Vectors - revision [96 marks]

1. [Maximum mark: 6] SPM.1.AHL.TZ0.11

A particle P moves with velocity $\boldsymbol{v} = \begin{pmatrix} -15 \\ 2 \\ 4 \end{pmatrix}$ in a magnetic field, $\boldsymbol{B} = \begin{pmatrix} 0 \\ d \\ 1 \end{pmatrix}$, $d \in \mathbb{R}$.

(a) Given that $\boldsymbol{v}$ is perpendicular to $\boldsymbol{B}$, find the value of d. [2]

(b) The force, $\boldsymbol{F}$, produced by P moving in the magnetic field is given by the vector equation $\boldsymbol{F} = a\boldsymbol{v} \times \boldsymbol{B}$, $a \in \mathbb{R}^+$.

Given that $|\boldsymbol{F}| = 14$, find the value of a. [4]

2. [Maximum mark: 14] SPM.2.AHL.TZ0.4

An aircraft's position is given by the coordinates (x, y, z), where x and y are the aircraft's displacement east and north of an airport, and z is the height of the aircraft above the ground. All displacements are given in kilometres.

The velocity of the aircraft is given as $\begin{pmatrix} -150 \\ -50 \\ -20 \end{pmatrix} \text{km h}^{-1}$.

At 13:00 it is detected at a position 30 km east and 10 km north of the airport, and at a height of 5 km. Let t be the length of time in hours from 13:00.

(a) Write down a vector equation for the displacement, $\boldsymbol{r}$ of the aircraft in terms of t. [2]

If the aircraft continued to fly with the velocity given

(b.i) verify that it would pass directly over the airport. [2]

(b.ii) state the height of the aircraft at this point. [1]

(b.iii) find the time at which it would fly directly over the airport. [1]

When the aircraft is 4 km above the ground it continues to fly on the same bearing but adjusts the angle of its descent so that it will land at the point (0, 0, 0).

(c.i) Find the time at which the aircraft is 4 km above the ground. [2]

(c.ii) Find the direct distance of the aircraft from the airport at this point. [3]

(d) Given that the velocity of the aircraft, after the adjustment of the angle of descent, is $\begin{pmatrix} -150 \\ -50 \\ a \end{pmatrix} \mathrm{km\,h^{-1}}$, find the value of a. [3]

3. [Maximum mark: 6] EXN.1.AHL.TZ0.7

The position of a helicopter relative to a communications tower at the top of a mountain at time t (hours) can be described by the vector equation below.

$$\boldsymbol{r} = \begin{pmatrix} 20 \\ -25 \\ 0 \end{pmatrix} + t\begin{pmatrix} 4.2 \\ 5.8 \\ -0.5 \end{pmatrix}$$

The entries in the column vector give the displacements east and north from the communications tower and above/below the top of the mountain respectively, all measured in kilometres.

(a) Find the speed of the helicopter. [2]

(b) Find the distance of the helicopter from the communications tower at $t=0$. [2]

(c) Find the bearing on which the helicopter is travelling. [2]

4. [Maximum mark: 17] EXN.2.AHL.TZ0.7

A ball is attached to the end of a string and spun horizontally. Its position relative to a given point, O, at time t seconds, $t \geq 0$, is given by the equation

$\boldsymbol{r}=\begin{pmatrix}1.5\cos\left(0.1t^2\right)\\1.5\sin\left(0.1t^2\right)\end{pmatrix}$ where all displacements are in metres.

(a) Show that the ball is moving in a circle with its centre at O and state the radius of the circle. [4]

(b.i) Find an expression for the velocity of the ball at time t. [2]

(b.ii) Hence show that the velocity of the ball is always perpendicular to the position vector of the ball. [2]

(c.i) Find an expression for the acceleration of the ball at time t. [3]

The string breaks when the magnitude of the ball's acceleration exceeds $20\,\mathrm{ms}^{-2}$.

(c.ii) Find the value of t at the instant the string breaks. [3]

(c.iii) How many complete revolutions has the ball completed from $t=0$ to the instant at which the string breaks? [3]

5. [Maximum mark: 10] 24M.1.AHL.TZ2.8

The quadrilateral ABCD has coordinates $\mathrm{A}\ (1,\ 3,\ 5)$, $\mathrm{B}\ (4,\ 7,\ 5)$, $\mathrm{C}\ (5,\ 8,\ 7)$ and $\mathrm{D}\ (2,\ 4,\ 7)$.

(a) Write down $\overrightarrow{\mathrm{AD}}$. [1]

(b) Calculate

(b.i) the size of $\mathrm{B\hat{A}D}$. [3]

(b.ii) the area of triangle BAD. [4]

(c) Show that ABCD is a parallelogram. [2]

6. [Maximum mark: 7] 24M.1.AHL.TZ2.12

A duck is sitting in a duck pond at point $\mathrm{A}(7,\ 4,\ 0)$ relative to an origin O, where lengths are measured in metres and time, t, is measured in seconds. It takes off and flies in a straight line with vector equation

$$\boldsymbol{d} = \begin{pmatrix} 7 \\ 4 \\ 0 \end{pmatrix} + t\begin{pmatrix} 6 \\ 6 \\ 3 \end{pmatrix}.$$

(a) Find the speed of the duck through the air (in $\mathrm{m\,s^{-1}}$). [2]

A hawk hovering at position vector $\begin{pmatrix} -38 \\ 134 \\ 315 \end{pmatrix}$, relative to O, sees the duck take off and immediately dives from its position with constant velocity vector $\begin{pmatrix} 15 \\ -20 \\ -60 \end{pmatrix}$ to intercept the duck.

