Sequences revision [54 marks]

1. [Maximum mark: 7] 24M.1.AHL.TZ1.6

On 1 January 2025, the Faber Car Company will release a new car to global markets. The company expects to sell 40 cars in January 2025. The number of cars sold each month can be modelled by a geometric sequence where $r = 1.1$.

(a) Use this model to find the number of cars that will be sold in December 2025. [2]

Markscheme

attempt to substitute into geometric sequence formula for twelfth term **OR** at least three correct terms of the sequence *(M1)*

$u_{12} = 40 \times 1.1^{12-1}$ **OR** $40,\ 44,\ 48.4\ldots$

$114\ (114.124\ldots)$ ***A1***

[2 marks]

(b) Use this model to find the total number of cars that will be sold in the year

(b.i) 2025. [2]

Markscheme

attempt to substitute into geometric series formula **OR** a sum of at least the first three terms *(M1)*

$S_{12} = \frac{40(1.1^{12}-1)}{1.1-1}$ **OR** $\sum_{1}^{12}(40 \times 1.1^{n-1})$ **OR** $40 + 44 + 48.4 + \ldots$

Note: Award ***M1*** for $u_1 = 40$ and $r = 1.1$ seen as part of a geometric series formula, or ***M1*** for sigma notation and their u_n formula (condone missing limits), or ***M1*** for the sum of at least the **correct** first three terms of the sequence.

$S_{12} = 855 \quad (855.371\ldots)$ ***A1***

[2 marks]

(b.ii) 2026. [3]

Markscheme

finding $S_{24} = 3539.89\ldots$ or attempt to find the sum between u_{13} and u_{24} ***(M1)***

Note: Award ***M1*** for $S_{24} = 3539.89\ldots$ **or** sigma notation that includes correct limits and their u_n formula **or** a substituted geometric series formula that includes $125.537\ldots$ and $n = 12$ **or** a list of terms that includes at least the 13^{th} term and the 24^{th} term.

$3539.89\ldots - 855.371\ldots$ **OR** $\sum_{13}^{24}\left(40 \times 1.1^{n-1}\right)$ **OR**

$(S_{13\text{ to }24} =)\frac{125.537\ldots(1.1^{12}-1)}{1.1-1}$ **OR**
$125.537 + \ldots + 358.172\ldots$ ***(A1)***

Note: Accept a calculation using $u_{13} = 125$ or 126.

$2680 \ (2684.52\ldots,\ 2684,\ 2685)$ ***A1***

Note: For $u_{13} = 125$, the sum is $2673.03\ldots$ and for $u_{13} = 126$, the sum is $2694.41\ldots$

[3 marks]

2. [Maximum mark: 8] 24M.1.AHL.TZ2.5

The annual growth of a tree is 80% of its growth during the previous year.

This year the tree is 42 m in height and one year ago its height was 37 m.

(a) Calculate the annual growth of the tree in the coming year. [2]

Markscheme

recognizing that the growth in one year is the difference in the two heights ***(M1)***

$5 \times 0.8 = 4\,(\text{m})$ ***A1***

[2 marks]

(b) Calculate the height of the tree 6 years from now. Give your answer correct to the nearest cm. [4]

Markscheme

recognizing geometric sequence, $r = 0.8$ ***(M1)***

attempt to find total height by adding initial height to a term in series
(M1)

EITHER

$42+\frac{4\left(1-(0.8)^{6}\right)}{1-(0.8)}$ *(A1)*

OR

$37+\frac{5\left(1-(0.8)^{7}\right)}{1-(0.8)}$ *(A1)*

THEN

$=56.7571\ldots$

$=56.76\,(\mathrm{m})$ **OR** $5676\,(\mathrm{cm})$ *A1*

Note: Accept an answer in cm or in m to two decimal places.

[4 marks]

If the tree continues to follow this pattern of growth, its height will never exceed k metres.

