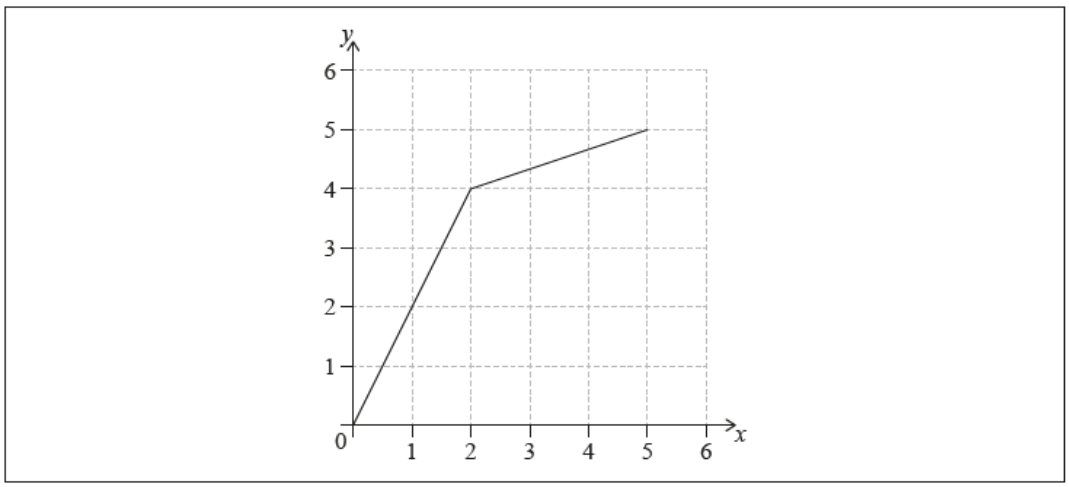
Functions revision [67 marks]

1. [Maximum mark: 7] 23N.1.AHL.TZ0.2

The graph of the function f is given in the following diagram.



(a) Write down $f(2)$. [1]

(b) On the axes, sketch $y = f^{-1}(x)$. [2]

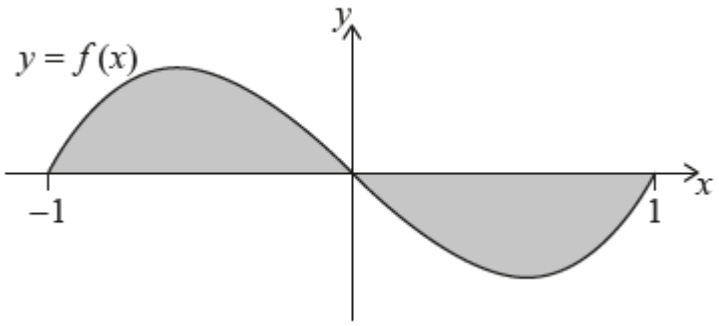
The function g is defined as $g(x) = 3x - 1$.

(c) Find an expression for $g^{-1}(x)$ [2]

(d) Find a value of x where $f^{-1}(x) = g^{-1}(x)$. [2]

2. [Maximum mark: 7] 23N.1.AHL.TZ0.11

Consider the function $f(x) = x^3 - x$, for $-1 \leq x \leq 1$. The shaded region, R, is bounded by the graph of $y = f(x)$ and the x-axis.

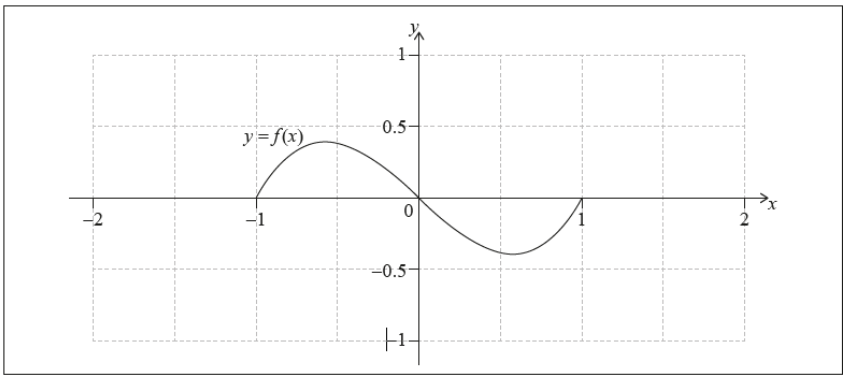


(a.i) Write down an integral that represents the area of R. [1]

(a.ii) Find the area of R. [1]

Another function, g, is defined such that $g(x) = 2f(x - 1)$.

