13 $3\sin x - 5\cos x \approx \sqrt{34}\cos(x + 3.68)$

14 **a** $2\sin x + \sqrt{3}\cos x \approx \sqrt{7}\sin(x + 0.714)$

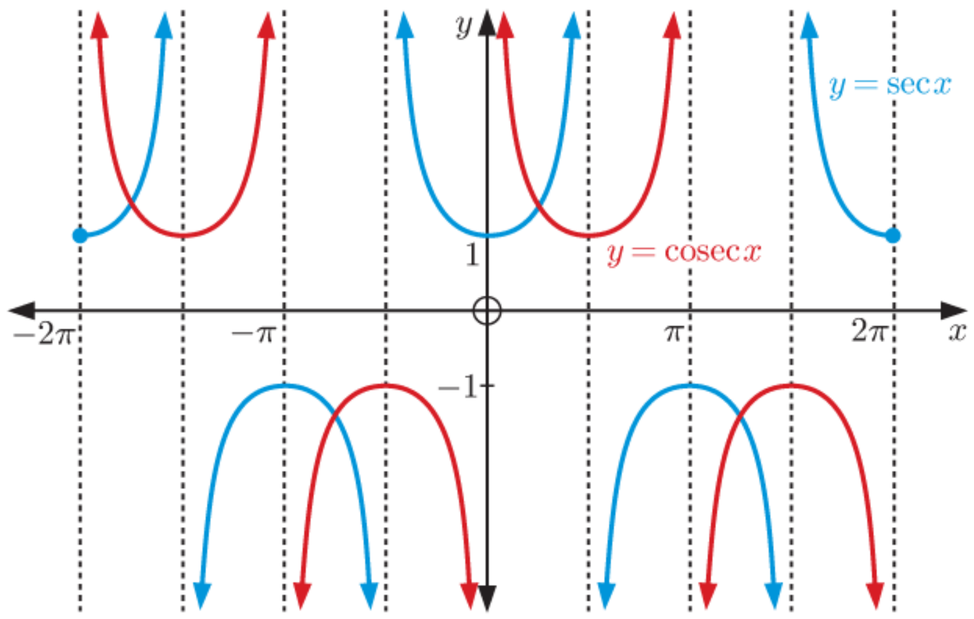
b **i** $A = \sqrt{7}$ **ii** $b \approx 2.43$

15 $\frac{\pi}{4}$

REVIEW SET 1B

1 $\sin x = -\frac{2\sqrt{2}}{3}$, $\tan x = 2\sqrt{2}$, $\operatorname{cosec} x = -\frac{3}{2\sqrt{2}}$, $\sec x = -3$, $\cot x = \frac{1}{2\sqrt{2}}$

2 **a**



b translation $\frac{\pi}{2}$ units right

3 $x = -\frac{5\pi}{6}$ or $\frac{\pi}{6}$ **4** **a** $x = \frac{\sqrt{3}}{2}$ **b** $x = 2 + \frac{1}{\sqrt{3}}$

5 **a** $\sec x$ **b** $\sin x$ **c** $\cos x$

6 **a** $\cos\theta$ **b** $-\sin\theta$ **c** $5\cos^2\theta$ **d** $-\cos\theta$
e $\operatorname{cosec}\theta$ **f** $\sin 2\theta$

7 **a** $\frac{120}{169}$ **b** $\frac{119}{169}$ **c** $\frac{120}{119}$

10 **a** $x = -\frac{2\pi}{3}, -\frac{\pi}{2}, -\frac{\pi}{3}$, or $\frac{\pi}{2}$ **b** $\theta = \frac{\pi}{3}$

11 $\sin\left(\theta + \frac{\pi}{6}\right) = \frac{3\sqrt{3}-\sqrt{7}}{8}$ **12** $\tan\theta = \frac{9}{19}$

13 $3\sin x + 4\cos x \approx 5\sin(x + 0.927)$ **14** 1.5 m

15 **b** $y = 2\sec 2x$ has range $\{y \mid y \leqslant -2 \text{ or } y \geqslant 2\}$

$$\therefore \quad \frac{1}{1+\sqrt{2}\sin x} + \frac{1}{1-\sqrt{2}\sin x} = 1 \quad \text{has no solutions.}$$

EXERCISE 2A

1 **a** $2^{\frac{1}{5}}$ **b** $2^{-\frac{1}{5}}$ **c** $2^{\frac{3}{2}}$ **d** $2^{\frac{5}{2}}$ **e** $2^{-\frac{1}{3}}$
f $2^{\frac{4}{3}}$ **g** $2^{\frac{3}{2}}$ **h** $2^{\frac{3}{2}}$ **i** $2^{-\frac{4}{3}}$ **j** $2^{-\frac{3}{2}}$

2 **a** $3^{\frac{1}{3}}$ **b** $3^{-\frac{1}{3}}$ **c** $3^{\frac{1}{4}}$ **d** $3^{\frac{3}{2}}$ **e** $3^{-\frac{5}{2}}$

3 **a** $7^{\frac{1}{3}}$ **b** $3^{\frac{3}{4}}$ **c** $2^{\frac{4}{5}}$ **d** $2^{\frac{5}{3}}$ **e** $7^{\frac{2}{7}}$
f $7^{-\frac{1}{3}}$ **g** $3^{-\frac{3}{4}}$ **h** $2^{-\frac{4}{5}}$ **i** $2^{-\frac{5}{3}}$ **j** $7^{-\frac{2}{7}}$

4 **a** $x^{\frac{1}{2}}$ **b** $x^{\frac{3}{2}}$ **c** $x^{-\frac{1}{2}}$ **d** $x^{\frac{5}{2}}$ **e** $x^{-\frac{3}{2}}$

5 **a** ≈ 2.28 **b** ≈ 0.435 **c** ≈ 1.68 **d** ≈ 1.93
e ≈ 0.523

6 **a** $\sqrt[3]{5}$ **b** $\frac{1}{\sqrt{3}}$ **c** $9\sqrt{3}$ **d** $m\sqrt{m}$ **e** $x^3\sqrt{x}$

7 **a** 8 **b** 32 **c** 8 **d** 125 **e** 4
f $\frac{1}{2}$ **g** $\frac{1}{27}$ **h** $\frac{1}{16}$ **i** $\frac{1}{81}$ **j** $\frac{1}{25}$

EXERCISE 2B

1 **a** 1 **b** x **c** $x^{\frac{1}{2}}$ or $\sqrt{x}$

2 **a** $x^5 + 2x^4 + x^2$ **b** $2^{2x} + 2^x$ **c** $x + 1$
d $7^{2x} + 2(7^x)$ **e** $2(3^x) - 1$ **f** $x^2 + 2x + 3$
g $1 + 5(2^{-x})$ **h** $5^x + 1$ **i** $x^{\frac{3}{2}} + x^{\frac{1}{2}} + 1$
j $3^{2x} + 5(3^x) + 1$ **k** $2x^{\frac{3}{2}} - x^{\frac{1}{2}} + 5$ **l** $2^{3x} - 3(2^{2x}) - 1$

