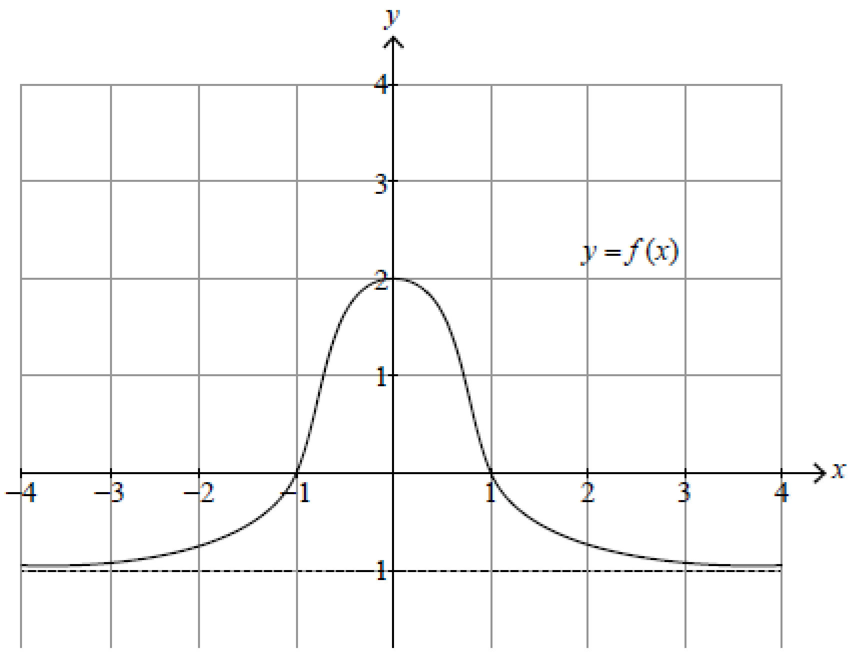
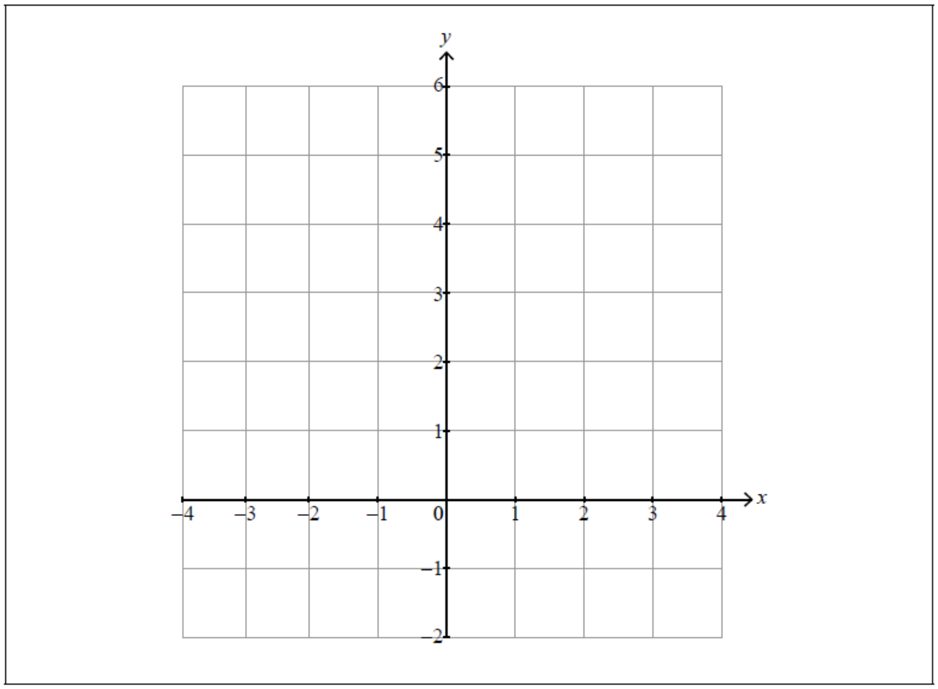
Revision set (Paper 1) [297 marks]

1. [Maximum mark: 5] SPM.1.AHL.TZ0.4

The following diagram shows the graph of $y = f(x)$. The graph has a horizontal asymptote at $y = -1$. The graph crosses the x-axis at $x = -1$ and $x = 1$, and the y-axis at $y = 2$.