(b) On the following set of axes, the graph of $y = f(x)$ has been drawn. On the same set of axes, sketch the graph of $y = g(x)$.



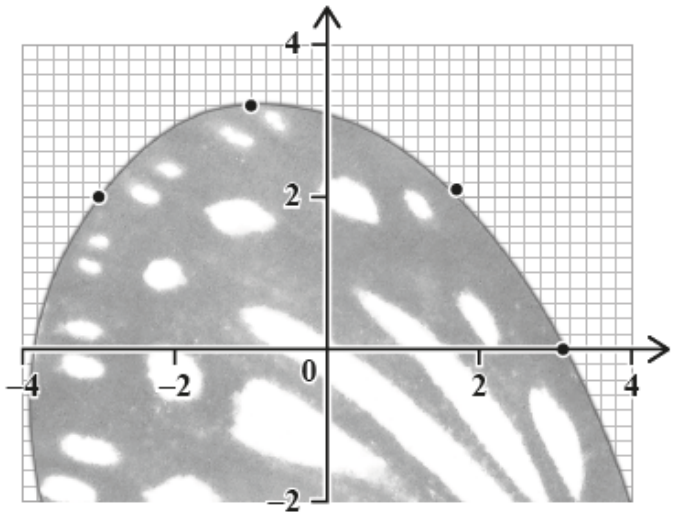
[2]

The region R from the original graph $y = f(x)$ is rotated through 2π radians about the x-axis to form a solid.

(c) Find the volume of the solid. [3]

3. [Maximum mark: 5] 22N.1.AHL.TZ0.11

Gloria wants to model the curved edge of a butterfly wing. She inserts a photo of the wing into her graphing software and finds the coordinates of four points on the edge of the wing.



x	y
−3	2
−1	3.2
1.7	2.1
3.1	0

Gloria thinks a cubic curve will be a good model for the butterfly wing.

[Source: Fleur, 2019. photo-1560263816-d704d83cce0f. [image online] Available at: https://unsplash.com/photos/SE2zTdS1MNo [Accessed 8 February 2022]. Source adapted.]

(a) Find the equation of the cubic regression curve for this data. [2]

For the photo of a second butterfly wing, Gloria finds the equation of the regression curve is $y = 0.0083x^3 - 0.075x^2 - 0.58x + 2.2$.

Gloria realizes that her photo of the second butterfly is an enlargement of the life-size butterfly, scale factor 2 and centred on $(0,\ 0)$.

(b) Find the equation of the cubic curve that models the life-size wing. [3]

4. [Maximum mark: 5] 22M.1.AHL.TZ2.10

The function $f(x) = \ln\left(\frac{1}{x-2}\right)$ is defined for $x > 2,\ x \in \mathbb{R}$.

(a) Find an expression for $f^{-1}(x)$. You are not required to state a domain.

[3]

(b) Solve $f(x) = f^{-1}(x)$. [2]