3 **a** $2^{2x} + 2^{x+1} - 3$ **b** $3^{2x} + 7(3^x) + 10$
c $5^{2x} - 6(5^x) + 8$ **d** $2^{2x} + 6(2^x) + 9$
e $3^{2x} - 2(3^x) + 1$ **f** $4^{2x} + 14(4^x) + 49$
g $x - 4$ **h** $4^x - 9$ **i** $x - \frac{1}{x}$ **j** $x^2 + 4 + \frac{4}{x^2}$
k $7^{2x} - 2 + 7^{-2x}$ **l** $25 - 10(2^{-x}) + 2^{-2x}$

4 **a** $5^x(5^x + 1)$ **b** $10(3^n)$ **c** $7^n(1 + 7^{2n})$
d $5(5^n - 1)$ **e** $6(6^{n+1} - 1)$ **f** $16(4^n - 1)$
g $2^n(2^n - 8)$ **h** $\frac{5}{2}(2^n)$ **i** $\frac{9}{2}(2^{2n})$

5 **a** $(3^x + 2)(3^x - 2)$ **b** $(2^x + 5)(2^x - 5)$
c $(4 + 3^x)(4 - 3^x)$ **d** $(5 + 2^x)(5 - 2^x)$
e $(3^x + 2^x)(3^x - 2^x)$ **f** $(2^x + 3)^2$
g $(3^x + 5)^2$ **h** $(2^x - 7)^2$ **i** $(5^x - 2)^2$

6 **a** $(2^x + 1)(2^x - 2)$ **b** $(3^x + 3)(3^x - 2)$
c $(2^x - 3)(2^x - 4)$ **d** $(2^x + 3)(2^x + 6)$
e $(2^x + 4)(2^x - 5)$ **f** $(3^x + 2)(3^x + 7)$
g $(3^x + 5)(3^x - 1)$ **h** $(5^x + 2)(5^x - 1)$
i $(7^x - 4)(7^x - 3)$

7 **a** 2^n **b** 10^a **c** 3^b **d** $\frac{1}{5^n}$ **e** 5^x
f $(\frac{3}{4})^a$ **g** $(\frac{8}{3})^k$ **h** 5 **i** 5^n

8 **a** $3^m + 1$ **b** $1 + 6^n$ **c** $4^n + 2^n$ **d** $4^x - 1$
e 6^n **f** 5^n **g** 4 **h** $2^n - 1$ **i** $\frac{1}{2}$

9 **a** $n\,2^{n+1}$ **b** -3^{n-1}

EXERCISE 2C

1 **a** $x = 5$ **b** $x = 2$ **c** $x = 4$ **d** $x = 0$
e $x = -1$ **f** $x = \frac{1}{2}$ **g** $x = -3$ **h** $x = 2$
i $x = -3$ **j** $x = -4$ **k** $x = 2$ **l** $x = \frac{3}{4}$

2 **a** $x = \frac{5}{3}$ **b** $x = -\frac{3}{2}$ **c** $x = -\frac{3}{2}$ **d** $x = -\frac{1}{2}$
e $x = -\frac{2}{3}$ **f** $x = -\frac{5}{4}$ **g** $x = \frac{3}{2}$ **h** $x = \frac{5}{2}$
i $x = \frac{1}{8}$ **j** $x = \frac{9}{2}$ **k** $x = -4$ **l** $x = -\frac{7}{2}$
m $x = 0$ **n** $x = \frac{7}{2}$ **o** $x = -\frac{2}{3}$ **p** $x = -6$

3 **a** $x = \frac{1}{7}$ **b** no solution **c** $x = \frac{5}{2}$
d $x = \frac{1}{3}$ **e** $x = -\frac{1}{4}$ **f** $x = -1$ or 3

4 **a** $x = 3$ **b** $x = 2$ **c** $x = -1$ **d** $x = 2$
e $x = -2$ **f** $x = -2$

5 **a** $x = 1$ or 2 **b** $x = 1$ **c** $x = 1$ or 2
d $x = 1$ **e** $x = 2$ **f** $x = 0$
g $x = 1$ **h** $x = 1$ or -1 **i** $x = 2$
j $x = -2$ or 1 **k** $x = 2$ **l** $x = \frac{1}{2}$

6 $x = \frac{15}{7}$, $y = \frac{10}{7}$

EXERCISE 2D

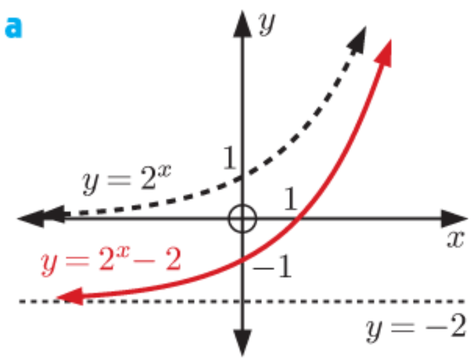
1 **a** **i** ≈ 1.4 **ii** ≈ 1.7 **iii** ≈ 2.8 **iv** ≈ 0.4

b **i** $x \approx 1.6$ **ii** $x \approx -0.7$

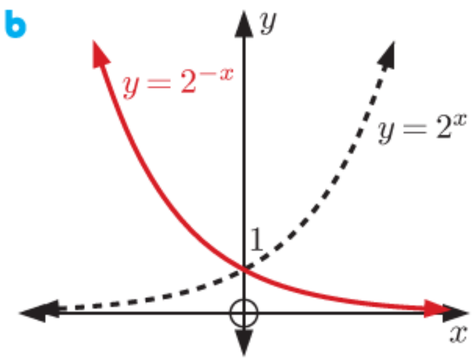
c $y = 2^x$ has a horizontal asymptote of $y = 0$.