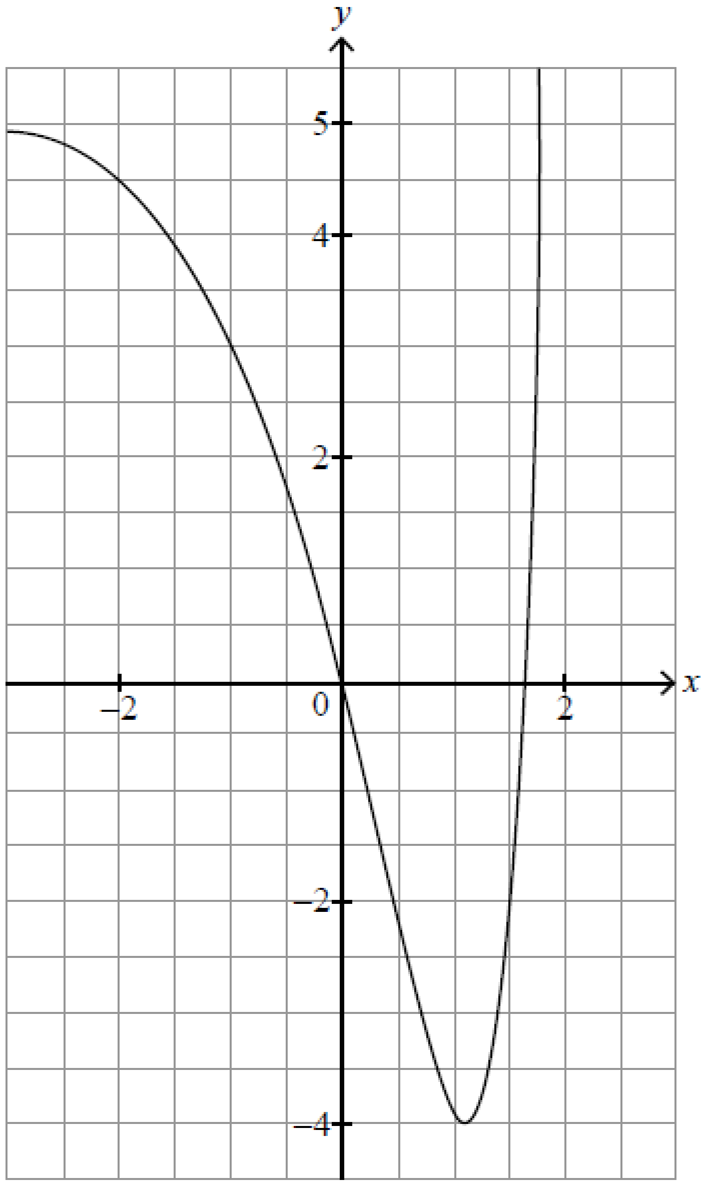
On the following set of axes, sketch the graph of $y = [f(x)]^2 + 1$, clearly showing any asymptotes with their equations and the coordinates of any local maxima or minima.



[5]

2. [Maximum mark: 8] SPM.1.AHL.TZ0.9

The function f is defined by $f\left(x\right) = \mathrm{e}^{2x} - 6\mathrm{e}^{x} + 5,\ x \in \mathbb{R},\ x \leqslant a$. The graph of $y = f\left(x\right)$ is shown in the following diagram.



(a) Find the largest value of a such that f has an inverse function. [3]

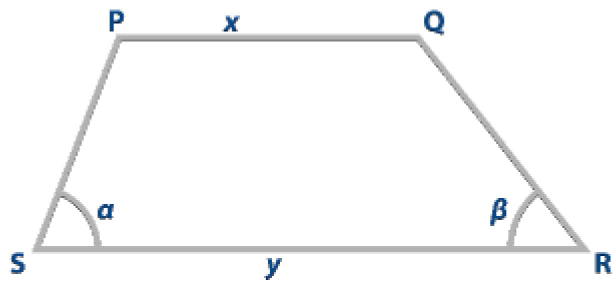
(b) For this value of a, find an expression for $f^{-1}(x)$, stating its domain. [5]

3. [Maximum mark: 5] EXN.1.AHL.TZ0.6

Use l'Hôpital's rule to determine the value of $\lim_{x \to 0}\left(\frac{2x\cos(x^2)}{5\tan x}\right)$. [5]

4. [Maximum mark: 5] EXN.1.AHL.TZ0.7

Consider quadrilateral PQRS where $[\mathrm{PQ}]$ is parallel to $[\mathrm{SR}]$.



In PQRS, $\mathrm{PQ} = x$, $\mathrm{SR} = y$, $\mathrm{R\hat{S}P} = \alpha$ and $\mathrm{Q\hat{R}S} = \beta$.

Find an expression for PS in terms of $x,\ y,\ \sin\beta$ and $\sin(\alpha+\beta)$. [5]

5. [Maximum mark: 21] EXN.1.AHL.TZ0.11

A function f is defined by $f(x) = \frac{3}{x^2+2},\ x \in \mathbb{R}$.

(a) Sketch the curve $y = f(x)$, clearly indicating any asymptotes with their equations and stating the coordinates of any points of intersection with the axes. [4]

The region R is bounded by the curve $y = f(x)$, the x-axis and the lines $x = 0$ and $x = \sqrt{6}$. Let A be the area of R.

(b) Show that $A = \frac{\sqrt{2}\pi}{2}$. [4]

The line $x = k$ divides R into two regions of equal area.

(c) Find the value of k. [4]

Let m be the gradient of a tangent to the curve $y = f(x)$.

(d) Show that $m = -\frac{6x}{(x^2+2)^2}$. [2]

(e) Show that the maximum value of m is $\frac{27}{32}\sqrt{\frac{2}{3}}$. [7]

6. [Maximum mark: 8] EXM.1.AHL.TZ0.1

Let $f(x) = \frac{1}{1-x^2}$ for $-1 < x < 1$. Use partial fractions to find $\int f(x)\, dx$. [8]

7. [Maximum mark: 9] EXM.1.AHL.TZ0.6

Let $f(x) = \frac{x^2-10x+5}{x+1}$, $x \in \mathbb{R}$, $x \neq -1$.

(a) Find the co-ordinates of all stationary points. [4]

(b) Write down the equation of the vertical asymptote. [1]

(c) With justification, state if each stationary point is a minimum, maximum or horizontal point of inflection. [4]

8. [Maximum mark: 11] EXM.1.AHL.TZ0.2

Consider the integral $\int_1^t \frac{-1}{x+x^2}\, dx$ for $t > 1$.

