# Vectors - revision [96 marks]

**1.** [Maximum mark: 6] SPM.1.AHL.TZ0.11

A particle P moves with velocity $\boldsymbol{v} = \begin{pmatrix} -15 \\ 2 \\ 4 \end{pmatrix}$ in a magnetic field, $\boldsymbol{B} = \begin{pmatrix} 0 \\ d \\ 1 \end{pmatrix}$, $d \in \mathbb{R}$.

(a) Given that $\boldsymbol{v}$ is perpendicular to $\boldsymbol{B}$, find the value of $d$. [2]

Markscheme

$15 \times 0 + 2d + 4 = 0$ ***(M1)***

$d = -2$ ***A1***

***[2 marks]***

(b) The force, $\boldsymbol{F}$, produced by P moving in the magnetic field is given by the vector equation $\boldsymbol{F} = a\boldsymbol{v} \times \boldsymbol{B}$, $a \in \mathbb{R}^+$.

Given that $|\boldsymbol{F}| = 14$, find the value of $a$. [4]

Markscheme

$a \begin{pmatrix} -15 \\ 2 \\ 4 \end{pmatrix} \times \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$ ***(M1)***

$= a \begin{pmatrix} 10 \\ 15 \\ 30 \end{pmatrix} \left( = 5a \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \right)$ ***A1***

magnitude is $5a\sqrt{2^2 + 3^2 + 6^2} = 14$ ***M1***

$a = \frac{14}{35} \ (= 0.4)$ ***A1***

***[4 marks]***

**2.** [Maximum mark: 14] SPM.2.AHL.TZ0.4

An aircraft's position is given by the coordinates $(x, y, z)$, where $x$ and $y$ are the aircraft's displacement east and north of an airport, and $z$ is the height of the aircraft above the ground. All displacements are given in kilometres.

The velocity of the aircraft is given as $\begin{pmatrix} -150 \\ -50 \\ -20 \end{pmatrix} \text{km h}^{-1}$.

At 13:00 it is detected at a position 30 km east and 10 km north of the airport, and at a height of 5 km. Let $t$ be the length of time in hours from 13:00.

(a) Write down a vector equation for the displacement, $\boldsymbol{r}$ of the aircraft in terms of $t$. [2]

Markscheme

$r = \begin{pmatrix} 30 \\ 10 \\ 5 \end{pmatrix} + t \begin{pmatrix} -150 \\ -50 \\ -20 \end{pmatrix}$ ***A1A1***

***[2 marks]***

If the aircraft continued to fly with the velocity given

(b.i) verify that it would pass directly over the airport. [2]

Markscheme

when $x = 0, t = \frac{30}{150} = 0.2$ ***M1***

***EITHER***

when $y = 0, t = \frac{10}{150} = 0.2$ ***A1***

since the two values of $t$ are equal the aircraft passes directly over the airport

***OR***

$t = 0.2, y = 0$ ***A1***

***[2 marks]***

(b.ii) state the height of the aircraft at this point. [1]

Markscheme

height = 5 – 0.2 × 20 = 1 km ***A1***

***[1 mark]***

(b.iii) find the time at which it would fly directly over the airport. [1]

Markscheme

time 13:12 ***A1***

***[1 mark]***

When the aircraft is 4 km above the ground it continues to fly on the same bearing but adjusts the angle of its descent so that it will land at the point (0, 0, 0).

(c.i) Find the time at which the aircraft is 4 km above the ground. [2]

Markscheme

$5-20t=4 \Rightarrow t=\frac{1}{20}$ (3 minutes) ***(M1)***

time 13:03 ***A1***

***[2 marks]***

(c.ii) Find the direct distance of the aircraft from the airport at this point. [3]

Markscheme

displacement is $\begin{pmatrix} 22.5 \\ 7.5 \\ 4 \end{pmatrix}$ ***A1***

distance is $\sqrt{22.5^2+7.5^2+4^2}$ ***(M1)***

= 24.1 km ***A1***

***[3 marks]***

(d) Given that the velocity of the aircraft, after the adjustment of the angle of descent, is $\begin{pmatrix} -150 \\ -50 \\ a \end{pmatrix} \mathrm{km\,h^{-1}}$, find the value of $a$. [3]

