Logarithms - revision [81 marks]

1. [Maximum mark: 6] EXN.1.SL.TZ0.5

The pH of a solution is given by the formula $pH = -\log_{10} C$ where C is the hydrogen ion concentration in a solution, measured in moles per litre (Ml^{-1}).

(a) Find the pH value for a solution in which the hydrogen ion concentration is 5.2×10^{-8}. [2]

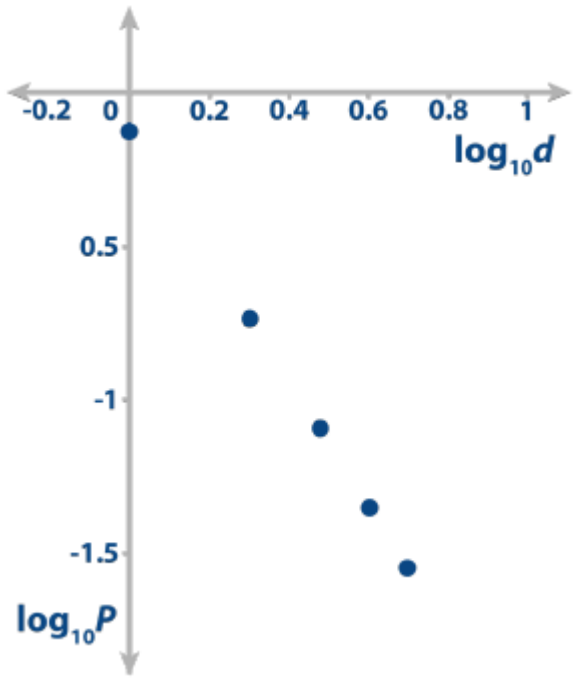
(b.i) Write an expression for C in terms of pH. [2]

(b.ii) Find the hydrogen ion concentration in a solution with pH 4.2. Give your answer in the form $a \times 10k$ where $1 \leq a < 10$ and k is an integer. [2]

2. [Maximum mark: 7] EXN.1.AHL.TZ0.12

It is believed that the power P of a signal at a point d km from an antenna is inversely proportional to d^n where $n \in \mathbb{Z}^+$.

The value of P is recorded at distances of $1\,\mathrm{m}$ to $5\,\mathrm{m}$ and the values of $\log_{10} d$ and $\log_{10} P$ are plotted on the graph below.



(a) Explain why this graph indicates that P is inversely proportional to d^n.

[2]

The values of $\log_{10} d$ and $\log_{10} P$ are shown in the table below.

$\log_{10} d$	0	0.301	0.477	0.602	0.699
$\log_{10} P$	-0.127	-0.740	-1.10	-1.36	-1.55

(b) Find the equation of the least squares regression line of $\log_{10} P$ against $\log_{10} d$.

[2]

(c.i) Use your answer to part (b) to write down the value of n to the nearest integer. [1]

(c.ii) Find an expression for P in terms of d. [2]

3. [Maximum mark: 7] EXM.1.AHL.TZ0.14

It is believed that two variables, v and w are related by the equation $v = kw^n$, where $k,\ n \in \mathbb{R}$. Experimental values of v and w are obtained. A graph of $\ln v$ against $\ln w$ shows a straight line passing through (−1.7, 4.3) and (7.1, 17.5).

Find the value of k and of n. [7]

4. [Maximum mark: 10] EXM.1.AHL.TZ0.15

Adesh wants to model the cooling of a metal rod. He heats the rod and records its temperature as it cools.

Time, t (seconds)	0	30	60	90	120	150
Temperature, T (°C)	75.6	62.2	53.3	47.4	42.3	38.5

He believes the temperature can be modeled by $T(t) = a\mathrm{e}^{bt} + 25$, where $a,\ b \in \mathbb{R}$.

(a) Show that $\ln(T - 25) = bt + \ln a$. [2]

(b) Find the equation of the regression line of $\ln(T - 25)$ on t. [3]

Hence

(c.i) find the value of a and of b. [3]

(c.ii) predict the temperature of the metal rod after 3 minutes. [2]

5. [Maximum mark: 5] EXM.1.AHL.TZ0.13

It is believed that two variables, m and p are related. Experimental values of m and p are obtained. A graph of $\ln m$ against p shows a straight line passing through (2.1, 7.3) and (5.6, 2.4).