(a) Very briefly, explain why the value of this integral must be negative. [1]

(b) Express the function $f(x) = \frac{-1}{x+x^2}$ in partial fractions. [6]

(c) Use parts (a) and (b) to show that $\ln(1+t) - \ln t < \ln 2$. [4]

9. [Maximum mark: 8] EXM.1.AHL.TZ0.4

Let $f(x) = \frac{2x+6}{x^2+6x+10}$, $x \in \mathbb{R}$.

(a) Show that $f(x)$ has no vertical asymptotes. [3]

(b) Find the equation of the horizontal asymptote. [2]

(c) Find the exact value of $\int_0^1 f(x)\, dx$, giving the answer in the form $\ln q,\ q \in \mathbb{Q}$. [3]

10. [Maximum mark: 9] EXM.1.AHL.TZ0.5

Let $f(x) = \frac{2x^2-5x-12}{x+2}$, $x \in \mathbb{R},\ x \neq -2$.

(a) Find all the intercepts of the graph of $f(x)$ with both the x and y axes. [4]

(b) Write down the equation of the vertical asymptote. [1]

(c) As $x \to \pm\infty$ the graph of $f(x)$ approaches an oblique straight line asymptote.

Divide $2x^2 - 5x - 12$ by $x + 2$ to find the equation of this asymptote. [4]

11. [Maximum mark: 5] 24M.1.AHL.TZ1.2

It is given that $\log_{10} a = \frac{1}{3}$, where $a > 0$.

Find the value of

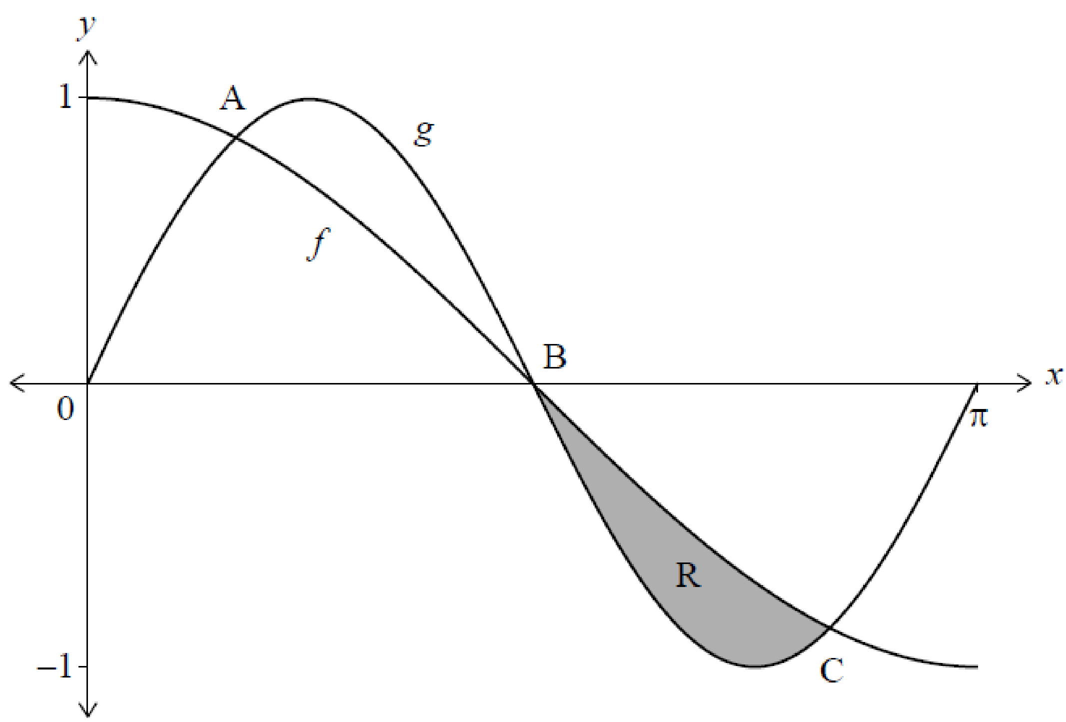
(a) $\log_{10}\left(\frac{1}{a}\right)$; [2]

(b) $\log_{1000} a$. [3]

12. [Maximum mark: 7] 24M.1.AHL.TZ1.4

Consider the functions $f(x) = \cos x$ and $g(x) = \sin 2x$, where $0 \leq x \leq \pi$.

The graph of f intersects the graph of g at the point A, the point $\mathrm{B}\left(\frac{\pi}{2}, 0\right)$ and the point C as shown on the following diagram.



(a) Find the x-coordinate of point A and the x-coordinate of point C. [3]

The shaded region R is enclosed by the graph of f and the graph of g between the points B and C.

(b) Find the area of R. [4]

13. [Maximum mark: 5] 24M.1.AHL.TZ1.5

Consider a geometric sequence with first term 1 and common ratio 10.

S_n is the sum of the first n terms of the sequence.

