = -3$ ***A1***

[2 marks]

(d) (i) attempt to find the vector equation of the line through A perpendicular to Π_1 ***(M1)***

$$(\boldsymbol{r} =) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ \frac{1}{2} \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix}$$

attempt to substitute their vector equation into Π_2 ***(M1)***

$$3(2 + 2\lambda) + 9\left(\frac{1}{2} + 6\lambda\right) - 3(1 - 2\lambda) \left(= -\frac{51}{2}\right)$$

$$6 + 6\lambda + \frac{9}{2} + 54\lambda - 3 + 6\lambda = -\frac{51}{2}$$ (or equivalent) ***(A1)***

$$66\lambda = -\frac{51}{2} - 6 - \frac{9}{2} + 3$$ (or equivalent)

$$\lambda = -\frac{1}{2}$$ ***A1***

$$\left(1, -\frac{5}{2}, 2\right)$$ ***A1***

continued…

Question 11 continued.

(ii) distance AB or $\left|\lambda\begin{pmatrix}2\\6\\-2\end{pmatrix}\right|$ ***(M1)***

$\sqrt{(2-1)^2+\left(\frac{1}{2}+\frac{5}{2}\right)^2+(1-2)^2}$ OR $\sqrt{1+9+1}$ ***A1***

$=\sqrt{11}$ ***AG***

[7 marks]

(e) Valid method to find a point C on Π_3 using $\overrightarrow{\mathrm{AC}}=\overrightarrow{\mathrm{BA}}$ or $\overrightarrow{\mathrm{BC}}=2\overrightarrow{\mathrm{BA}}$ or A as the midpoint of BC. ***(M1)***

$\lambda=\frac{1}{2}$, $\overrightarrow{\mathrm{OC}}=\begin{pmatrix}2\\\frac{1}{2}\\1\end{pmatrix}+\begin{pmatrix}1\\3\\-1\end{pmatrix},\overrightarrow{\mathrm{OC}}=\begin{pmatrix}1\\-\frac{5}{2}\\2\end{pmatrix}+\begin{pmatrix}2\\6\\-2\end{pmatrix},\overrightarrow{\mathrm{OA}}=\frac{1}{2}\left(\overrightarrow{\mathrm{OB}}+\overrightarrow{\mathrm{OC}}\right)$

point on Π_3 is $\left(3,\frac{7}{2},0\right)$ ***A1***

attempt to substitute their $\left(3,\frac{7}{2},0\right)$ into $\Pi_3: x+3y-z=d$ (or equivalent) ***(M1)***

$1\times3+3\times\frac{7}{2}-1\times0\left(=\frac{27}{2}\right)$

$\Pi_3: x+3y-z=\frac{27}{2}$ $(2x+6y-2z=27)$ (or equivalent) ***A1***

[4 marks]

Total [19 marks]

12. (a) $n=1$: $LHS = f^{(1)}(x) = -\frac{1}{2} \times -a(1-ax)^{-\frac{3}{2}} \left(= \frac{a}{2}(1-ax)^{-\frac{3}{2}}\right)$ ***A1***

$RHS = \frac{a(1)!(1-ax)^{-\frac{3}{2}}}{2^1(0)!}$ therefore true for $n=1$ ***R1***

assume (that the result is) true for $n=k$ ***M1***

Note: Do not award ***M1*** for statements such as "let $n=k$" or "assume that $n=k$ is true". The assumption of truth must be clear.

$$f^{(k)}(x) = \frac{a^k(2k-1)!(1-ax)^{-\frac{(2k+1)}{2}}}{2^{2k-1}(k-1)!}$$

attempt to differentiate the right-hand side with respect to x: ***M1***

$$f^{(k+1)}(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(f^{(k)}(x)\right)$$

$$= \frac{-(2k+1)\times -a}{2} \times \frac{a^k(2k-1)!(1-ax)^{-\frac{(2k+1)}{2}-1}}{2^{2k-1}(k-1)!}$$ (or equivalent) ***A1***

attempt to multiply top and bottom by $2k$ ***M1***

$$= \frac{(2k+1)}{2} \times \frac{a^{k+1}(2k-1)!(1-ax)^{-\frac{2k+3}{2}}}{2^{2k-1}(k-1)!} \times \frac{2k}{2k}$$

$$= \frac{a^{k+1}(2k+1)!(1-ax)^{-\frac{2k+3}{2}}}{2^{2k+1}(k)!}$$ ***A1***

hence if the result is true for $n=k$ then it is true for $n=k+1$ and as it is true for $n=1$ it is true for all $n \in \mathbb{Z}^+$ ***R1***

Note: To obtain the final ***R1***, at least five of the previous marks must have been awarded.

[8 marks]

continued…

Question 12 continued.

(b) $f(x) = f(0) + xf'(0) + \frac{x^2}{2} f''(0) + \ldots$

$f''(x) = \frac{a^2 (3)!(1-ax)^{-\frac{5}{2}}}{2^3 (1)!}$ OR $\frac{3}{4} a^2 (1-ax)^{-\frac{5}{2}}$ OR $f''(0) = \frac{a^2 (3)!}{2^3}$ ***A1***

$f(0) = 1$, $f'(0) = \frac{a}{2}$, $f''(0) = \frac{6a^2}{8}$ ***A1***

$f(x) = 1 + \frac{1}{2} ax + \frac{3}{8} a^2 x^2$ + …. ***AG***

[2 marks]

continued…

Question 12 continued.

(c) attempt to use $a=2$ or $a=4$ in the expansion ***M1***

$$(1-2x)^{-\frac{1}{2}}=1+\frac{2x}{2}+\frac{3\times4x^2}{8}\left(=1+x+\frac{3}{2}x^2+\ldots\right)$$

$$(1-4x)^{-\frac{1}{2}}=1+\frac{4x}{2}+\frac{3\times16x^2}{8}\left(=1+2x+6x^2+\ldots\right)$$ ***A1***

Note: Award ***A1*** for at least one correct.

attempt to multiply their two expansions together ***M1***

$$\left(1+x+\frac{3}{2}x^2+\ldots\right)\left(1+2x+6x^2+\ldots\right)=1+2x+6x^2+x+2x^2+\frac{3}{2}x^2+...$$

$$=1+3x+\frac{19}{2}x^2+\ldots \text{ OR } \frac{2+4x+12x^2+2x+4x^2+3x^2+...}{2}$$ ***A1***

$$(1-2x)^{-\frac{1}{2}}(1-4x)^{-\frac{1}{2}}\approx\frac{2+6x+19x^2}{2}$$ ***AG***

[4 marks]

(d) $|x|<\frac{1}{4}$ ***A1***

[1 mark]

continued…

Question 12 continued.

(e)

$$\left(\frac{2+6x+19x^2}{2}\right)=\frac{2+0.6+0.19}{2}\left(=\frac{279}{200}\right)$$ (or equivalent) ***A1***

attempt to substitute $x=\frac{1}{10}$ into $(1-2x)^{-\frac{1}{2}}(1-4x)^{-\frac{1}{2}}$ ***(M1)***

$$\left((1-2x)^{-\frac{1}{2}}(1-4x)^{-\frac{1}{2}}=\right)\frac{10}{\sqrt{48}}$$ (or equivalent) ***A1***

$$\frac{10}{4\sqrt{3}}\approx\frac{279}{200}$$ (or equivalent in terms of $\sqrt{3}$) ***A1***

$$\frac{1}{\sqrt{3}}\approx\frac{279}{500}$$

$$\sqrt{3}\approx\frac{500}{279}$$ ***A1***

[5 marks]

Total [20 marks]