Markscheme

***METHOD 1***

time until landing is $12-3=9$ minutes ***M1***

height to descend = $4\ \mathrm{km}$

$a=\frac{-4}{\frac{9}{60}}$ ***M1***

$=-26.7$ ***A1***

***METHOD 2***

$\begin{pmatrix} -150 \\ -50 \\ a \end{pmatrix} = s\begin{pmatrix} 22.5 \\ 7.5 \\ 4 \end{pmatrix}$ ***M1***

$-150=22.5\,s \Rightarrow s=-\frac{20}{3}$ ***M1***

$a=-\frac{20}{3}\times 4$

$=-26.7$ ***A1***

***[3 marks]***

**3.** [Maximum mark: 6] EXN.1.AHL.TZ0.7

The position of a helicopter relative to a communications tower at the top of a mountain at time $t$ (hours) can be described by the vector equation below.

$$\boldsymbol{r}=\begin{pmatrix}20\\-25\\0\end{pmatrix}+t\begin{pmatrix}4.2\\5.8\\-0.5\end{pmatrix}$$

The entries in the column vector give the displacements east and north from the communications tower and above/below the top of the mountain respectively, all measured in kilometres.

(a) Find the speed of the helicopter. [2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

$|v|=\sqrt{4.2^2+5.8^2+0.5^2}$ **(M1)**

$7.18\ (7.1784\ldots)\ \left(\text{kmh}^{-1}\right)$ **A1**

**[2 marks]**

(b) Find the distance of the helicopter from the communications tower at $t=0$. [2]

Markscheme

$$\boldsymbol{r}=\begin{pmatrix}20\\-25\\0\end{pmatrix}$$

$|\boldsymbol{r}|=\sqrt{20^2+25^2}$ **(M1)**

$=\sqrt{1025}=32.0\ (32.0156\ldots)\ (\text{km})$ **A1**

**[2 marks]**

(c) Find the bearing on which the helicopter is travelling. [2]

Markscheme

Bearing is $\arctan\left(\frac{4.2}{5.8}\right)$ or $90^\circ$-$\arctan\left(\frac{5.8}{4.2}\right)$ **(M1)**

$035.9^\circ\ \ (35.909\ldots)$ **A1**

**[2 marks]**

**4.** [Maximum mark: 17] EXN.2.AHL.TZ0.7

A ball is attached to the end of a string and spun horizontally. Its position relative to a given point, $\mathrm{O}$, at time $t$ seconds, $t \geq 0$, is given by the equation

$\boldsymbol{r} = \begin{pmatrix} 1.5\cos\left(0.1t^2\right) \\ 1.5\sin\left(0.1t^2\right) \end{pmatrix}$ where all displacements are in metres.

(a) Show that the ball is moving in a circle with its centre at $\mathrm{O}$ and state the radius of the circle. [4]

Markscheme

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$|\boldsymbol{r}| = \sqrt{1.5^2\cos^2\left(0.1t^2\right) + 1.5^2\sin^2\left(0.1t^2\right)}$ **M1**

$= 1.5$ as $\sin^2\theta + \cos^2\theta = 1$ **R1**

**Note:** use of the identity needs to be explicitly stated.

Hence moves in a circle as displacement from a fixed point is constant. **R1**

Radius $= 1.5\,\mathrm{m}$ **A1**

**[4 marks]**

(b.i) Find an expression for the velocity of the ball at time $t$. [2]

Markscheme

$\boldsymbol{v} = \begin{pmatrix} -0.3t\sin\left(0.1t^2\right) \\ 0.3t\cos\left(0.1t^2\right) \end{pmatrix}$ **M1A1**

**Note: M1** is for an attempt to differentiate each term

**[2 marks]**

(b.ii) Hence show that the velocity of the ball is always perpendicular to the position vector of the ball. [2]

Markscheme

$$\boldsymbol{v} \bullet \boldsymbol{r} = \begin{pmatrix} 1.5\cos(0.1t^2) \\ 1.5\sin(0.1t^2) \end{pmatrix} \bullet \begin{pmatrix} -0.3t\sin(0.1t^2) \\ 0.3t\cos(0.1t^2) \end{pmatrix}$$ **M1**