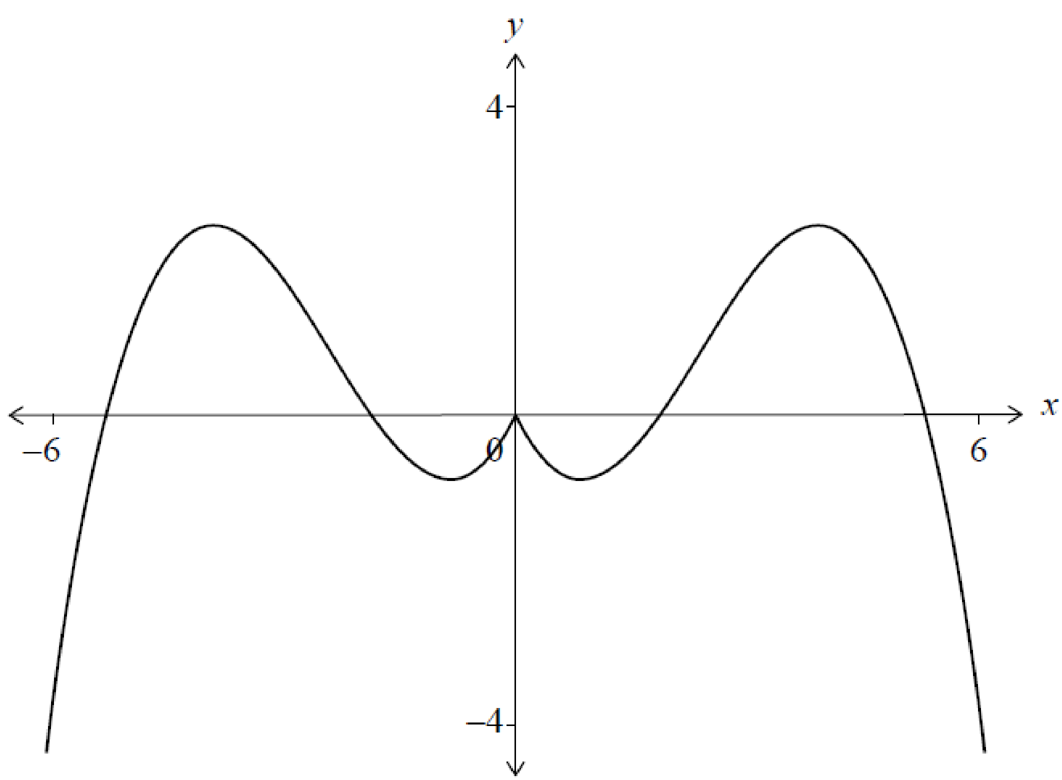
(a) Find an expression for S_n in the form $\frac{a^n-1}{b}$, where $a,\ b \in \mathbb{Z}^+$. [1]

(b) Hence, show that

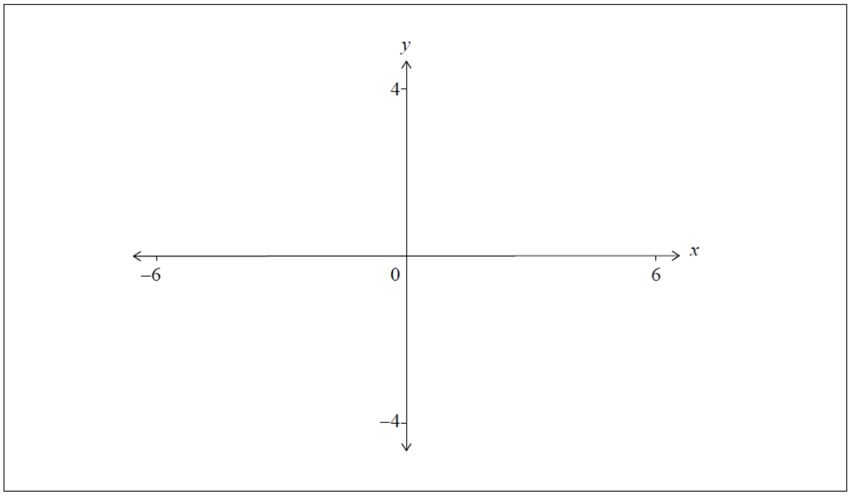
$S_1 + S_2 + S_3 + \ldots + S_n = \frac{10(10^n-1)-9n}{81}$. [4]

14. [Maximum mark: 6] 24M.1.AHL.TZ1.9

The graph of $y = f(|x|)$ for $-6 \leq x \leq 6$ is shown in the following diagram.



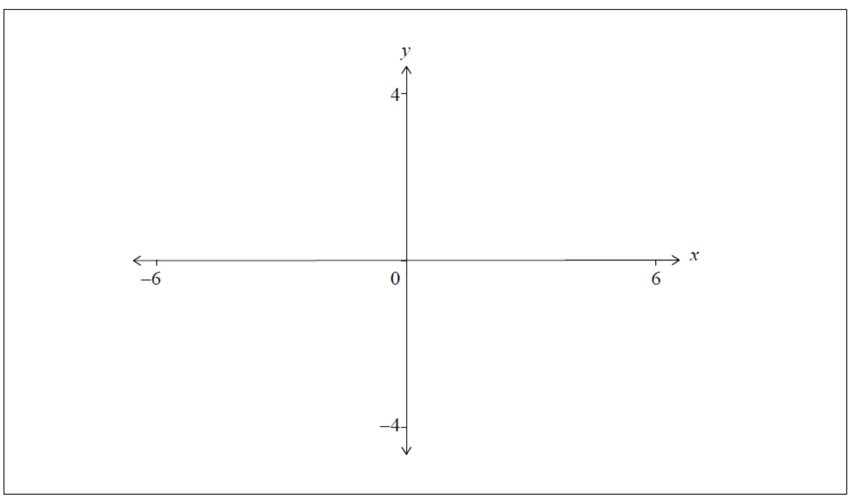
(a) On the following axes, sketch the graph of $y = |f(|x|)|$ for $-6 \leq x \leq 6$.



[2]

It is given that f is an odd function.

(b) On the following axes, sketch the graph of $y = f(x)$ for $-6 \leq x \leq 6$.