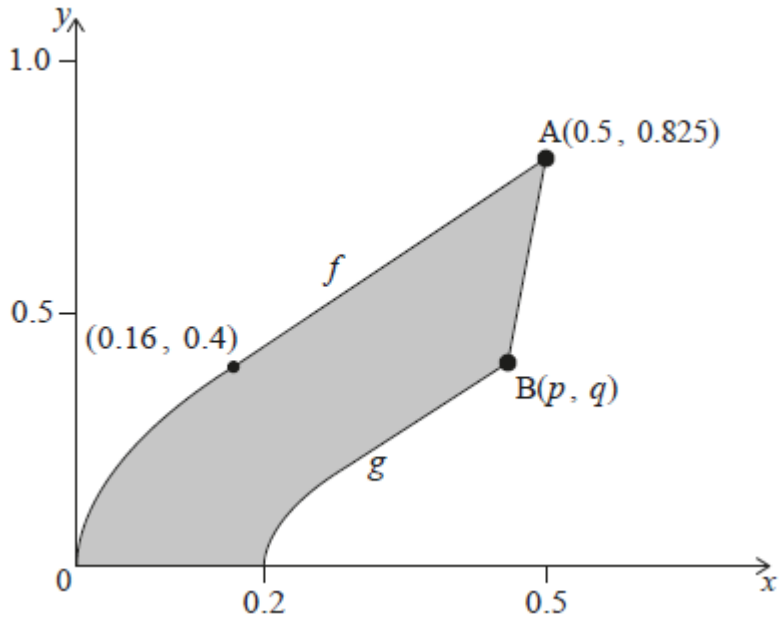
5. [Maximum mark: 18] 22M.2.AHL.TZ1.6

Consider the curve $y = \sqrt{x}$.

(a.i) Find $\frac{dy}{dx}$. [2]

(a.ii) Hence show that the equation of the tangent to the curve at the point $(0.16,\ 0.4)$ is $y = 1.25x + 0.2$. [2]

The shape of a piece of metal can be modelled by the region bounded by the functions f, g, the x-axis and the line segment $[AB]$, as shown in the following diagram. The units on the x and y axes are measured in metres.



The piecewise function f is defined by

$$f(x) = \begin{cases} \sqrt{x} & 0 \leq x \leq 0.16 \\ 1.25x + 0.2 & 0.16 < x \leq 0.5 \end{cases}$$

The graph of g is obtained from the graph of f by:

- a stretch scale factor of $\frac{1}{2}$ in the x direction,
- followed by a stretch scale factor $\frac{1}{2}$ in the y direction,
- followed by a translation of 0.2 units to the right.

Point A lies on the graph of f and has coordinates $(0.5,\ 0.825)$. Point B is the image of A under the given transformations and has coordinates $(p,\ q)$.

(b) Find the value of p and the value of q. [2]

The piecewise function g is given by

$$g(x) = \begin{cases} h(x) & 0.2 \leq x \leq a \\ 1.25x + b & a < x \leq p \end{cases}$$

(c.i) Find an expression for $h(x)$. [2]

(c.ii) Find the value of a. [1]

(c.iii) Find the value of b. [2]

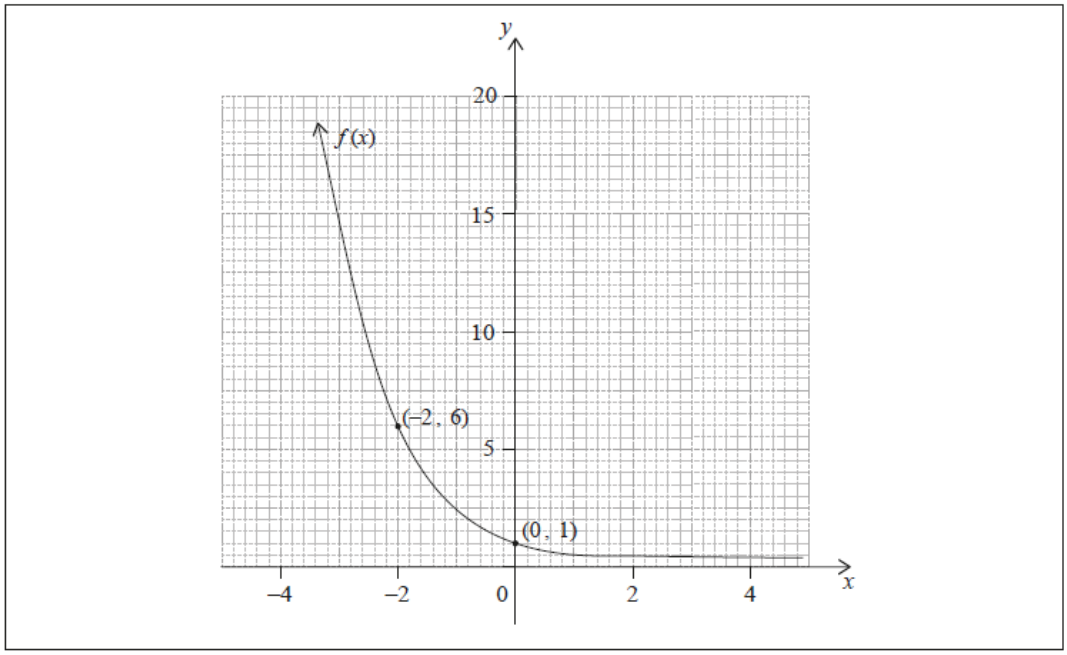
(d.i) Find the area enclosed by $y = f(x)$, the x-axis and the line $x = 0.5$. [3]

The area enclosed by $y = g(x)$, the x-axis and the line $x = p$ is $0.0627292\,\mathrm{m}^2$ correct to six significant figures.