(a) Find the equation of the straight line, giving your answer in the form $\ln m = ap + b$, where $a,\ b \in \mathbb{R}$. [3]

Hence, find

(b.i) a formula for m in terms of p. [1]

(b.ii) the value of m when $p = 0$. [1]

6. [Maximum mark: 7] 24M.1.SL.TZ2.7

The pH scale is a measure of the acidity of a solution. Its value is given by the formula

$$\text{pH} = -\log_{10}\left[\text{H}^+\right],$$

where $\left[\text{H}^+\right]$ is the concentration of hydrogen ions in the solution (measured in moles per litre).

(a) Calculate the pH value if the concentration of hydrogen ions is 0.0003. [2]

The pH of milk is 6.6.

(b) Calculate the concentration of hydrogen ions in milk. [2]

The strength of an acid is measured by its concentration of hydrogen ions.

A lemon has a pH value of 2 and a tomato has a pH value of 4.5.

(c) Calculate how many times stronger the acid in a lemon is when compared to the acid in a tomato. [3]

7. [Maximum mark: 7] 24M.1.AHL.TZ1.9

(a) Find $\int \frac{8}{2x+3}\,\mathrm{dx}$. [3]

(b) Hence find the exact area between the curve $y = \frac{8}{2x+3}$, the x-axis and the lines $x = 0$ and $x = 6$. Give your answer in the form $a \ln b$, where $a,\ b \in \mathbb{N}$. [4]

8. [Maximum mark: 7] 23N.1.AHL.TZ0.10

The decay of a chemical isotope over five years is recorded in **Table 1**. The mass of the chemical M is measured to the nearest gram at the beginning of each year t of the experiment.

Table 1

Time t (years)	1	2	3	4	5
Mass M (grams)	1000	660	517	435	381

It is believed that the decay of the isotope can be modelled by an equation of the form $M = a \times t^b$.

(a) Use power regression on your graphic display calculator to find the value of a and the value of b. [2]

The values of t and M can be transformed such that $x = \ln\ t$ and $y = \ln\ M$. **Table 2** shows data for x and y to three decimal places.

Table 2

x	0	0.693	1.099	1.386	1.609
y	6.908	6.492	6.248	6.075	5.943

(b) Find the linear regression equation of y on x, in the form $y = cx + d$. Give the values of c and d to three decimal

places. [2]

(c) Hence, show that this linear regression is equivalent to the power regression found in part (a). [3]

9. [Maximum mark: 7] 23M.1.SL.TZ1.8

"Password entropy" is a measure of the predictability of a computer password. The higher the entropy, the more difficult it is to guess the password.

The relationship between the password entropy, p, (measured in bits) and the number of guesses, G, required to decode the password is given by $0.301p = \log_{10} G$.

(a) Calculate the value of p for a password that takes 5000 guesses to decode. [2]

(b) Write down G as a function of p. [1]

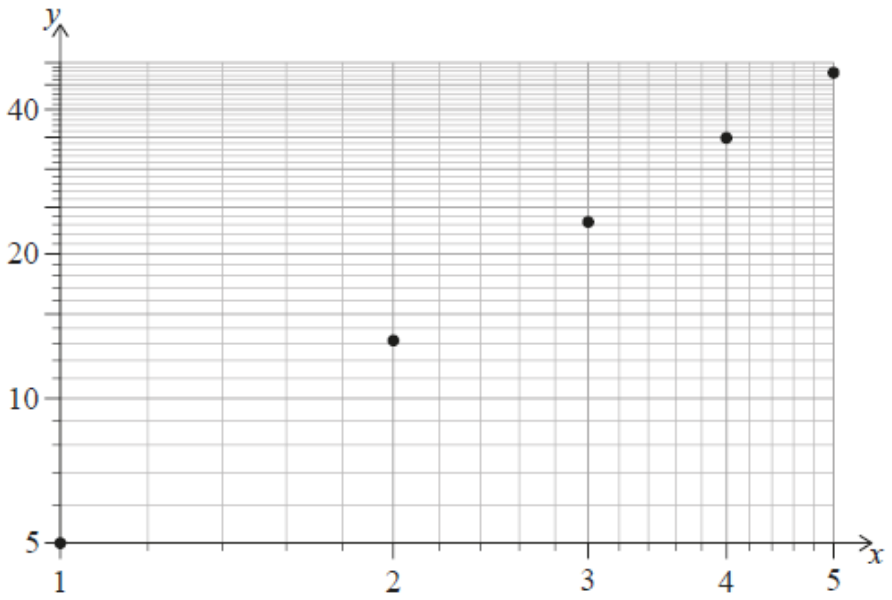
(c) Find the number of guesses required to decode a password that has an entropy of 28 bits. Write your answer in the form $a \times 10^k$, where $1 \leq a < 10, k \in \mathbb{Z}$. [3]

There is a point on the graph of the function $G(p)$ with coordinates $(0,\ 1)$.