(b) Write down the vector equation for $\boldsymbol{h}$, that models the flight of the hawk. [1]

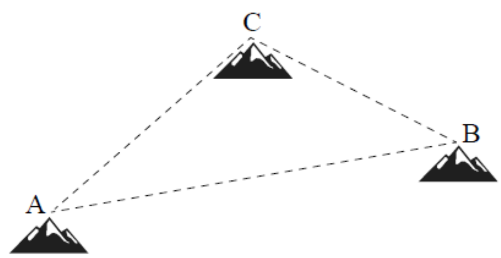
(c) Find the position vector at which the hawk intercepts the duck. [4]

7. [Maximum mark: 12] 24M.2.AHL.TZ1.2

A national park contains three mountains whose summits are at points A, B and C.

According to a coordinate system, the position of A is $(0,\ 0,\ 2.8)$ and the position of B is $(7.2,\ 5.1,\ 2.4)$. All the values are in kilometres.

diagram not to scale



(a.i) Find the vector $\overrightarrow{\mathrm{AB}}$. [1]

(a.ii) Hence find AB, the distance between A and B. [2]

The vector $\overrightarrow{\mathrm{AC}}$ is parallel to the vector $\begin{pmatrix} 1.1 \\ 8.4 \\ 0.2 \end{pmatrix}$.

(b) Find the angle between $\begin{pmatrix} 1.1 \\ 8.4 \\ 0.2 \end{pmatrix}$ and $\overrightarrow{\mathrm{AB}}$. [5]

The angle between $\overrightarrow{\mathrm{BA}}$ and $\overrightarrow{\mathrm{BC}}$ is 55.2°.

(c) Use the sine rule to find AC. [4]

8. [Maximum mark: 7] 23N.1.AHL.TZ0.14

A straight line L has vector equation $\boldsymbol{r} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and point Q has coordinates $(11, \ -1, \ 3)$.

Point P is the point on L closest to Q.

(a) Find the coordinates of P. [4]

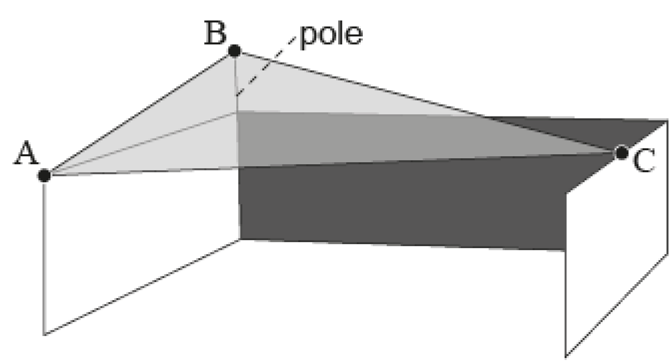
(b) Find a vector that is perpendicular to both L and the line passing through points P and Q. [3]

9. [Maximum mark: 9] 23M.1.AHL.TZ1.7

A triangular cover is positioned over a walled garden to provide shade. It is anchored at points A and C, located at the top of a $2\,\mathrm{m}$ wall, and at a point B, located at the top of a $1\,\mathrm{m}$ vertical pole fixed to a top corner of the wall.

The three edges of the cover can be represented by the vectors

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}$, $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix}$ and $\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 7 \\ -3 \\ -1 \end{pmatrix}$, where distances are measured in metres.



(a) Calculate the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$. [2]

(b) Hence find the area of the triangular cover. [2]

The point X on $[AC]$ is such that $[BX]$ is perpendicular to $[AC]$.

(c) Use your answer to part (b) to find the distance BX. [3]

(d) Find the angle the cover makes with the horizontal plane. [2]

10. [Maximum mark: 8] 23M.1.AHL.TZ2.14

In this question, $\boldsymbol{i}$ denotes a unit vector due east, and $\boldsymbol{j}$ denotes a unit vector due north.

Two ships, A and B, are each moving with constant velocities.

The position vector of ship A, at time t hours, is given as $\boldsymbol{r}_A = (1 + 2t)\boldsymbol{i} + (3 - 3t)\boldsymbol{j}$.

The position vector of ship B, at time t hours, is given as $\boldsymbol{r}_B = (-2 + 4t)\boldsymbol{i} + (-4 + t)\boldsymbol{j}$.

(a) Find the bearing on which ship A is sailing. [3]

(b) Find the value of t when ship B is directly south of ship A. [2]

(c) Find the value of t when ship B is directly south-east of ship A. [3]