[2]

It is also given that $\int_0^4 f(|x|)\ \mathrm{d}\ x = 1.6$.

(c) Write down the value of

(c.i) $\int_{-4}^{0} f(x)\ \mathrm{d}\ x$; [1]

(c.ii) $\int_{-4}^{4}(f(|x|) + f(x))\ \mathrm{d}\ x$. [1]

15. [Maximum mark: 16] 24M.1.AHL.TZ1.10

Consider the function $f(x) = \frac{4x+2}{x-2},\ x \neq 2$.

(a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]

(b) Write down the range of f. [1]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\frac{1}{2}$ and p, where $p \in \mathbb{Q}$.

(c) Show that $p = \frac{9}{2}$. [1]

(d) Find the value of b and the value of c. [3]

(e) Find the y-coordinate of the vertex of the graph of $y = g(x)$. [2]

(f) Find the product of the solutions of the equation $f(x) = g(x)$. [4]

16. [Maximum mark: 17] 24M.1.AHL.TZ1.11

Consider the polynomial $P(x) = 3x^3 + 5x^2 + x - 1$.

(a) Show that $(x + 1)$ is a factor of $P(x)$. [2]

(b) Hence, express $P(x)$ as a product of three linear factors. [3]

Now consider the polynomial $Q(x) = (x + 1)(2x + 1)$.

(c) Express $\frac{1}{Q(x)}$ in the form $\frac{A}{x+1} + \frac{B}{2x+1}$, where $A,\ B \in \mathbb{Z}$. [3]

(d) Hence, or otherwise, show that
$\frac{1}{(x+1)Q(x)} = \frac{4}{2x+1} - \frac{2}{x+1} - \frac{1}{(x+1)^2}$. [2]

(e) Hence, find $\int \frac{1}{(x+1)^2(2x+1)}\ \mathrm{d}\ x$. [4]

Consider the function defined by $f(x) = \frac{P(x)}{(x+1)Q(x)}$, where $x \neq -1,\ x \neq -\frac{1}{2}$.

(f) Find

(f.i) $\lim_{x \to -1} f(x)$; [2]

(f.ii) $\lim_{x \to \infty} f(x)$. [1]

17. [Maximum mark: 4] 24M.1.AHL.TZ2.1

Solve $\tan\ (2x - 5^\circ) = 1$ for $0^\circ \leq x \leq 180^\circ$. [4]

18. [Maximum mark: 7] 24M.1.AHL.TZ2.7

A function $g(x)$ is defined by $g(x) = 2x^3 - 7x^2 + dx - e$, where $d,\ e \in \mathbb{R}$.

$\alpha,\ \beta$ and γ are the three roots of the equation $g(x) = 0$ where $\alpha,\ \beta,\ \gamma \in \mathbb{R}$.

(a) Write down the value of $\alpha + \beta + \gamma$. [1]

A function $h(z)$ is defined by
$h(z) = 2z^5 - 11z^4 + rz^3 + sz^2 + tz - 20$, where $r,\ s,\ t \in \mathbb{R}$.

$\alpha,\ \beta$ and γ are also roots of the equation $h(z) = 0$.

It is given that $h(z) = 0$ is satisfied by the complex number $z = p + 3\text{i}$.

(b) Show that $p = 1$. [3]

It is now given that $h\left(\frac{1}{2}\right) = 0$, and $\alpha,\ \beta \in \mathbb{Z}^+,\ \alpha < \beta$ and $\gamma \in \mathbb{Q}$.

(c.i) Find the value of the product $\alpha\beta$. [2]

(c.ii) Write down the value of α and the value of β. [1]

19. [Maximum mark: 6] 24M.1.AHL.TZ2.8

Use l'Hôpital's rule to find $\lim\limits_{x \to 0} \frac{\sec^4 x - \cos^2 x}{x^4 - x^2}$. [6]

20. [Maximum mark: 16] 24M.1.AHL.TZ2.10

Consider the arithmetic sequence $a,\ p,\ q \ldots$, where $a,\ p,\ q \neq 0$.

(a) Show that $2p - q = a$. [2]

Consider the geometric sequence $a,\ s,\ t \ldots$, where $a,\ s,\ t \neq 0$.

(b) Show that $s^2 = at$. [2]

The first term of both sequences is a.

It is given that $q = t = 1$.

(c) Show that $p > \frac{1}{2}$. [2]

Consider the case where $a = 9,\ s > 0$ and $q = t = 1$.

(d) Write down the first four terms of the (d.i) arithmetic sequence; [2]

(d.ii) geometric sequence. [2]

The arithmetic and the geometric sequence are used to form a new arithmetic sequence u_n.

The first three terms of u_n are $u_1 = 9 + \ln 9,\ u_2 = 5 + \ln 3$, and $u_3 = 1 + \ln 1$.

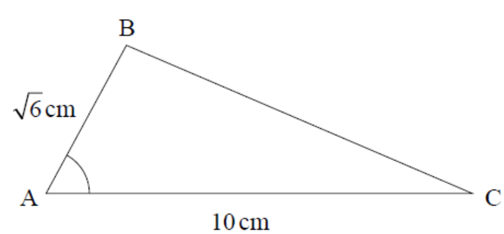
(e.i) Find the common difference of the new sequence in terms of $\ln 3$. [3]

(e.ii) Show that $\sum_{i=1}^{10} = -90 - 25 \ln 3$. [3]

21. [Maximum mark: 6] 23N.1.AHL.TZ1.4

In the following triangle ABC, $\mathrm{AB} = \sqrt{6 \text{ cm}}$, $\mathrm{AC} = 10 \text{ cm}$ and $\cos \mathrm{B\hat{A}C} = \frac{1}{5}$.

diagram not to scale



Find the area of triangle ABC. [6]

22. [Maximum mark: 5] 23N.1.AHL.TZ1.7

It is given that $z = 5 + q\mathrm{i}$ satisfies the equation $z^2 + \mathrm{i}z = -p + 25\mathrm{i}$, where $p,\ q \in \mathbb{R}$.

