Mathematics: applications and interpretation

Practice paper 1 HL markscheme

1.

(a) $\frac{181}{740}\times 25 = 6.11486...$ **M1(A1)**

6 (students) **A1**

[3 marks]

(b) Quota and convenience **A1A1**

[2 marks]

Note: Award **A1A0** for one correct and one incorrect answer.

Total [5 marks]

2.

(a) 760 − 420 = 340 (g) **(M1)A1**

[2 marks]

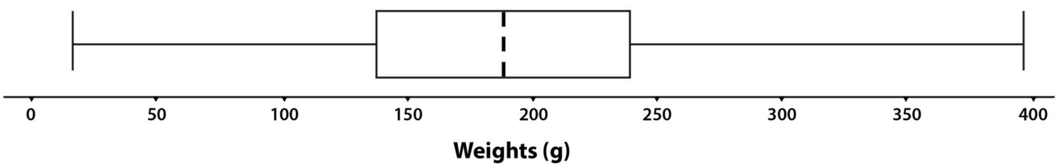
(b) (i) Median = 190 (g) **A1**

(ii) Lower quartile = 135–140 (g) **A1**

(iii) Upper quartile = 242–247 (g) **A1**

[3 marks]

(c)



Note: The **M1** is for a box and whisker plot and the **A1** for all 5 statistics in the right places.

M1A1

[2 marks]

Total [7 marks]

3.

(a) $(T = 1.517...m + 3.679...)$ $T = 1.52m + 3.68$ **A1A1**

[2 marks]

(b) 11.3 $(11.2671...)$ (°C) **(M1)A1**

[2 marks]

(c) (i) Because the line should only be used to estimate *T* from *m* and not *m* from *T*. **R1**

(ii) Because the temperatures are no longer going up at a steady rate, **or** because we know that winter is approaching so the temperature will go down, **or** temperatures are not likely to continue increasing. **R1**

[2 marks]

(d) Trigonometric or sinusoidal **A1**

[1 mark]

Total [7 marks]

4.

(a) Number of time periods $12 \times 5 = 60$ **(A1)**

N = 60
I% = 3.1
PV = 0
PMT = 200
P/Y = 12
C/Y = 12
Value ($)12 961.91 **(M1)A1**

[3 marks]

(b) **METHOD 1**

Real interest rate = $3.6 - 1.2 = 2.4\%$ **(M1)A1**

METHOD 2

$$\frac{1+0.036}{1+0.012} = 1.02371...$$ **(M1)**

2.37% (accept 2.4%) **A1**

[2 marks]

(c) **EITHER**

N = 5
I% = 2.4
PV = 1 000
PMT = 0
(P/Y = 1)
C/Y = 12 **(M1)**

OR

$$\left(1+\frac{0.024}{12}\right)^{60}$$ **(M1)**

THEN

($)1 127.36 **A1**

Note: Accept any answer which rounds to 1 130 and does not have more than two decimal places.

[2 marks]
Total [7 marks]

5.

(a)

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{pmatrix} 0 & 0.5 & 0 & 0 \\ 1 & 0 & 0.5 & 0.5 \\ 0 & 0.25 & 0 & 0.5 \\ 0 & 0.25 & 0.5 & 0 \end{pmatrix} \end{array}$$

(M1)A1A1

Note: Award **A1A0** if there is one error in the matrix. **A0A0** for more than one error.

[3 marks]

(b) Steady state column matrix is $\begin{pmatrix} 0.2 \\ 0.4 \\ 0.2 \\ 0.2 \end{pmatrix}$ **(M1)**

Probability it is in room B is 0.4 **A1**

[2 marks]
[Total 5 marks]

6. **Note:** Accept probabilities written as percentages throughout.

(a) 0.27 **A1**

[1 mark]

(b) $0.22\times0.28\times0.25\times0.73$ **(M1)**

$=0.0234\ (0.02338336)$ **A1**

[2 marks]

(c) $1-0.02338336$ **(M1)**

$=0.977\ (0.97661664)$ **A1**

[2 marks]

(d) $\mathrm{P}(\text{rains during lunch} \mid \text{rains at least once}) = \dfrac{\mathrm{P}(\text{rains during lunch})}{\mathrm{P}(\text{rains at least once})}$ **M1A1**

$\dfrac{0.27}{0.97661664} = 0.276\ (0.276464)$ **A1**

[3 marks]

Total [8 marks]

7.