(d) Explain what these coordinate values mean in the context of computer passwords. [1]

10. [Maximum mark: 6] 23M.1.AHL.TZ1.14

Petra examines two quantities, x and y, and plots data points on a log-log graph.



She observes that on this graph the data points follow a perfect straight line. Given that the line passes through the points $(2,\ 13.1951)$ and $(4,\ 34.822)$, find the equation of the relationship connecting x and y. Your final answer should not include logarithms. [6]

11. [Maximum mark: 6] 22N.1.SL.TZ0.10

Stars are classified by their brightness. The brightest stars in the sky have a magnitude of 1. The magnitude, m, of another star can be modelled as a function of its brightness, b, relative to a star of magnitude 1, as shown by the following equation.

$$m = 1 - 2.5 \log_{10}(b)$$

The star called Acubens has a brightness of 0.0525.

(a) Find the magnitude of Acubens. [2]

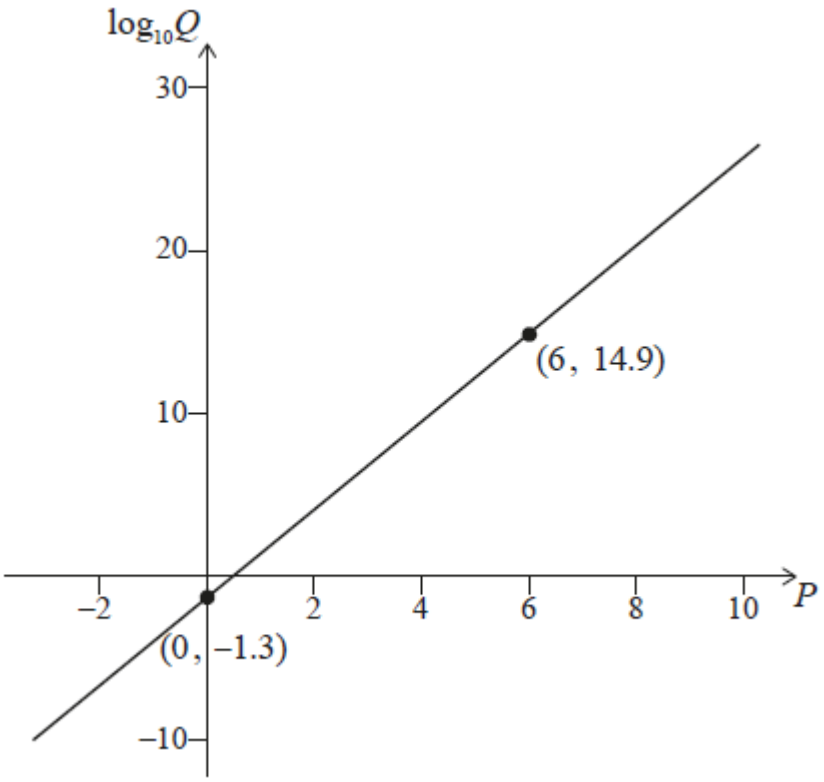
Ceres has a magnitude of 7 and is the least bright star visible without magnification.

(b) Find the brightness of Ceres. [2]

(c) Find how many times brighter Acubens is compared to Ceres. [2]

12. [Maximum mark: 6] 22N.1.AHL.TZ0.13

Gen is investigating the relationship between two sets of data, labelled P and Q, that she collected. She created a scatter plot with P on the x-axis and $\log_{10} Q$ on the y-axis. Gen noticed that the points had a strong linear correlation, so she drew a line of best fit, as shown in the diagram. The line passes through the points $(0,\ -1.3)$ and $(6,\ 14.9)$.



(a) Find an equation for Q in terms of P. [3]

Gen also investigates the relationship between the same data, Q, and some new data, R. She believes that the data can be modelled by $Q = a \ln R + b$ and she decides to create a scatter plot to verify her belief.

(b) State what expression Gen should plot on each axis to verify her belief. [1]

The scatter plot has a linear relationship and Gen finds $a = 4.3$ and $b = 12.1$.

(c) Find an equation for P in terms of R. [2]