**Note: M1** is for an attempt to find $\boldsymbol{v} \bullet \boldsymbol{r}$

$$= 1.5\cos(0.1t^2) \times (-0.3t\sin(0.1t^2)) + 1.5\sin(0.1t^2) \times 0.3t\sin(0.1t^2) = 0$$ **A1**

Hence velocity and position vector are perpendicular. **AG**

**[2 marks]**

(c.i) Find an expression for the acceleration of the ball at time $t$. [3]

Markscheme

$$\boldsymbol{a} = \begin{pmatrix} -0.3\sin(0.1t^2) - 0.06t^2\cos(0.1t^2) \\ 0.3\cos(0.1t^2) - 0.06t^2\sin(0.1t^2) \end{pmatrix}$$ **M1A1A1**

**[3 marks]**

The string breaks when the magnitude of the ball's acceleration exceeds $20\,\mathrm{ms}^{-2}$.

(c.ii) Find the value of $t$ at the instant the string breaks. [3]

Markscheme

$$(-0.3\sin(0.1t^2) - 0.06t^2\cos(0.1t^2))^2 + (0.3\cos(0.1t^2) - 0.06t^2\sin(0.1t^2))^2 = 400$$
**(M1)(A1)**

**Note: M1** is for an attempt to equate the magnitude of the acceleration to $20$.

$t = 18.3\ (18.256\ldots)\ (\mathrm{s})$ **A1**

**[3 marks]**

(c.iii) How many complete revolutions has the ball completed from $t = 0$ to the instant at which the string breaks? [3]

Markscheme

Angle turned through is $0.1 \times 18.256^2 =$ **M1**

$= 33.329\ldots$ **A1**

$\frac{33.329}{2\pi}$ **M1**

$\frac{33.329}{2\pi} = 5.30\ldots$

$5$ complete revolutions **A1**

**[4 marks]**

**5.** [Maximum mark: 10] 24M.1.AHL.TZ2.8

The quadrilateral $\mathrm{ABCD}$ has coordinates $\mathrm{A}\ (1,\ 3,\ 5)$, $\mathrm{B}\ (4,\ 7,\ 5)$, $\mathrm{C}\ (5,\ 8,\ 7)$ and $\mathrm{D}\ (2,\ 4,\ 7)$.

(a) Write down $\overrightarrow{\mathrm{AD}}$. [1]

Markscheme

$\begin{pmatrix}1\\1\\2\end{pmatrix}$ ***A1***

**Note:** Accept any valid vector notation. Do not accept $(1,\ 1,\ 2)$.

***[1 mark]***

(b) Calculate

(b.i) the size of $\mathrm{B\hat{A}D}$. [3]

Markscheme

**EITHER**

use of scalar product formula to find angle $\mathrm{B\hat{A}D}$ ***(M1)***

$\cos \mathrm{B\hat{A}D} = \frac{\begin{pmatrix}3\\4\\0\end{pmatrix} \bullet \begin{pmatrix}1\\1\\2\end{pmatrix}}{5\sqrt{6}}$ ***(A1)***

**OR**

use of cosine rule to find $\text{B}\widehat{\text{A}}\text{D}$ *(M1)*

$\cos \text{B}\widehat{\text{A}}\text{D} = \frac{6+25-17}{2\times\sqrt{6}\times5}$ *(A1)*

**THEN**

$\text{B}\widehat{\text{A}}\text{D} = 55.1^\circ \quad (55.1417\ldots^\circ,\ 0.92405\ldots)$ ***A1***

**Note:** If the direction of one of the vectors is reversed, leading to an obtuse angle $(124.858\ldots^\circ)$ between the vectors, then award ***M1A1A0***.