Find the value of p and the value of q. [5]

23. [Maximum mark: 9] 23N.1.AHL.TZ1.8

(a) Find $\int x\ (\ln\ x)^2\ \mathrm{d}x$ [6]

(b) Hence, show that

$\int_1^4\ x(\ln\ x)^2 \mathrm{d}x = 32(\ln\ 2)^2 - 16\ \ln\ 2 + \frac{15}{4}$. [3]

24. [Maximum mark: 8] 23N.1.AHL.TZ1.9

Consider the function $f(x) = \frac{\sin^2\ (kx)}{x^2}$, where $x \neq 0$ and $k \in \mathbb{R}^+$.

(a) Show that f is an even function. [2]

(b) Given that $\lim_{x \to 0} f(x) = 16$, find the value of k. [6]

25. [Maximum mark: 15] 23N.1.AHL.TZ1.10

The functions f and g are defined by

$f(x) = \ln\ (2x - 9)$, where $x > \frac{9}{2}$

$g(x) = 2\ \ln\ x - \ln\ d$, where $x > 0,\ d \in \mathbb{R}^+$.

(a) State the equation of the vertical asymptote to the graph of $y = g(x)$. [1]

The graphs of $y = f(x)$ and $y = g(x)$ intersect at two distinct points.

(b.i) Show that, at the points of intersection,

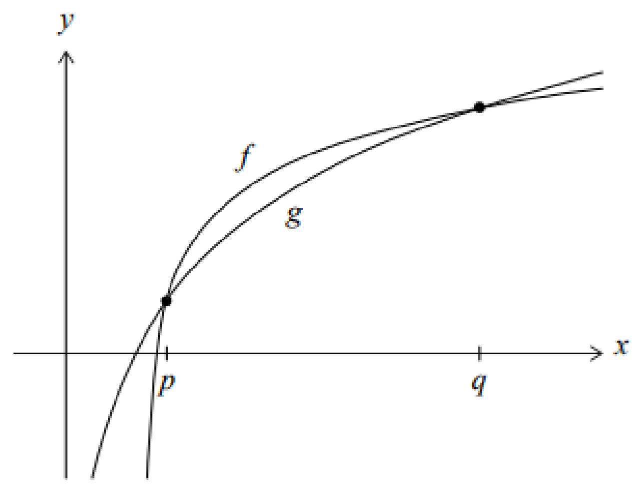
$x^2 - 2dx + 9d = 0$.

[4]

(b.ii) Hence show that $d^2 - 9d > 0$. [3]

(b.iii) Find the range of possible values of d. [2]

The following diagram shows part of the graphs of $y = f(x)$ and $y = g(x)$.



The graphs intersect at $x = p$ and $x = q$, where $p < q$.

(c) In the case where $d = 10$. find the value of $q - p$. Express your answer in the form of $a\sqrt{b}$, where, $a,\ b \in \mathbb{Z}^+$. [5]

26. [Maximum mark: 6] 23M.1.AHL.TZ1.3

Solve $\cos 2x = \sin x$, where $-\pi \leq x \leq \pi$. [6]

27. [Maximum mark: 5] 23M.1.AHL.TZ1.6

The side lengths, x cm, of an equilateral triangle are increasing at a rate of $4\ \text{cm s}^{-1}$.

Find the rate at which the area of the triangle, $A\ \mathrm{cm}^2$, is increasing when the side lengths are $5\sqrt{3}\ \mathrm{cm}$.

[5]

28. [Maximum mark: 6] 23M.1.AHL.TZ1.7

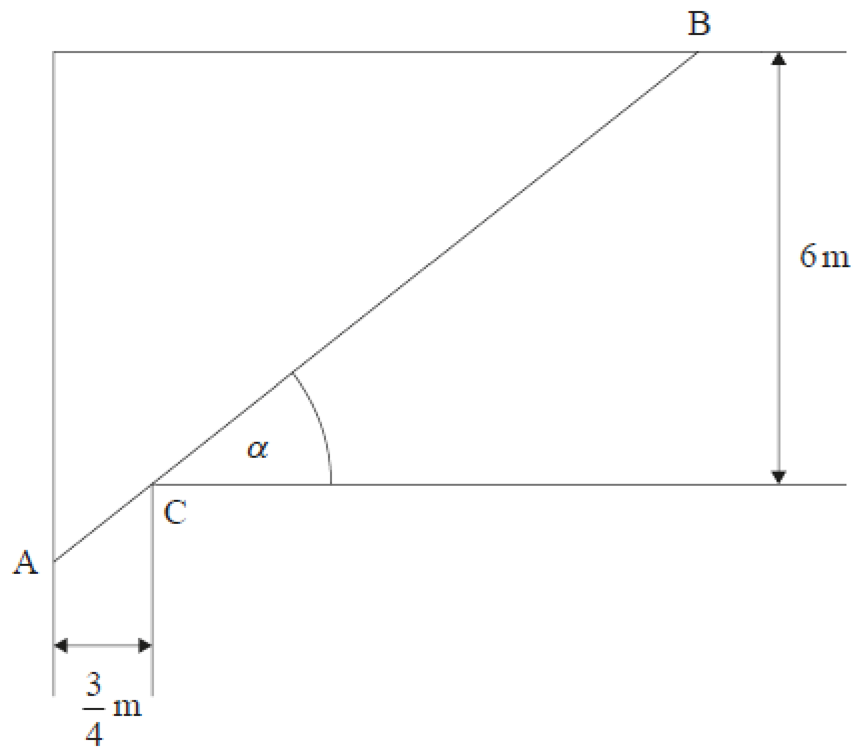
Consider $P(z) = 4m - mz + \frac{36}{m}z^2 - z^3$, where $z \in \mathbb{C}$ and $m \in \mathbb{R}^+$.