2 **a** **C** **b** **B** **c** **E** **d** **A** **e** **D**

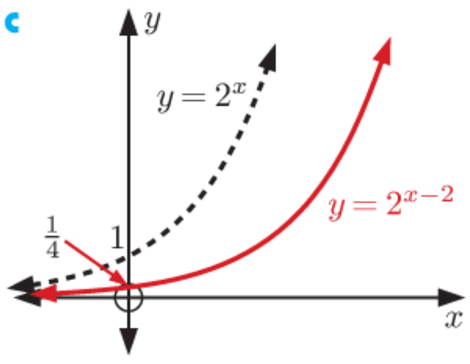
3 **a**



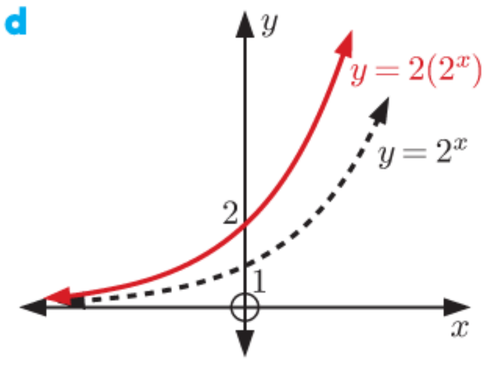
b



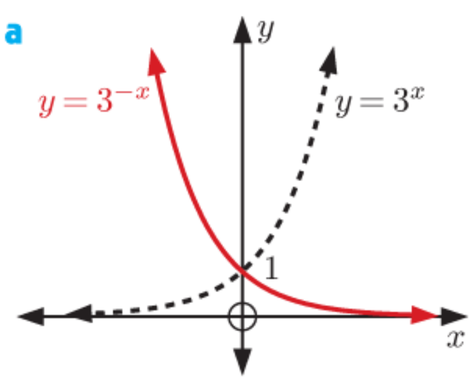
c



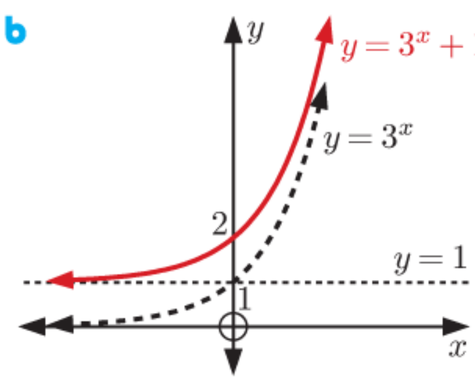
d



4 **a**



b



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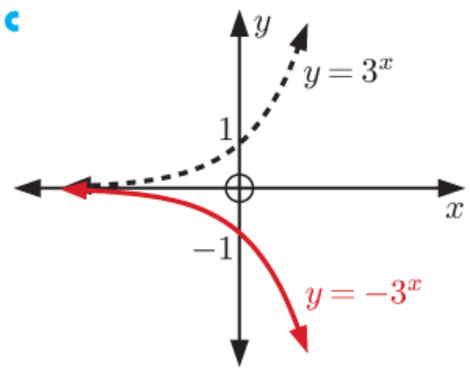
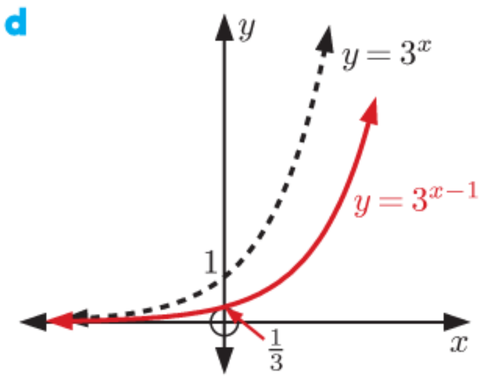


d



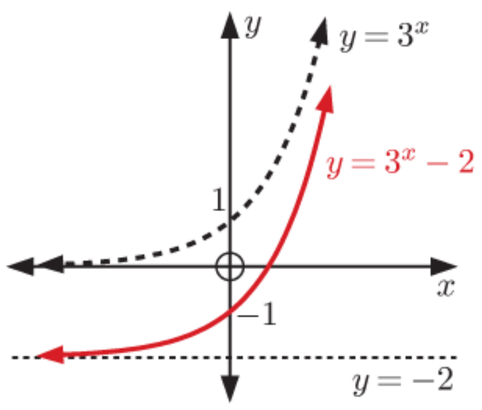
5 **a** $y = 0$ **b** $y = -1$ **c** $y = 3$ **d** $y = 2$
e $y = 0$ **f** $y = -4$

6 **a** **i** -1 **ii** 7
iii $-\frac{17}{9} = -1\frac{8}{9}$

b $y = -2$

d Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y > -2\}$

c

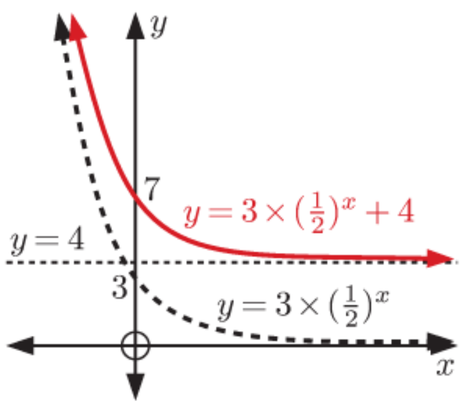


7 **a** **i** 7
ii $\frac{19}{4} = 4\frac{3}{4}$
iii 16

b $y = 4$

d Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y > 4\}$

c

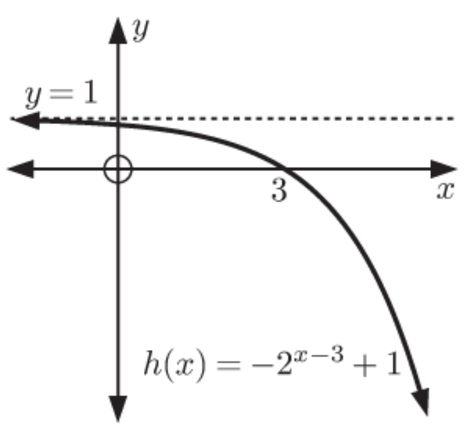


8 **a** **i** $\frac{7}{8}$ **ii** 0
iii -7

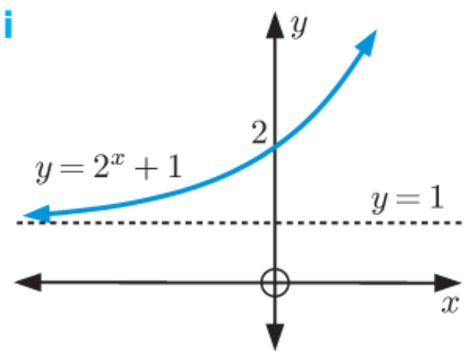
b $y = 1$

d Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y < 1\}$

c



9 **a** **i**



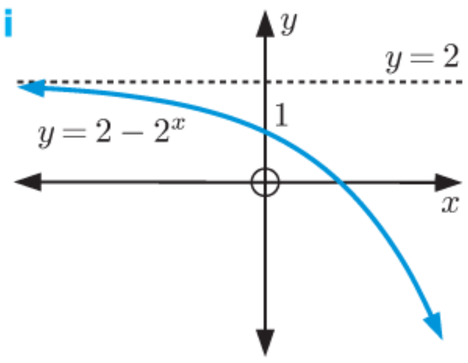
ii Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y > 1\}$

iii $y \approx 3.67$

iv as $x \to \infty$, $y \to \infty$
as $x \to -\infty$, $y \to 1^+$

v $y = 1$

b **i**



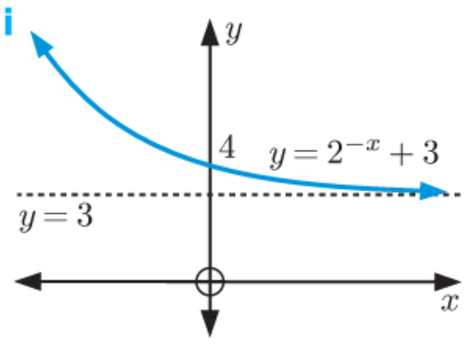
ii Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y < 2\}$