(c) Find the smallest possible value of k. [2]

Markscheme

attempt to use infinite geometric series to find the total growth of the tree
(M1)

e.g. $\frac{5}{1-0.8}$ **OR** $\frac{4}{1-0.8}$

$\left(S_{\infty}=37+\frac{5}{1-0.8},\ S_{\infty}=42+\frac{4}{1-0.8}\right)$

$k = 62$ ***A1***

[2 marks]

3. [Maximum mark: 14] 24M.2.AHL.TZ2.6

The k th triangle number, T_k, is defined as $T_k = \sum_{r=1}^{k} r$.

(a.i) Calculate the value of the fifth triangle number, T_5. [1]

Markscheme

15 ***A1***

[1 mark]

(a.ii) Determine the formula for T_k in the form $ak^2 + bk$. [3]

Markscheme

EITHER

attempt to use arithmetic series formula ***(M1)***

OR

attempt to set up simultaneous equations ***(M1)***

OR

attempt to use quadratic regression ***(M1)***

$(T_k =)\frac{1}{2}k^2 + \frac{1}{2}k$ ***A1A1***

Note: Condone variable change (eg in quadratic regression).

Accept $a = \frac{1}{2},\ b = \frac{1}{2}$.

[3 marks]

(b.i) Find the value of $T_5 + T_4$. [1]

Markscheme

$(15 + 10 =)\ 25$ ***A1***

[1 mark]

(b.ii) Find the simplest expression for $T_k + T_{k-1}$. [2]

Markscheme

$\frac{k(k+1)}{2} + \frac{(k-1)((k-1)+1)}{2}$ **OR**
$\frac{1}{2}k^2 + \frac{1}{2}k + \frac{1}{2}(k-1)^2 + \frac{1}{2}(k-1)$ ***(A1)***

$= k^2$ ***A1***

[2 marks]

A bag contains 15 red discs and 10 blue discs, all identical except for colour. Two discs are chosen at random from the bag without replacement.

(c) Calculate the probability that the two discs are different colours. [3]

Markscheme

one correct product of probabilities seen: $\frac{15}{25} \times \frac{10}{24}$ **OR** $\frac{10}{25} \times \frac{15}{24}$ ***(A1)***

adding their products ***(M1)***

$\frac{15}{25} \times \frac{10}{24} + \frac{10}{25} \times \frac{15}{24}$

$= \frac{1}{2}$ ***A1***

[3 marks]

A bag contains T_k red discs and T_{k-1} blue discs, all identical except for colour. Two discs are chosen at random from the bag without replacement.

(d) Show that the probability that the two discs are different colours is independent of k. [4]

Markscheme

attempt to add two products of probabilities involving k only ***M1***

(these may be incorrect or in terms of T_k)

$\frac{\frac{k}{2}(k+1)}{k^2} \times \frac{\frac{k}{2}(k-1)}{k^2-1} + \frac{\frac{k}{2}(k-1)}{k^2} \times \frac{\frac{k}{2}(k+1)}{k^2-1}$ ***A1***

further simplification consistent with given answer ***A1***

$= \frac{1}{2}$ ***A1***

hence independent of k ***AG***

[4 marks]

4. [Maximum mark: 5] 22M.1.AHL.TZ2.7

The sum of an infinite geometric sequence is 9.

The first term is 4 more than the second term.

Find the third term. Justify your answer. [5]

Markscheme

METHOD 1

$\frac{u_1}{1-r} = 9$ ***A1***

therefore $u_1 = 9 - 9r$

$u_1 = 4 + u_1 r$ ***A1***

substitute or solve graphically: ***M1***

$9 - 9r = 4 + (9 - 9r)r$ **OR** $\frac{4}{(1-r)^2} = 9$

$9r^2 - 18r + 5 = 0$

$r = \frac{1}{3}$ or $r = \frac{5}{3}$

only $r = \frac{1}{3}$ is possible as the sum to infinity exists ***R1***

then $u_1 = 9 - \left(9 \times \frac{1}{3}\right) = 6$

$u_3 = 6 \times \frac{1}{3}^2 = \frac{2}{3}$ ***A1***

METHOD 2

$\frac{u_1}{1-r} = 9$ ***A1***

$r = \frac{u_1 - 4}{u_1}$ ***A1***

attempt to solve ***M1***

$\frac{u_1}{1-\left(\frac{u_1-4}{u_1}\right)} = 9$

$\frac{u_1}{\left(\frac{4}{u_1}\right)} = 9$

$(u_1)^2 = 36$

$u_1 = \pm 6$

attempting to solve both possible sequences

$6,\ 2,\ \ldots$ or $-6,\ -10\ \ldots$

$r = \frac{1}{3}$ or $r = \frac{5}{3}$

only $r = \frac{1}{3}$ is possible as the sum to infinity exists ***R1***

$u_3 = 6 \times \left(\frac{1}{3}\right)^2 = \frac{2}{3}$ ***A1***

[5 marks]