Given that $z - 3\mathrm{i}$ is a factor of $P(z)$, find the roots of $P(z) = 0$. [6]

29. [Maximum mark: 19] 23M.1.AHL.TZ1.11

Consider the following diagram, which shows the plan of part of a house.

diagram not to scale



A narrow passageway with $\frac{3}{4}\ \mathrm{m}$ width is perpendicular to a room of width $6\ \mathrm{m}$. There is a corner at point C. Points A and B are variable points on the base of the walls such that A, C and B lie on a straight line.

Let L denote the length AB in metres.

Let α be the angle that $[\mathrm{AB}]$ makes with the room wall, where $0 < \alpha < \frac{\pi}{2}$.

(a) Show that $L = \frac{3}{4}\sec \alpha + 6 \operatorname{cosec} \alpha$. [2]

(b.i) Find $\frac{\mathrm{d}L}{\mathrm{d}\alpha}$. [1]

(b.ii) When $\frac{\mathrm{d}L}{\mathrm{d}\alpha} = 0$, show that $\alpha = \arctan 2$. [4]

(c.i) Find $\frac{\mathrm{d}^2L}{\mathrm{d}\alpha^2}$. [3]

(c.ii) When $\alpha = \arctan 2$, show that $\frac{\mathrm{d}^2L}{\mathrm{d}\alpha^2} = \frac{45}{4}\sqrt{5}$. [4]

(d.i) Hence, justify that L is a minimum when $\alpha = \arctan 2$. [1]

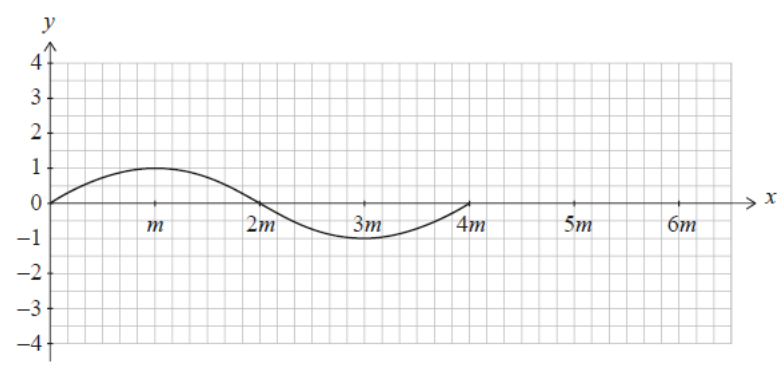
(d.ii) Determine this minimum value of L. [2]

Two people need to carry a pole of length $11.25\,\mathrm{m}$ from the passageway into the room. It must be carried horizontally.

(e) Determine whether this is possible, giving a reason for your answer. [2]

30. [Maximum mark: 6] 23M.1.AHL.TZ1.5

The function f is defined by $f(x) = \sin qx$, where $q > 0$. The following diagram shows part of the graph of f for $0 \le x \le 4m$, where x is in radians. There are x-intercepts at $x = 0, 2m$ and $4m$.



(a) Find an expression for m in terms of q. [2]

The function g is defined by $g\left(x\right) = 3\sin\frac{2qx}{3}$, for $0 \le x \le 6m$.

(b) On the axes above, sketch the graph of g. [4]

31. [Maximum mark: 14] 23M.1.AHL.TZ1.10

Consider the arithmetic sequence $u_1, u_2, u_3, \ldots$.

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

(a.i) Find the sum of the first five terms. [2]

(a.ii) Given that $S_6 = 60$, find u_6. [2]

(b) Find u_1. [2]

(c) Hence or otherwise, write an expression for u_n in terms of n. [3]