iii $y \approx -0.665$

iv as $x \to \infty$, $y \to -\infty$
as $x \to -\infty$, $y \to 2^-$

v $y = 2$

c **i**



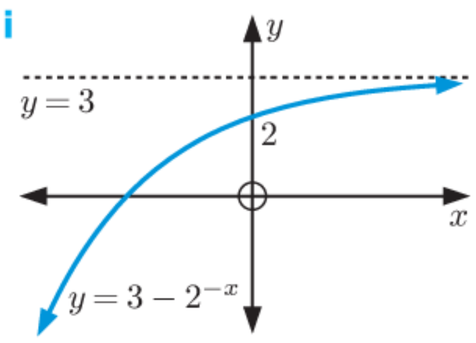
ii Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y > 3\}$

iii $y \approx 3.38$

iv as $x \to \infty$, $y \to 3^+$
as $x \to -\infty$, $y \to \infty$

v $y = 3$

d **i**



ii Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y < 3\}$

iii $y \approx 2.62$

iv as $x \to \infty$, $y \to 3^-$
as $x \to -\infty$, $y \to -\infty$

v $y = 3$

10 **a** $a = 5$, $b = -10$ **b** $y = 310$

11 **a** P$(0, 2.5)$ **b** $a = 1.5$ **c** $y = 3.5$

12 **a** Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y \geqslant 2\}$
b Domain is $\{x \mid x \neq 0\}$, Range is $\{y \mid y > 0,\ y < -1\}$
c Domain is $\{x \mid x \geqslant 1\}$, Range is $\{y \mid y \geqslant 0\}$

13 **a** $(f \circ g)(x) = 3^{\sqrt{x}} - 9$
Domain is $\{x \mid x \geqslant 0\}$, Range is $\{y \mid y \geqslant -8\}$

b $(g \circ f)(x) = \sqrt{3^x - 9}$
Domain is $\{x \mid x \geqslant 2\}$, Range is $\{y \mid y \geqslant 0\}$

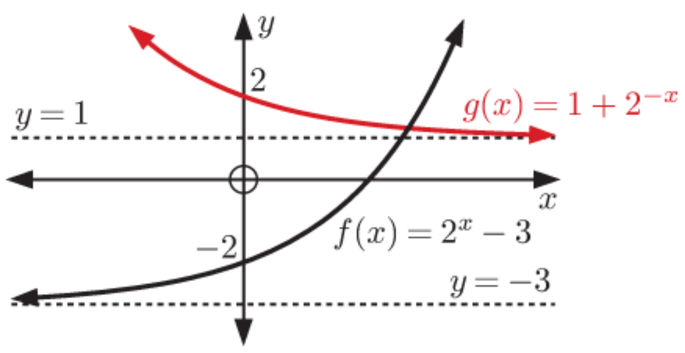
c **i** $x = 4$ **ii** $x = 3$

14 **a** **i** $f(x)$: $y = -3$, $g(x)$: $y = 1$

ii $f(x)$: Range is $\{y \mid y > -3\}$
$g(x)$: Range is $\{y \mid y > 1\}$

iii $f(x)$: y-intercept -2, $g(x)$: y-intercept 2

b

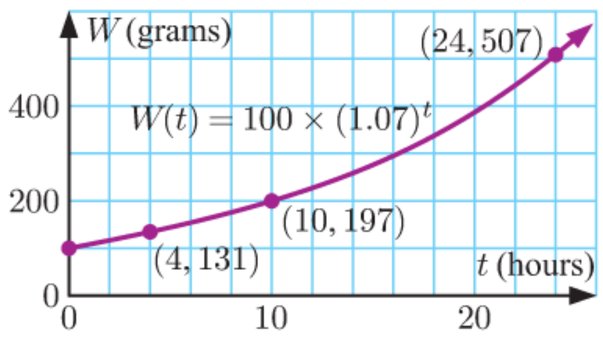


c $-1 + \sqrt{5}$

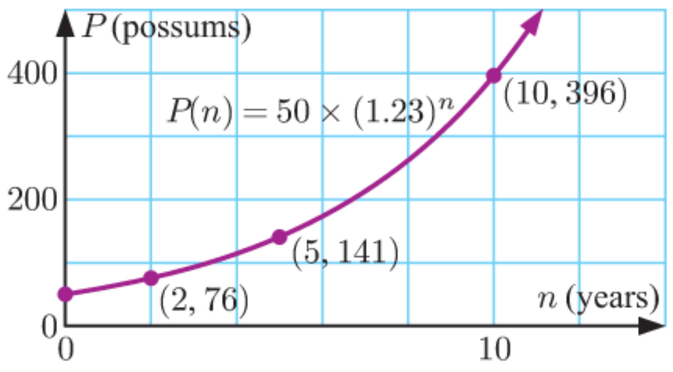
15 **a** $x \approx 3.46$ **b** $x \approx 2.46$ **c** $x \approx 1.16$
d $x \approx -0.738$ **e** $x \approx 1.85$ **f** $x \approx 0.0959$
g $x \approx 6.03$ **h** $x \approx 50.0$ **i** $x \approx 31.0$

EXERCISE 2E.1

1 **a** 100 grams
b **i** ≈ 131 g **ii** ≈ 197 g **iii** ≈ 507 g
c



2 **a** $P_0 = 50$
b **i** ≈ 76 possums **ii** ≈ 141 possums
iii ≈ 396 possums
c



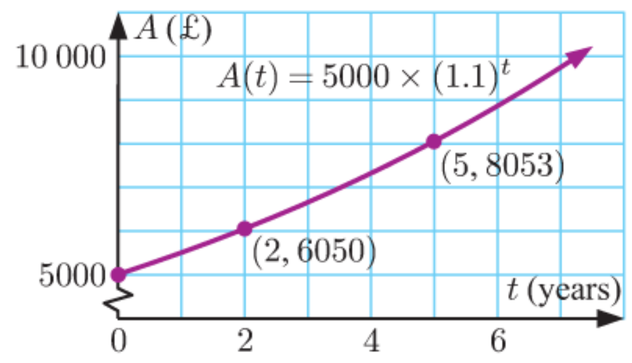
d ≈ 11 years **e** ≈ 11.1 years

3 **a** 4 people **b** ≈ 393 people **c** ≈ 19.9 days

4 **a** $B_0 = 200$
b $a = 1.1$, the bear population is increasing by 10% every year.
c ≈ 1350 bears **d** $\approx 159\%$ increase **e** ≈ 24.2 years

5 **a** **i** V_0 **ii** $2V_0$ **b** 100%
c $\approx 183\%$ increase, it is the percentage increase at 50°C compared with 20°C.