***[3 marks]***

(b.ii) the area of triangle $\text{BAD}$. [4]

Markscheme

**EITHER**

an attempt at using vector product *(M1)*

$\frac{1}{2}\left|\begin{pmatrix}1\\1\\2\end{pmatrix}\times\begin{pmatrix}3\\4\\0\end{pmatrix}\right|$ or equivalent

attempt to apply formula (e.g. one correct component) *(M1)*

$= \frac{1}{2}((1\times0-4\times2)\boldsymbol{i}-(1\times0-2\times3)\boldsymbol{j}-(1\times4-1\times3)\boldsymbol{k})$

$\frac{1}{2}\left|\begin{pmatrix}-8\\6\\1\end{pmatrix}\right|$ *(A1)*

$= 5.02 \quad \left(5.02493,\ \frac{\sqrt{101}}{2}\right)$ ***A1***

**OR**

use of formula for the area of a triangle *(M1)*

area$= \frac{1}{2}\left|\begin{pmatrix}3\\4\\0\end{pmatrix}\right|\left|\begin{pmatrix}1\\1\\2\end{pmatrix}\right|\sin 55.1417\ldots$ *(A1)(A1)*

$= 5.02$ ***A1***

**Note:** Award ***A1*** for the lengths $\text{AB}$ and $\text{AD}$ and ***A1*** for the angle.

***[4 marks]***

(c) Show that $\text{ABCD}$ is a parallelogram. [2]

Markscheme

**EITHER**

$\overrightarrow{\mathrm{AD}} = \overrightarrow{\mathrm{BC}} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ **OR** $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{DC}} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ ***A1***

One pair of opposite sides have equal length AND are parallel ***R1***

**OR**

$\overrightarrow{\mathrm{AD}} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \ \overrightarrow{\mathrm{BC}} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \quad \overrightarrow{\mathrm{AB}} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}, \ \overrightarrow{\mathrm{DC}} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ ***A1***

$\overrightarrow{\mathrm{AD}} = \overrightarrow{\mathrm{BC}} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{DC}} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$

both pairs of opposite sides are parallel (or have equal length) ***R1***

**Note:** Both pairs of opposite angles are equal is also valid.

**THEN**

hence $\mathrm{ABCD}$ is a parallelogram ***AG***

***[2 marks]***

**6.** [Maximum mark: 7] 24M.1.AHL.TZ2.12

A duck is sitting in a duck pond at point $\mathrm{A}(7,\ 4,\ 0)$ relative to an origin $\mathrm{O}$, where lengths are measured in metres and time, $t$, is measured in seconds. It takes off and flies in a straight line with vector equation

$$\boldsymbol{d} = \begin{pmatrix} 7 \\ 4 \\ 0 \end{pmatrix} + t \begin{pmatrix} 6 \\ 6 \\ 3 \end{pmatrix}.$$

(a) Find the speed of the duck through the air (in $\mathrm{m\,s^{-1}}$). [2]

Markscheme

$\sqrt{6^2 + 6^2 + 3^2}$ ***(A1)***

$= 9\left(\mathrm{m\,s^{-1}}\right)$ ***A1***

***[2 marks]***

A hawk hovering at position vector $\begin{pmatrix} -38 \\ 134 \\ 315 \end{pmatrix}$, relative to $\mathrm{O}$, sees the duck take off and immediately dives from its position with constant velocity vector $\begin{pmatrix} 15 \\ -20 \\ -60 \end{pmatrix}$ to intercept the duck.

(b) Write down the vector equation for $\boldsymbol{h}$, that models the flight of the hawk. [1]

Markscheme

$$\boldsymbol{h} = \begin{pmatrix} -38 \\ 134 \\ 315 \end{pmatrix} + t \begin{pmatrix} 15 \\ -20 \\ -60 \end{pmatrix}$$ ***A1***

***[1 mark]***

(c) Find the position vector at which the hawk intercepts the duck. [4]