(d.ii) Find the area of the shaded region on the diagram. [4]

6. [Maximum mark: 4] 21N.1.AHL.TZ0.10

The graph of $y = f(x)$ is given on the following set of axes. The graph passes through the points $(-2,\ 6)$ and $(0,\ 1)$, and has a horizontal asymptote at $y = 0$.



Let $g(x) = 2f(x-2) + 4$.

(a) Find $g(0)$. [2]

(b) On the same set of axes draw the graph of $y = g(x)$, showing any intercepts and asymptotes. [2]

7. [Maximum mark: 7] 21M.1.AHL.TZ1.17

The graph of the function $f(x) = \ln x$ is translated by $\begin{pmatrix} a \\ b \end{pmatrix}$ so that it then passes through the points $(0,\ 1)$ and $\left(e^3,\ 1 + \ln 2\right)$.

Find the value of a and the value of b. [7]

8. [Maximum mark: 7] 21M.1.AHL.TZ2.2

A function is defined by $f(x) = 2 - \frac{12}{x+5}$ for $-7 \leq x \leq 7,\ x \neq -5$.

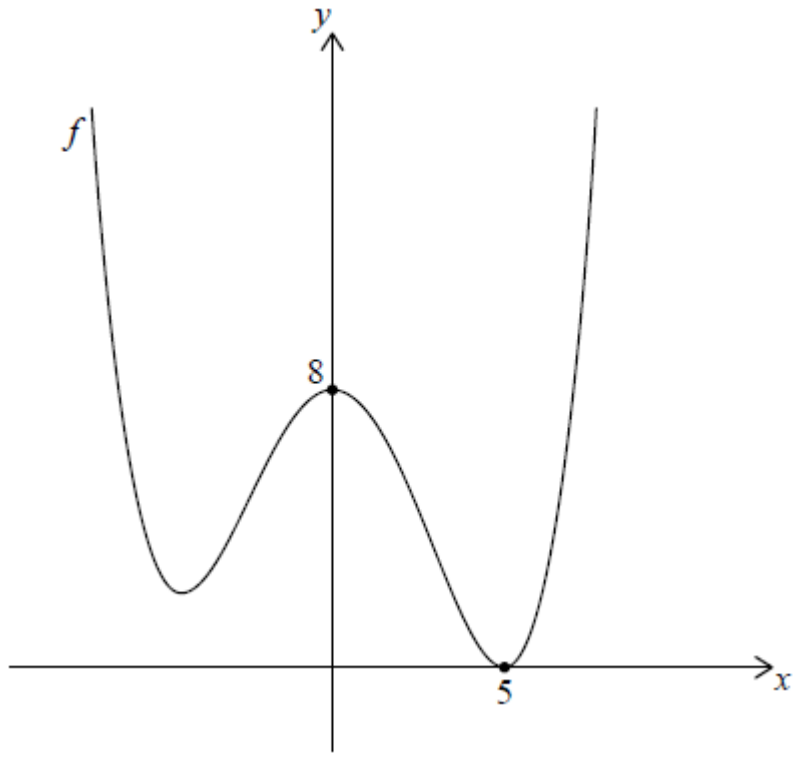
(a) Find the range of f. [3]

(b) Find an expression for the inverse function $f^{-1}(x)$. The domain is not required. [3]

(c) Write down the range of $f^{-1}(x)$. [1]

9. [Maximum mark: 7] 19M.1.SL.TZ2.S_4

The following diagram shows part of the graph of f with x-intercept (5, 0) and y-intercept (0, 8).



(a.i) Find the y-intercept of the graph of $f(x) + 3$. [1]

(a.ii) Find the y-intercept of the graph of $f(4x)$. [2]

(b) Find the x-intercept of the graph of $f(2x)$. [2]

(c) Describe the transformation $f(x + 1)$. [2]