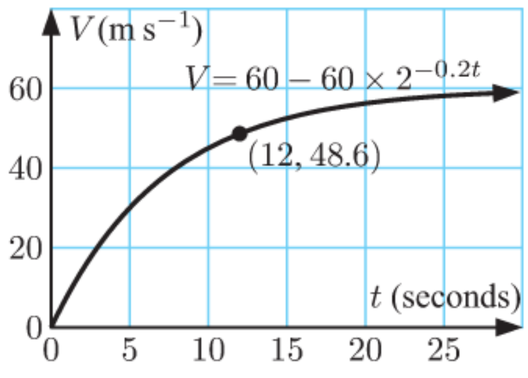
6 **a** $A(t) = 5000 \times (1.1)^t$ **b** **i** £6050 **ii** £8052.55
c



d ≈ 4.93 years

7 **a** $a = 1.08$, the expected value of the house is increasing by 8% per year.
$k = 375\,000$, the original value of the house was \$375 000.
b ≈ 4.98 years

8 **a** When $t = 0$, $V = c - 60 = 0$
$\therefore \quad c = 60$
b $k = -\frac{1}{5} = -0.2$
c $\approx 48.6 \text{ m s}^{-1}$ **d**

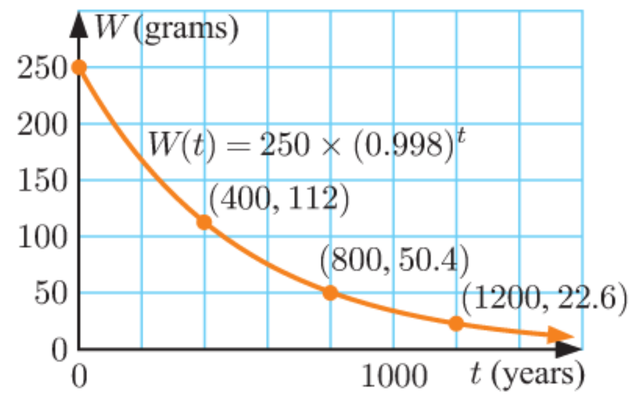


e The parachutist accelerates rapidly until he approaches his terminal velocity of 60 m s^{-1}.

9 ≈ 2.27 hours

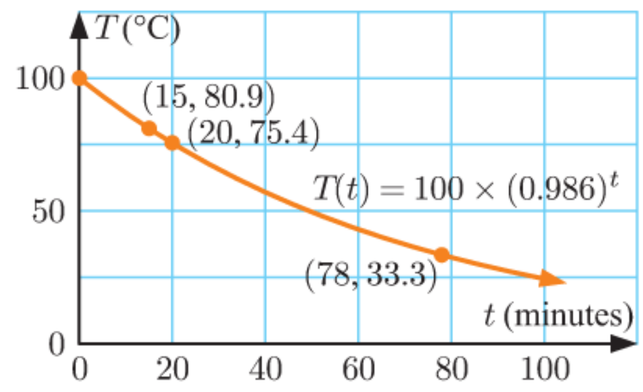
EXERCISE 2E.2

1 **a** 250 g **b** **i** ≈ 112 g **ii** ≈ 50.4 g **iii** ≈ 22.6 g
c

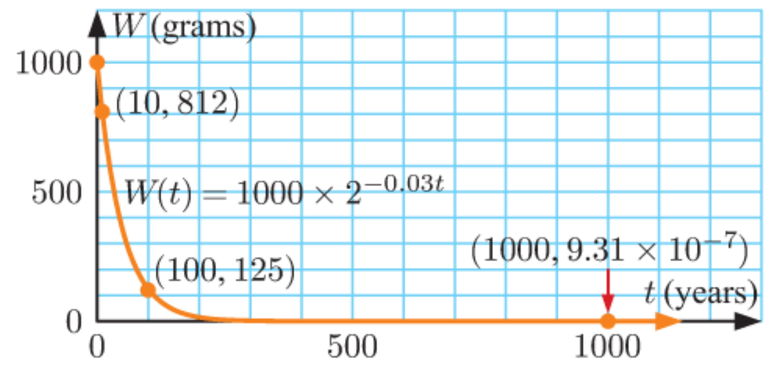


d ≈ 346 years

2 **a** 100°C
b **i** ≈ 80.9°C **ii** ≈ 75.4°C **iii** ≈ 33.3°C
c

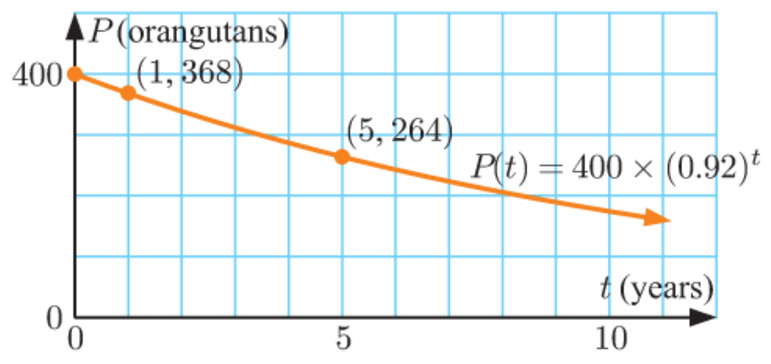


3 **a** 1000 g
b **i** ≈ 812 g **ii** 125 g **iii** $\approx 9.31 \times 10^{-7}$ g
c



d ≈ 221 years **e** $1000(1 - 2^{-0.03t})$ grams

4 **a** $P(t) = 400 \times (0.92)^t$
b **i** 368 orangutans **ii** ≈ 264 orangutans
c

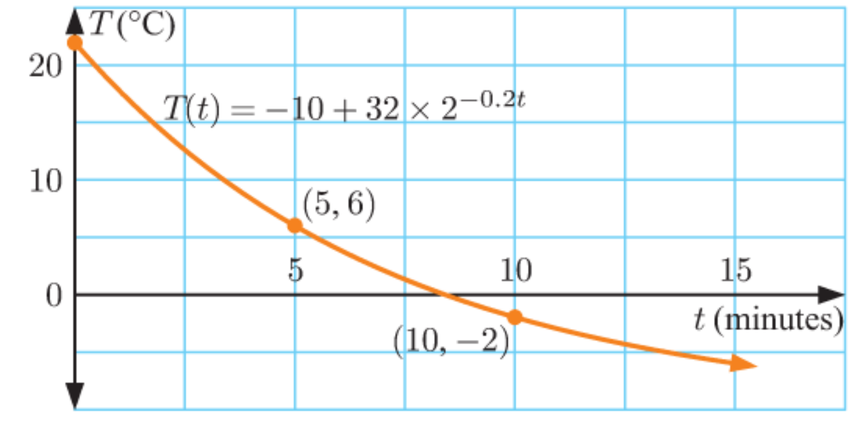


d ≈ 8.31 years, or ≈ 8 years 114 days

5 **a** $L_0 = 10$ units **b** ≈ 2.77 units **c** ≈ 17.9 m
d between ≈ 23.5 m and ≈ 44.9 m

6 **a** \$24 000 **b** $r = 0.85$ **c** 7 years

7 **a** **i** 22°C **ii** 6°C **iii** −2°C
b



c ≈ 8.39 min or ≈ 8 min 23 s
d No, as $32 \times 2^{-0.2t} > 0$ for any value of t.

8 **a** W_0 **b** $\approx 12.9\%$ **c** 45 000 years

9 **a** $A(t) = 150 \times (1.48)^{\frac{t}{3}}$, $B(t) = 400 \times (0.8)^t + 100$
b **i** $t \approx 4.16$ years **ii** $t \approx 3.45$ years **iii** $t \approx 1.69$ years

10 **a** The initial weight of the isotope is 10 mg.