Markscheme

equating one component from each ***(M1)***

e.g. $7 + 6t = -38 + 15t$

$t = 5$ ***(A1)***

substituting their t-value into either equation ***(M1)***

$$\begin{pmatrix} -38 \\ 134 \\ 315 \end{pmatrix} + 5 \begin{pmatrix} 15 \\ -20 \\ -60 \end{pmatrix}$$

$$= \begin{pmatrix} 37 \\ 34 \\ 15 \end{pmatrix}$$ ***A1***

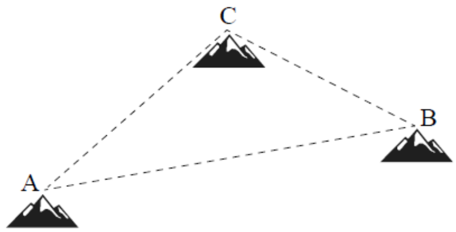
***[4 marks]***

**7.** [Maximum mark: 12] 24M.2.AHL.TZ1.2

A national park contains three mountains whose summits are at points $\mathrm{A}, \mathrm{B}$ and $\mathrm{C}$.

According to a coordinate system, the position of $\mathrm{A}$ is $(0,\ 0,\ 2.8)$ and the position of $\mathrm{B}$ is $(7.2,\ 5.1,\ 2.4)$. All the values are in kilometres.

diagram not to scale




(a.i) Find the vector $\overrightarrow{AB}$. [1]

Markscheme

$\begin{pmatrix} 7.2 \\ 5.1 \\ 2.4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 2.8 \end{pmatrix} = \begin{pmatrix} 7.2 \\ 5.1 \\ -0.4 \end{pmatrix}$ ***A1***

**Note:** Accept alternate vector notation, e.g. $(7.2,\ 5.1,\ -0.4)$ or $<7.2,\ 5.1,\ -0.4>$

***[1 mark]***

(a.ii) Hence find $AB$, the distance between $A$ and $B$. [2]

Markscheme

use of correct formula to find $\left|\overrightarrow{AB}\right|$ ***(M1)***

$\sqrt{7.2^2 + 5.1^2 + (-0.4)^2}$

$8.83\,(\text{km})\ (8.83232\ldots)$ ***A1***

***[2 marks]***

The vector $\overrightarrow{AC}$ is parallel to the vector $\begin{pmatrix} 1.1 \\ 8.4 \\ 0.2 \end{pmatrix}$.

(b) Find the angle between $\begin{pmatrix} 1.1 \\ 8.4 \\ 0.2 \end{pmatrix}$ and $\overrightarrow{AB}$. [5]

Markscheme

magnitude of $\begin{pmatrix} 1.1 \\ 8.4 \\ 0.2 \end{pmatrix}$ is

$\sqrt{1.1^2+8.4^2+0.2^2}\ \ (=8.47407\ldots)$ ***(A1)***

**EITHER**

$\begin{pmatrix}1.1\\8.4\\0.2\end{pmatrix}\bullet\begin{pmatrix}7.2\\5.1\\-0.4\end{pmatrix}$ ***(M1)***

$1.1\times7.2+8.4\times5.1-0.2\times0.4\ \ (=50.68)$ ***(A1)***

**Note:** The ***M*** mark can be implied by a partially correct ***A1*** line.

angle$=\ \arccos\left(\frac{50.68}{8.83232\ldots\times8.47407\ldots}\right)$ ***(M1)***

**OR**

Attempt to find $\begin{pmatrix}1.1\\8.4\\0.2\end{pmatrix}\times\begin{pmatrix}7.2\\5.1\\-0.4\end{pmatrix}$ ***(M1)***

$\left|\begin{pmatrix}1.1\\8.4\\0.2\end{pmatrix}\times\begin{pmatrix}7.2\\5.1\\-0.4\end{pmatrix}\right|=\sqrt{4.38^2+1.88^2+54.87^2}\ \ (=55.0766\ldots)$ ***(A1)***

angle$=\ \arcsin\left(\frac{55.0766\ldots}{8.83232\ldots\times8.47407\ldots}\right)$ ***(M1)***

**THEN**

$47.4^\circ\ \ (47.3805\ldots)$ **OR** $0.827\ \ (0.826947\ldots)$ ***A1***

***[5 marks]***

The angle between $\overrightarrow{\mathrm{BA}}$ and $\overrightarrow{\mathrm{BC}}$ is $55.2^\circ$.