5. [Maximum mark: 5] 21N.1.AHL.TZ0.6

An infinite geometric sequence, with terms u_n, is such that $u_1 = 2$ and $\sum_{k=1}^{\infty} u_k = 10$.

(a) Find the common ratio, r, for the sequence. [2]

Markscheme

$10 = \frac{2}{1-r}$ ***(M1)***

$r = 0.8$ ***A1***

[2 marks]

(b) Find the least value of n such that $u_n < \frac{1}{2}$. [3]

Markscheme

$2 \times (0.8)^{n-1} < 0.5$ **OR** $2 \times (0.8)^{n-1} = 0.5$ ***(M1)***

$(n >)\ 7.212\ldots$ ***(A1)***

$n = 8$ ***A1***

Note: If $n = 7$ is seen, with or without seeing the value $7.212\ldots$ then award ***M1A1A0***.

[3 marks]

6. [Maximum mark: 6] 21M.1.AHL.TZ2.7

A meteorologist models the height of a hot air balloon launched from the ground. The model assumes the balloon travels vertically upwards and travels 450 metres in the first minute.

Due to the decrease in temperature as the balloon rises, the balloon will continually slow down. The model suggests that each minute the balloon will travel only 82% of the distance travelled in the previous minute.

(a) Find how high the balloon will travel in the first 10 minutes after it is launched. [3]

Markscheme

recognition of geometric sequence *eg* $r = 0.82$ ***(M1)***

$S_{10} = \frac{450(1-0.82^{10})}{1-0.82}$ ***(A1)***

$= 2160\,\text{m}\ \ (2156.37\ldots)$ ***A1***

[3 marks]

(b) The balloon is required to reach a height of at least 2520 metres.

Determine whether it will reach this height. [2]

Markscheme

$S_\infty = \frac{450}{1-0.82}$ ***(M1)***

$= 2500 < 2520$ so the balloon will not reach the required height. ***A1***

[2 marks]

(c) Suggest a limitation of the given model. [1]

Markscheme

horizontal motion not taken into account,

rate of cooling will not likely be linear,

balloon is considered a point mass / size of balloon not considered,

effects of wind/weather unlikely to be consistent,

a discrete model has been used, whereas a continuous one may offer greater accuracy ***R1***

Note: Accept any other sensible answer.

[1 mark]

7. [Maximum mark: 4] 19M.1.AHL.TZ2.H_1

In an arithmetic sequence, the sum of the 3rd and 8th terms is 1.

Given that the sum of the first seven terms is 35, determine the first term and the common difference. [4]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

attempting to form two equations involving u_1 and d ***M1***

$(u_1 + 2d) + (u_1 + 7d) = 1$ and $\frac{7}{2}[2u_1 + 6d] = 35$

$2u_1 + 9d = 1$

$14u_1 + 42d = 70 \quad (2u_1 + 6d = 10)$ ***A1***

Note: Award ***A1*** for any two correct equations

attempting to solve their equations: ***M1***

$u_1 = 14$, $d = -3$ ***A1***

[4 marks]

8. [Maximum mark: 5] 18N.2.AHL.TZ0.H_1

Consider a geometric sequence with a first term of 4 and a fourth term of –2.916.

(a) Find the common ratio of this sequence. [3]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$u_4 = u_1 r^3 \Rightarrow -2.916 = 4r^3$ ***(A1)***

solving, $r = -0.9$ ***(M1)A1***

[3 marks]

(b) Find the sum to infinity of this sequence. [2]

Markscheme

$S_\infty = \frac{4}{1-(-9)}$ ***(M1)***

$= \frac{40}{19}\ (= 2.11)$ ***A1***

[2 marks]