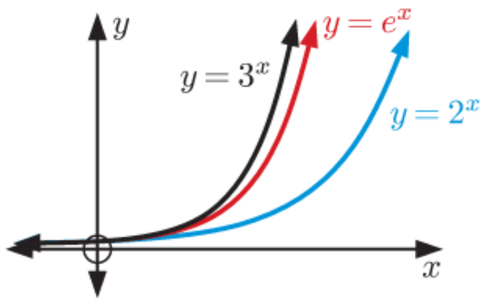
b $a \approx 0.7937$; each day the isotope's weight is decreasing by $\approx 20.63\%$.

c ≈ 6.30 mg **d** **i** ≈ 5.21 days **ii** ≈ 9.00 days

11 ≈ 33.2 minutes or ≈ 33 minutes 13.2 seconds

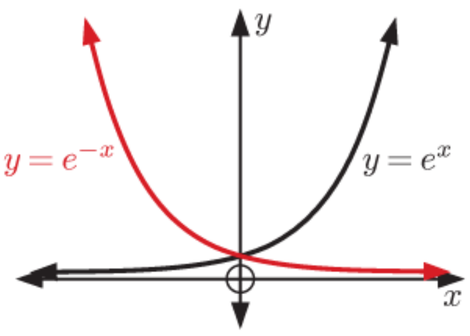
EXERCISE 2F

1



The graph of $y = e^x$ lies between $y = 2^x$ and $y = 3^x$.

2



One is the other reflected in the y-axis.

3 p

4 **a** $e^x > 0$ for all x

b **i** $y \approx 4.12 \times 10^{-9}$ **ii** $y \approx 9.70 \times 10^8$

5 **a** ≈ 7.39 **b** ≈ 20.1 **c** ≈ 2.01 **d** ≈ 1.65 **e** ≈ 0.368

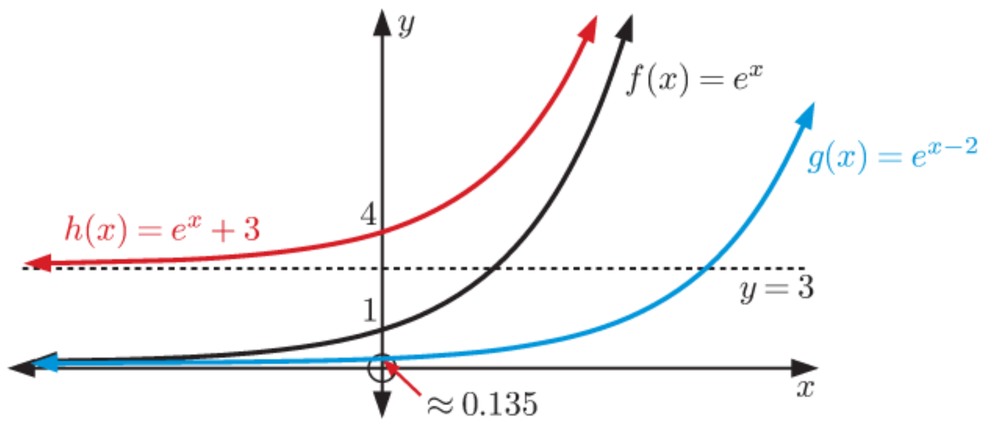
6 **a** $e^{\frac{1}{2}}$ **b** $e^{-\frac{1}{2}}$ **c** e^{-2} **d** $e^{\frac{3}{2}}$

7 **a** ≈ 10.074 **b** $\approx 0.099\,261$ **c** ≈ 125.09
d $\approx 0.007\,994\,5$ **e** ≈ 41.914 **f** ≈ 42.429
g ≈ 3540.3 **h** $\approx 0.006\,342\,4$

8 **a** $e^{2x} + 2e^x + 1$ **b** $1 - e^{2x}$ **c** $1 - 3e^x$

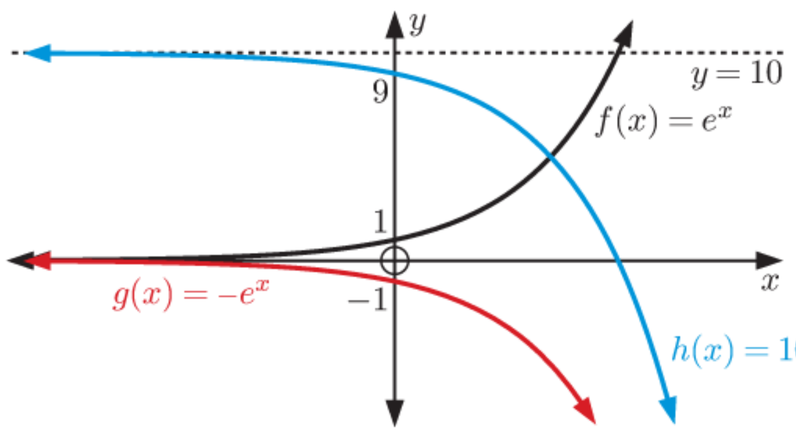
9 **a** $e^x(e^x + 1)$ **b** $(e^x + 4)(e^x - 4)$ **c** $(e^x - 6)(e^x - 2)$

10 **a**



b Domain of f, g, and h is $\{x \mid x \in \mathbb{R}\}$
Range of f is $\{y \mid y > 0\}$, Range of g is $\{y \mid y > 0\}$,
Range of h is $\{y \mid y > 3\}$

11 **a**



b Domain of f, g, and h is $\{x \mid x \in \mathbb{R}\}$
Range of f is $\{y \mid y > 0\}$, Range of g is $\{y \mid y < 0\}$,
Range of h is $\{y \mid y < 10\}$

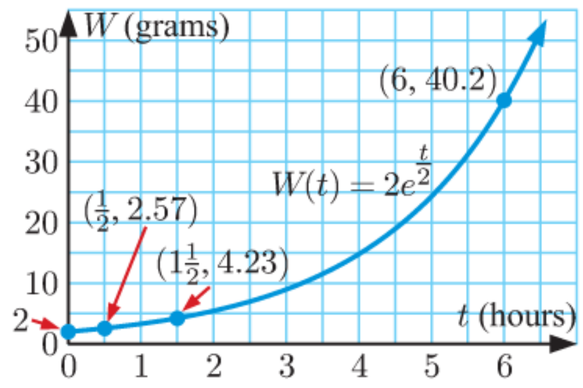
c For f: as $x \to \infty$, $y \to \infty$
as $x \to -\infty$, $y \to 0^+$
For g: as $x \to \infty$, $y \to -\infty$
as $x \to -\infty$, $y \to 0^-$
For h: as $x \to \infty$, $y \to -\infty$
as $x \to -\infty$, $y \to 10^-$

12 **a** $(f \circ g)(x) = e^{\frac{1}{x}} - 1$
Domain is $\{x \mid x \neq 0\}$
Range is $\{y \mid -1 < y < 0 \text{ or } y > 0\}$

b $(g \circ f)(x) = \dfrac{1}{e^x - 1}$
Domain is $\{x \mid x \neq 0\}$
Range is $\{y \mid y < -1 \text{ or } y > 0\}$

