Sequences revision [54 marks]

1. [Maximum mark: 7] 24M.1.AHL.TZ1.6

On 1 January 2025, the Faber Car Company will release a new car to global markets. The company expects to sell 40 cars in January 2025. The number of cars sold each month can be modelled by a geometric sequence where $r = 1.1$.

(a) Use this model to find the number of cars that will be sold in December 2025. [2]

(b) Use this model to find the total number of cars that will be sold in the year

(b.i) 2025. [2]

(b.ii) 2026. [3]

2. [Maximum mark: 8] 24M.1.AHL.TZ2.5

The annual growth of a tree is $80\,\%$ of its growth during the previous year.

This year the tree is $42\,\text{m}$ in height and one year ago its height was $37\,\text{m}$.

(a) Calculate the annual growth of the tree in the coming year. [2]

(b) Calculate the height of the tree 6 years from now. Give your answer correct to the nearest cm. [4]

If the tree continues to follow this pattern of growth, its height will never exceed k metres.

(c) Find the smallest possible value of k. [2]

3. [Maximum mark: 14] 24M.2.AHL.TZ2.6

The k th triangle number, T_k, is defined as $T_k = \sum_{r=1}^{k} r$.

(a.i) Calculate the value of the fifth triangle number, T_5. [1]

(a.ii) Determine the formula for T_k in the form $ak^2 + bk$. [3]

(b.i) Find the value of $T_5 + T_4$. [1]

(b.ii) Find the simplest expression for $T_k + T_{k-1}$. [2]

A bag contains 15 red discs and 10 blue discs, all identical except for colour. Two discs are chosen at random from the bag without replacement.

(c) Calculate the probability that the two discs are different colours. [3]

A bag contains T_k red discs and T_{k-1} blue discs, all identical except for colour. Two discs are chosen at random from the bag without replacement.

(d) Show that the probability that the two discs are different colours is independent of k. [4]

4. [Maximum mark: 5] 22M.1.AHL.TZ2.7

The sum of an infinite geometric sequence is 9.

The first term is 4 more than the second term.

Find the third term. Justify your answer. [5]

5. [Maximum mark: 5] 21N.1.AHL.TZ0.6

An infinite geometric sequence, with terms u_n, is such that $u_1 = 2$ and $\sum_{k=1}^{\infty} u_k = 10$.

(a) Find the common ratio, r, for the sequence. [2]

(b) Find the least value of n such that $u_n < \frac{1}{2}$. [3]

6. [Maximum mark: 6] 21M.1.AHL.TZ2.7

A meteorologist models the height of a hot air balloon launched from the ground. The model assumes the balloon travels vertically upwards and travels 450 metres in the first minute.

Due to the decrease in temperature as the balloon rises, the balloon will continually slow down. The model suggests that each minute the balloon will travel only 82% of the distance travelled in the previous minute.

(a) Find how high the balloon will travel in the first 10 minutes after it is launched. [3]

(b) The balloon is required to reach a height of at least 2520 metres.

Determine whether it will reach this height. [2]

(c) Suggest a limitation of the given model. [1]

7. [Maximum mark: 4] 19M.1.AHL.TZ2.H_1

In an arithmetic sequence, the sum of the 3rd and 8th terms is 1.

Given that the sum of the first seven terms is 35, determine the first term and the common difference. [4]

8. [Maximum mark: 5] 18N.2.AHL.TZ0.H_1

Consider a geometric sequence with a first term of 4 and a fourth term of −2.916.

(a) Find the common ratio of this sequence. [3]

(b) Find the sum to infinity of this sequence. [2]