Matrices revision [62 marks]

1. [Maximum mark: 4]

A and B are 2×2 matrices, where $A = \begin{bmatrix} 5 & 2 \\ 2 & 0 \end{bmatrix}$ and

$BA = \begin{bmatrix} 11 & 2 \\ 44 & 8 \end{bmatrix}$. Find B

[4]

2. [Maximum mark: 6]

Consider the matrix $A = \begin{pmatrix} e^x & e^{-x} \\ 2 + e^x & 1 \end{pmatrix}$, where $x \in \mathbb{R}$.

Find the value of x for which A is singular. [6]

3. [Maximum mark: 5]

If $A = \begin{pmatrix} 1 & 2 \\ k & -1 \end{pmatrix}$ and A^2 is a matrix whose entries are all 0, find k.

[5]

4. [Maximum mark: 6]

Given that $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$, find X if $BX = A - AB$.

[6]

5. [Maximum mark: 6]

The matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ -3 & 1 & -1 \\ 2 & -2 & 1 \end{pmatrix}$ has inverse $A^{-1} = \begin{pmatrix} -1 & -2 & -2 \\ 3 & 1 & 1 \\ a & 6 & b \end{pmatrix}$.

(a.i)   Write down the value of a.   [1]

(a.ii)   Write down the value of b.   [1]

Consider the simultaneous equations

$$x + 2y = 7$$

$$-3x + y - z = 10$$

$$2x - 2y + z = -12$$

(b)   Write these equations as a matrix equation.   [1]

(c)   Solve the matrix equation.   [3]

6.   [Maximum mark: 6]

Let $A = \begin{pmatrix} 1 & x & -1 \\ 3 & 1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 \\ x \\ 2 \end{pmatrix}$.

(a)   Find AB.   [3]

(b)   The matrix $C = \begin{pmatrix} 20 \\ 28 \end{pmatrix}$ and $2AB = C$. Find the value of x.   [3]

7.   [Maximum mark: 6]

(a)

Find the inverse of the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 4 \end{pmatrix}$.

[2]

(b)   **Hence** solve the system of equations

$$x + 2y + z = 0$$
$$x + y + 2z = 7$$
$$2x + y + z = 17$$

[4]

8.   [Maximum mark: 16]

The drivers of a delivery company can park their vans overnight either at its headquarters or at home.

Urvashi is a driver for the company. If Urvashi has parked her van overnight at headquarters on a given day, the probability that she parks her van at headquarters on the following day is 0.88. If Urvashi has parked her van overnight at her home on a given day, the probability that she parks her van at home on the following day is 0.92.

(a)   Write down a transition matrix, T, that shows the movement of Urvashi's van between headquarters and home.

[2]

On Monday **morning** she collected her van from headquarters where it was parked overnight.

(b)   Find the probability that Urvashi's van will be parked at home at the end of the week on Friday **evening**.

[3]

(c)   Write down the characteristic polynomial for the matrix T. Give your answer in the form $\lambda^2 + b\lambda + c$.

[2]

(d)   Calculate eigenvectors for the matrix T.

[4]

(e) Write down matrices P and D such that $T = PDP^{-1}$,
where D is a diagonal matrix. [2]

(f) Hence find the long-term probability that Urvashi's van is
parked at home. [3]

9. [Maximum mark: 7]

The eating habits of students in a school are studied over a number of months.
The focus of the study is whether non-vegetarians become vegetarians, and
whether vegetarians remain vegetarians.

Each month, students choose between the vegetarian or non-vegetarian lunch
options.
Once they have chosen for the month, they cannot change the option until the
next month.

In any month during the study, it is noticed that the probability of a non-
vegetarian becoming vegetarian the following month is 0.1, and that the
probability of a vegetarian remaining a vegetarian the following month is 0.8.

This situation can be represented by the transition matrix

$$T = \begin{pmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{pmatrix}.$$

(a) Interpret the value 0.9 in T in terms of the changes in the
eating habits of the students in the school. [1]

(b) Find the eigenvalues of matrix T. [3]

One of the eigenvectors of T is $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

(c) Find another, non-parallel, eigenvector and interpret it in
context.