13 **a** **i** 2 g **ii** ≈ 2.57 g **iii** ≈ 4.23 g **iv** ≈ 40.2 g

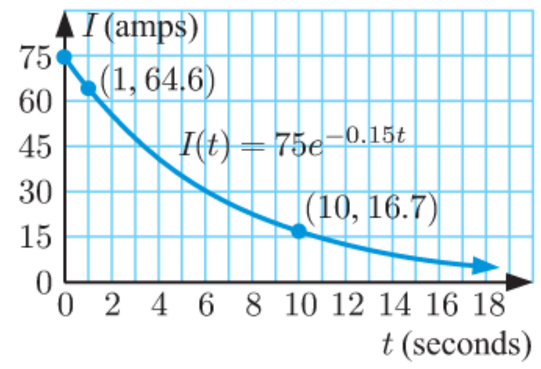
b



14 **a** $x = \frac{1}{2}$ **b** $x = -4$ **c** $x = 0$

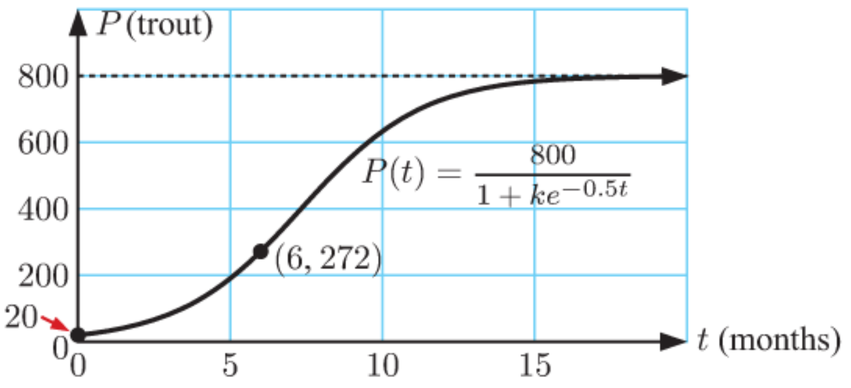
15 **a** **i** ≈ 64.6 amps **ii** ≈ 16.7 amps
c ≈ 28.8 seconds

b



16 **a** $k = 39$ **b** ≈ 272 trout

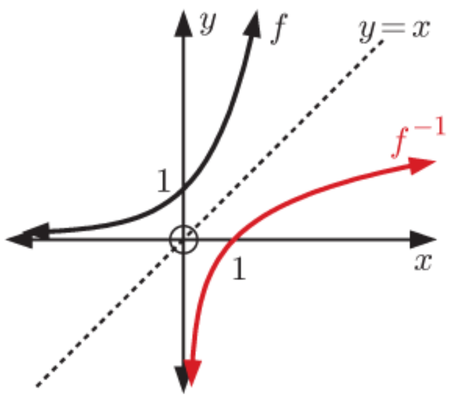
c



d As t increases, the population approaches a limiting value of 800 trout.

e ≈ 9.52 months

17 **a** $f^{-1}(x) = \log_e x$



b Domain of f^{-1} is $\{x \mid x > 0\}$
Range of f^{-1} is $\{y \mid y \in \mathbb{R}\}$

18 $e^1 \approx \sum_{i=0}^{19} \dfrac{1}{k!}\, 1^k \approx 2.718\,281\,828$

REVIEW SET 2A

1 **a** 4 **b** $\frac{1}{9}$ **c** $\frac{1}{3}$

2 **a** $x = -2$ **b** $x = \frac{3}{4}$ **c** $x = -\frac{1}{4}$

3 **a** $1 + e^{2x}$ **b** $2^{2x} + 10(2^x) + 25$ **c** $x - 49$

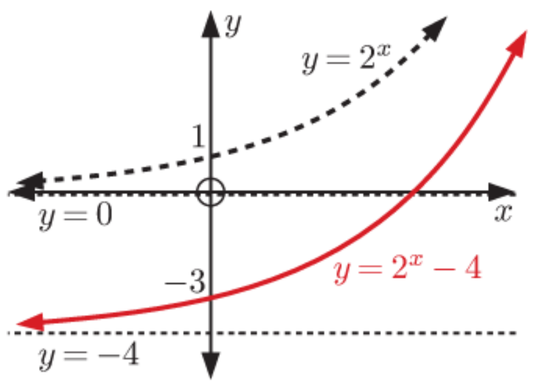
4 **a** $x = 5$ **b** $x = -\frac{5}{2}$ **c** $x = 3$ **5** $k = \frac{3}{2}$

6 **a** **i** ≈ 2.2 **ii** ≈ 0.6

b **i** $x \approx 1.45$ **ii** $x \approx -0.6$ **iii** $x \approx 1.1$

7 **a** 3 **b** 24 **c** $\frac{3}{4}$

8

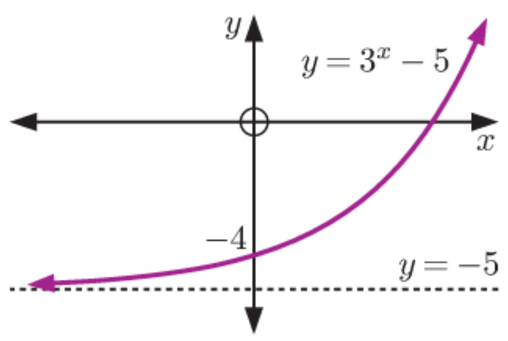


9 **a**

x	-2	-1	0	1	2
y	$-4\frac{8}{9}$	$-4\frac{2}{3}$	-4	-2	4

b as $x \to \infty$, $y \to \infty$
as $x \to -\infty$, $y \to -5^+$

c



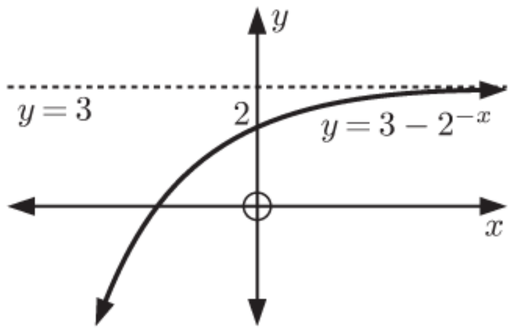
d $y = -5$

10 **a**

x	-2	-1	0	1	2
y	-1	1	2	$2\frac{1}{2}$	$2\frac{3}{4}$

b as $x \to \infty$, $y \to 3^-$
as $x \to -\infty$, $y \to -\infty$

c



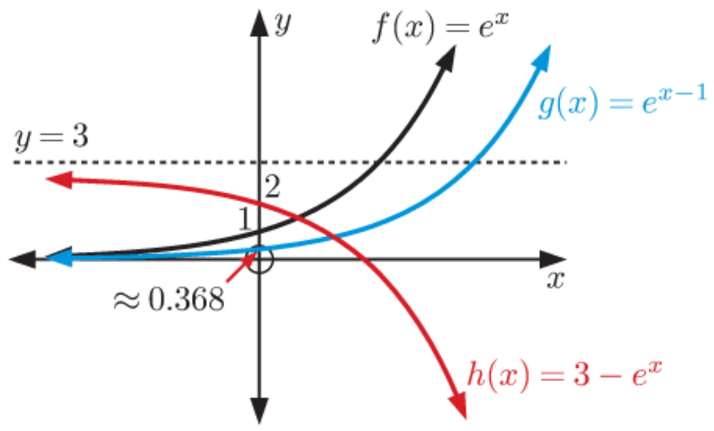
d $y = 3$

11 **a** $(f \circ g)(x) = 2^{3-x^2}$
Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid 0 < y \leqslant 8\}$