(c) Use the sine rule to find $\mathrm{AC}$. [4]

Markscheme

using sum of angles in a triangle equals $180$ ***(M1)***

$\mathrm{A\hat{C}B}=180-47.3805-55.2\ (=77.4194\ldots^\circ)$ ***(A1)***

$\frac{\mathrm{AC}}{\sin(55.2)}=\frac{8.83232\ldots}{\sin(77.4194\ldots)}$ ***(A1)***

$7.43\,(\mathrm{km})\ \ (7.43107\ldots)$ ***A1***

***[4 marks]***

**8.** [Maximum mark: 7] 23N.1.AHL.TZ0.14

A straight line $L$ has vector equation $\boldsymbol{r} = \begin{pmatrix}1\\3\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\2\end{pmatrix}$ and point $\mathrm{Q}$ has coordinates $(11,\ -1,\ 3)$.

Point $\mathrm{P}$ is the point on $L$ closest to $\mathrm{Q}$.

(a) Find the coordinates of $\mathrm{P}$. [4]

Markscheme

vector from $Q$ to any point in $L$ or vice versa

$$= \begin{pmatrix}1 + \lambda\\3 + \lambda\\2\lambda\end{pmatrix} - \begin{pmatrix}11\\-1\\3\end{pmatrix} = \begin{pmatrix}-10 + \lambda\\4 + \lambda\\2\lambda - 3\end{pmatrix}$$ ***(M1)***

**EITHER** (scalar product)

attempt to use scalar product ***(M1)***

$$\begin{pmatrix}-10 + \lambda\\4 + \lambda\\2\lambda - 3\end{pmatrix} \bullet \begin{pmatrix}1\\1\\2\end{pmatrix} = 0$$

$$-10 + \lambda + 4 + \lambda + 4\lambda - 6 = 0$$

**OR** (distance formula)

attempt to use distance formula ***(M1)***

minimizing $(-10 + \lambda)^2 + (4 + \lambda)^2 + (-3 + 2\lambda)^2$

**THEN**

$\lambda = 2$ ***(A1)***

point $\mathrm{P}\,(3,\ 5,\ 4)$ ***A1***

**Note:** Do not award final ***A1*** for P given as a vector.

***[4 marks]***

(b) Find a vector that is perpendicular to both $L$ and the line passing through points $\mathrm{P}$ and $\mathrm{Q}$. [3]

Markscheme

$\overrightarrow{PQ} = \begin{pmatrix} 8 \\ -6 \\ -1 \end{pmatrix}$ *(A1)*

attempt to use vector product *(M1)*

(perpendicular vector =) $\begin{pmatrix} 8 \\ -6 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

$\begin{pmatrix} -11 \\ -17 \\ 14 \end{pmatrix}$ ***A1***

**Note:** Award final ***A1*** for any multiple (positive or negative) of the answer given here.

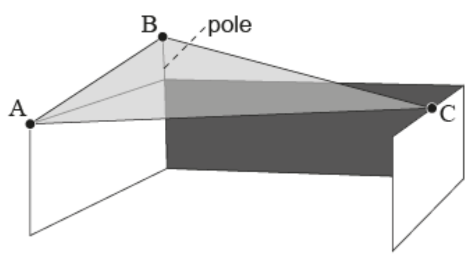
***[3 marks]***

**9.** [Maximum mark: 9] 23M.1.AHL.TZ1.7

A triangular cover is positioned over a walled garden to provide shade. It is anchored at points $\mathrm{A}$ and $\mathrm{C}$, located at the top of a $2\,\mathrm{m}$ wall, and at a point $\mathrm{B}$, located at the top of a $1\,\mathrm{m}$ vertical pole fixed to a top corner of the wall.

The three edges of the cover can be represented by the vectors

$\overrightarrow{AB} = \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 7 \\ -3 \\ -1 \end{pmatrix}$, where distances are measured in metres.