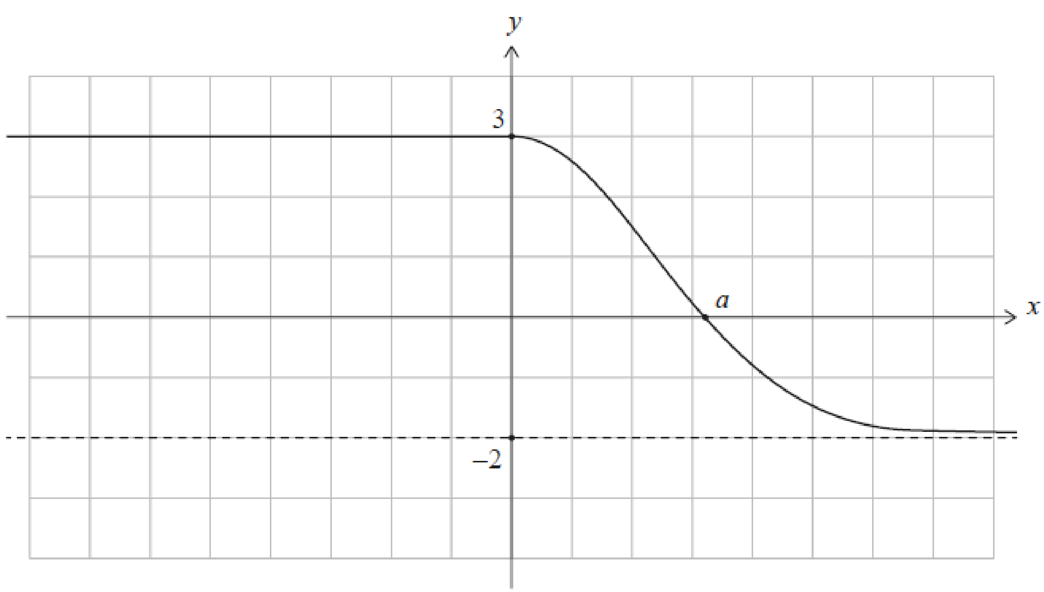
Consider a geometric sequence, v_n, where $v_2 = u_1$ and $v_4 = u_6$.

(d) Find the possible values of the common ratio, r. [3]

(e) Given that $v_{99} < 0$, find v_5. [2]

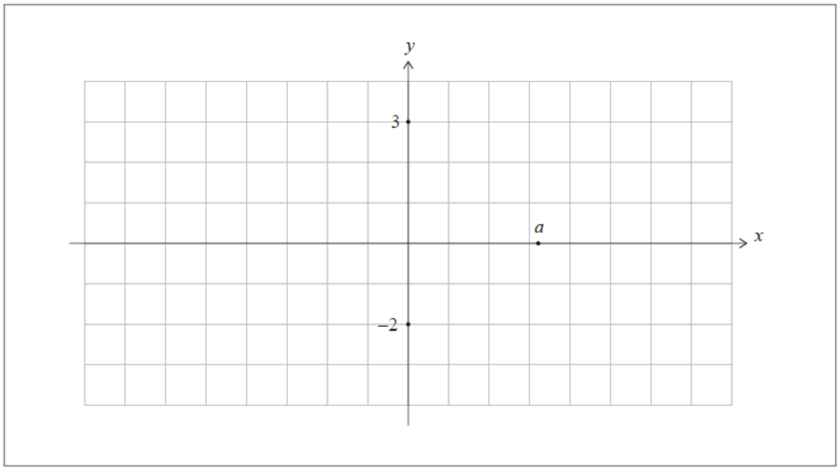
32. [Maximum mark: 7] 23M.1.AHL.TZ1.8

Part of the graph of a function, f, is shown in the following diagram. The graph of $y = f(x)$ has a y-intercept at $(0,\ 3)$, an x-intercept at $(a,\ 0)$ and a horizontal asymptote $y = -2$.



Consider the function $g(x) = |f(|x|)|$.

(a) On the following grid, sketch the graph of $y = g(x)$, labelling any axis intercepts and giving the equation of the asymptote.



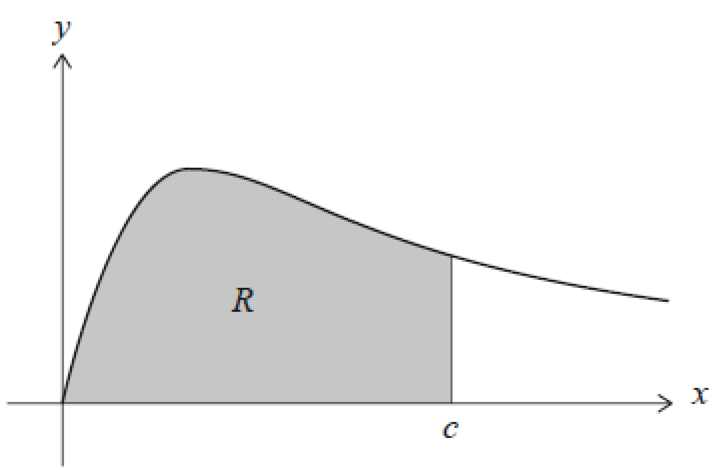
[4]

(b) Find the possible values of k such that $\left(g(x)\right)^2 = k$ has exactly two solutions.

[3]

33. [Maximum mark: 6] 23M.1.AHL.TZ2.4

The following diagram shows part of the graph of $y = \frac{x}{x^2+2}$ for $x \geq 0$.



The shaded region R is bounded by the curve, the x-axis and the line $x = c$.

The area of R is $\ln 3$.

[6]

Find the value of c.

34. [Maximum mark: 7] 23M.1.AHL.TZ2.8

The functions f and g are defined by

$$f(x) = \cos x, 0 \leq x \leq \frac{\pi}{2}$$

$$g(x) = \tan x, 0 \leq x < \frac{\pi}{2}.$$

The curves $y = f(x)$ and $y = g(x)$ intersect at a point P whose x-coordinate is k, where $0 < k < \frac{\pi}{2}$.

(a) Show that $\cos^2 k = \sin k$. [1]

(b) Hence, show that the tangent to the curve $y = f(x)$ at P and the tangent to the curve $y = g(x)$ at P intersect at right angles. [3]

(c) Find the value of $\sin k$. Give your answer in the form $\frac{a+\sqrt{b}}{c}$, where $a, c \in \mathbb{Z}$ and $b \in \mathbb{Z}^+$. [3]