b $(g \circ f)(x) = 3 - 2^{2x} = 3 - 4^x$
Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y < 3\}$

c **i** $x = \pm\sqrt{2}$ **ii** $x = 2$

12 **a**



b For $f(x)$: domain is $\{x \mid x \in \mathbb{R}\}$, range is $\{y \mid y > 0\}$
For $g(x)$: domain is $\{x \mid x \in \mathbb{R}\}$, range is $\{y \mid y > 0\}$
For $h(x)$: domain is $\{x \mid x \in \mathbb{R}\}$, range is $\{y \mid y < 3\}$

c For $f(x)$: as $x \to \infty$, $f(x) \to \infty$
as $x \to -\infty$, $f(x) \to 0^+$
For $g(x)$: as $x \to \infty$, $g(x) \to \infty$
as $x \to -\infty$, $g(x) \to 0^+$
For $h(x)$: as $x \to \infty$, $h(x) \to -\infty$
as $x \to -\infty$, $h(x) \to 3^-$

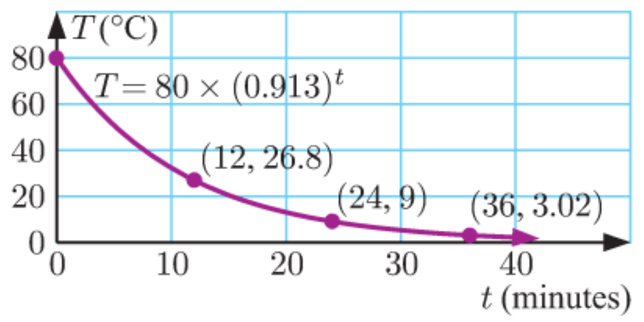
13 about every ≈ 7.92 days

14 **a** 80°C

b **i** ≈ 26.8°C
ii ≈ 9.00°C
iii ≈ 3.02°C

d ≈ 12.8 min

c



REVIEW SET 2B

1 **a** ≈ 3.95 **b** ≈ 0.517 **c** ≈ 3.16

2 **a** $9 - 6e^x + e^{2x}$ **b** $x - 2 - x^{-1}$ **c** $2^x + 1$

3 **a** $8(3^x)$ **b** $(2^x - 4)(2^x + 3)$ **c** $(e^x + 5)(e^x - 3)$

4 **a** $x = -4$ **b** $x = 0$ or 2 **c** $x = -1$ or -2

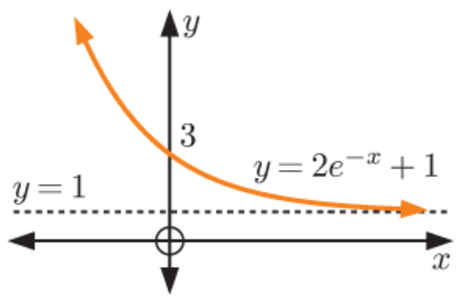
5 **a** $\frac{1}{\sqrt{2}} + 1 \approx 1.71$ **b** $a = -1$

6 **a**

x	-2	-1	0	1	2
y	15.8	6.44	3	1.74	1.27

b as $x \to \infty$, $y \to 1^+$
as $x \to -\infty$, $y \to \infty$

c



d $y = 1$

7 **a** clock: £525, vase: £428

b clock: $V(t) = 500 \times (1.05)^t$
vase: $V(t) = 400 \times (1.07)^t$

c clock $\approx$ £1039.46, vase $\approx$ £1103.61 $\therefore$ the vase

d $500 \times (1.05)^t = 400 \times (1.07)^t$ and solve for t;
$t \approx 11.8$ years

8 Domain is $\{x \mid x \geqslant -1\}$, Range is $\{y \mid y \geqslant 1\}$

9 **a** $f(x)$: y-intercept 2, $g(x)$: y-intercept -1

b $-\frac{5}{2} + \frac{1}{2}\sqrt{5}$ units

10 **a** **i** 81 **ii** $\frac{1}{3}$ **b** $k = 9$

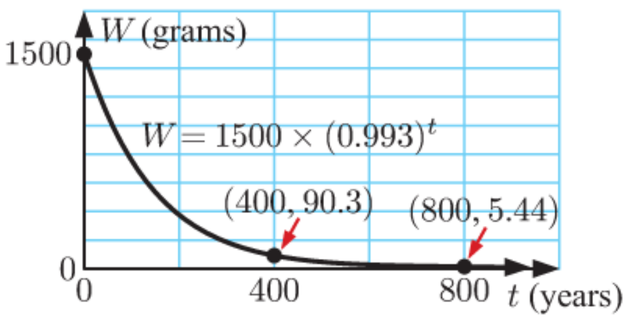
11 **a** y^2 **b** y^{-1} **c** $\frac{1}{\sqrt{y}}$ or $y^{-\frac{1}{2}}$

12 **a** 1500 g

b **i** ≈ 90.3 g
ii ≈ 5.44 g

d ≈ 386 years

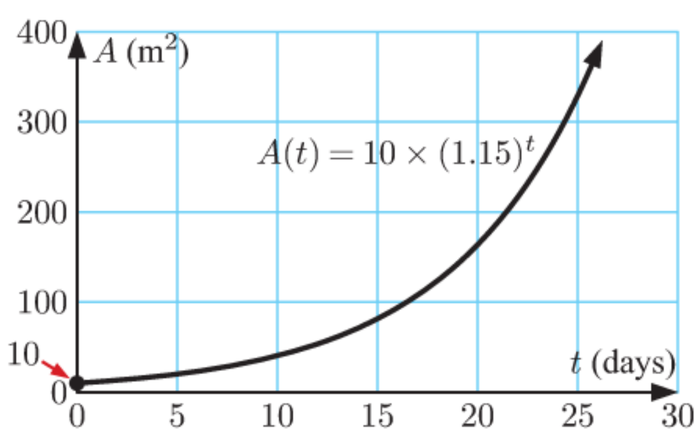
c



13 **a** $A(t) = 10 \times (1.15)^t$

b **i** 13.225 m^2 **ii** ≈ 20.1 m^2

c



d ≈ 24.3 days

EXERCISE 3A

1 **a** 4 **b** -3 **c** 1 **d** 0 **e** $\frac{1}{2}$ **f** $\frac{1}{3}$
g $-\frac{1}{4}$ **h** $1\frac{1}{2}$ **i** $\frac{2}{3}$ **j** $1\frac{1}{2}$ **k** $1\frac{1}{3}$ **l** $3\frac{1}{2}$