(a) Calculate the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$. [2]

Markscheme

attempt to find the vector product (e.g. one term correct) *(M1)*

$\begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix} \times \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 7 \\ -42 \end{pmatrix}$ ***A1***

*[2 marks]*

(b) Hence find the area of the triangular cover. [2]

Markscheme

**METHOD 1**

attempt to use the vector product formula for the area of triangle

(condone incorrect signs and missing $\frac{1}{2}$) ***(M1)***

area $= \frac{1}{2}\sqrt{3^2 + 7^2 + 42^2}$

$= 21.3\left(\text{m}^2\right)\ \left(21.3424\ldots,\ \frac{1}{2}\sqrt{1822}\right)$ ***A1***

**METHOD 2**

find $\theta$ using $\overrightarrow{\text{AB}} \times \overrightarrow{\text{AC}} = \left|\overrightarrow{\text{AB}}\right|\left|\overrightarrow{\text{AC}}\right| \sin\theta$ ***(M1)***

$\theta = 67.1$ ($67.1350^\circ\ldots,\ 1.171728\ldots$ radians)

then area $= \frac{1}{2}\left|\overrightarrow{\text{AB}}\right|\left|\overrightarrow{\text{AC}}\right| \sin\theta$

$= 21.3\left(\text{m}^2\right)\ \left(21.3424\ldots,\ \frac{1}{2}\sqrt{1822}\right)$ ***A1***

*[2 marks]*

The point $\text{X}$ on $[\text{AC}]$ is such that $[\text{BX}]$ is perpendicular to $[\text{AC}]$.

(c) Use your answer to part (b) to find the distance $\text{BX}$. [3]

Markscheme

$AC = 7.61577\ldots\ \left(\sqrt{58}\right)$ ***(A1)***

setting the area formula $\frac{1}{2}\times$base$\times$height equal to their part (b) ***(M1)***

$\text{BX} = \frac{2\times 21.3424\ldots}{\sqrt{58}}$

$= 5.60\ (5.60480\ldots)$ ***A1***

**Note:** Award ***A1*** for $5.6$.

Award ***A1*** for $5.59$ $(5.5936\ldots)$ from the use of $21.3$ to $3$ sf.

***[3 marks]***

(d) Find the angle the cover makes with the horizontal plane. [2]

Markscheme

attempting to set up a trig ratio ***(M1)***

angle is $\arcsin\left(\frac{1}{\text{BX}}\right)$

$10.3^\circ$ $(10.2776\ldots^\circ, 0.179378$ radians) ***A1***

***[2 marks]***

**10.** [Maximum mark: 8] 23M.1.AHL.TZ2.14

In this question, $\boldsymbol{i}$ denotes a unit vector due east, and $\boldsymbol{j}$ denotes a unit vector due north.

Two ships, $\text{A}$ and $\text{B}$, are each moving with constant velocities.

The position vector of ship $\text{A}$, at time $t$ hours, is given as $\boldsymbol{r}_A = (1+2t)\boldsymbol{i} + (3-3t)\boldsymbol{j}$.

The position vector of ship $\text{B}$, at time $t$ hours, is given as $\boldsymbol{r}_B = (-2+4t)\boldsymbol{i} + (-4+t)\boldsymbol{j}$.

(a) Find the bearing on which ship $\text{A}$ is sailing. [3]

Markscheme

$\boldsymbol{v}_B = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ ***(A1)***

attempt to find any relevant angle ***(M1)***

$\tan^{-1}\left(\frac{3}{2}\right)$ $(= 56.3099\ldots^\circ)$

$(90^\circ + 56.3099\ldots^\circ =)$ $146^\circ$ $(146.3099\ldots^\circ)$ ***A1***

***[3 marks]***

(b) Find the value of $t$ when ship $\text{B}$ is directly south of ship $\text{A}$. [2]

Markscheme

setting $1 + 2t = -2 + 4t$ ***(M1)***

$t = 1.5$ (hrs.) ***A1***

***[2 marks]***

(c) Find the value of $t$ when ship $\mathrm{B}$ is directly south-east of ship $\mathrm{A}$. [3]

Markscheme

$\boldsymbol{r}_B - \boldsymbol{r}_A = (-3 + 2t)\boldsymbol{i} + (-7 + 4t)\boldsymbol{j}$ ***(M1)***

$-3 + 2t = -(-7 + 4t)$ ***(M1)***

$t = 1.67\ (\text{hrs.})\ \left(1.66666\ldots,\ \frac{5}{3}\right)$ ***A1***

***[3 marks]***