(a) $|\boldsymbol{v}| = \sqrt{4.2^2 + 5.8^2 + 0.5^2}$ **(M1)**

$7.18\ (7.1784...)$ (kmh^{-1}) **A1**

[2 marks]

(b) $\boldsymbol{r} = \begin{pmatrix} 20 \\ -25 \\ 0 \end{pmatrix}$

$|\boldsymbol{r}| = \sqrt{20^2 + 25^2}$ **(M1)**

$= \sqrt{1025} = 32.0\ (32.0156...)$ (km) **A1**

[2 marks]

(c) Bearing is $\arctan\left(\dfrac{4.2}{5.8}\right)$ or $90^\circ - \arctan\left(\dfrac{5.8}{4.2}\right)$ **(M1)**

$035.9^\circ\ (35.909...)$ **A1**

[2 marks]

[Total 6 marks]

8.

(a) $a = 3, c = 1$ **A1A1**

$b = \frac{2\pi}{2}$ **(M1)**

$= \pi$ **A1**

$f(x) = 3\cos(\pi(x-1))$ **[4 marks]**

(b) Vertical stretch, scale factor 3 **A1**

Horizontal stretch, scale factor $\frac{1}{\pi} \approx 0.318$ **A1**

Horizontal translation of 1 unit to the right **A1**

Note: The vertical stretch can be at any position in the order of transformations. If the order of the final two transformations are reversed the horizontal translation is π units to the right.

[3 marks]
[Total 7 marks]

9.

(a) Area $= \frac{1}{2} \times 110 \times 85 \times \sin 55^\circ$ **(M1)(A1)**

= $3830\ (3829.53...)\,\mathrm{m}^2$ **A1**

[3 marks]

Note: units must be given for the final **A1** to be awarded.

(b) $BC^2 = 110^2 + 85^2 - 2 \times 110 \times 85 \times \cos 55^\circ$ **(M1)A1**

$BC = 92.7\ (92.731...)$ (m) **A1**

METHOD 1

Because the height and area of each triangle are equal they must have the same length base **R1**

D must be placed half-way along BC **A1**

$BD = \frac{92.731...}{2} \approx 46.4$ (m) **A1**

Note: the final two marks are dependent on the **R1** being awarded.

METHOD 2

Let $C\hat{B}A = \theta^\circ$

$\frac{\sin\theta}{110} = \frac{\sin 55^\circ}{92.731...}$ **M1**

$\Rightarrow \theta = 76.3^\circ\ (76.3354...)$

Use of area formula

$\frac{1}{2} \times 85 \times BD \times \sin(76.33...^\circ) = \frac{3829.53...}{2}$ **A1**

$BD = 46.4\ (46.365...)(m)$ **A1**

[6 marks]

Total [9 marks]

10.

(a) (i) $1252.7... \approx 1250$ (barrels per day) **A1**

(ii) This is the increase (change) in *P* (production per day) between $t = 0$ and $t = 5$ (or during the first 5 days) **A1**

[2 marks]

(b) **METHOD 1**

$P = 1000\ln(2+t) + c$ **(M1)A1**

$c = 20000 - 1000\ln 2 \approx 19306.8...$ **(M1)A1**

$P = 1000\ln(2+t) + 19300$

METHOD 2

$\int_{20000}^{P} \mathrm{d}P = \int_{0}^{t} \frac{1000}{2+x}\mathrm{d}x$ **(M1)**

$\left[P\right]_{20000}^{P} = 1000\left[\ln(2+x)\right]_{0}^{t}$ **A1**

Note: A1 is for the correct integral, with the correct limits.

$P - 20000 = 1000(\ln(2+t) - \ln 2)$ **(M1)A1**

$P = 1000\ln\left(\frac{2+t}{2}\right) + 20000$ **[4 marks]**

(c) 8847883 ≈ 8850000 (barrels) **A1**

Total production of oil in barrels in the first year (or first 365 days) **A1**

Note: For the final **A1** "barrels"' must be present either in the statement or as the units.

Accept any value which rounds correctly to 8850000

[2 marks]
Total [8 marks]

11. Odd vertices are B, F, H and I **(M1)A1**

Pairing the vertices **M1**

BF and HI   9 + 3 = 12
BH and FI   4 + 11 = 15
BI and FH   3 +8 = 11 **A2**
Note: award **A1** for two correct totals.

Shortest time is 105+11=116 (minutes) **M1A1**

Total [7 marks]

12.
(a) a staight line with a negative gradient **A1A1**
[2 marks]

(b) $\log P = -2.040...\log d - 0.12632... \approx -2.04\log d - 0.126$ **A1A1**

Note: A1 for each correct term.

[2 marks]

(c) (i) $n = 2$ **A1**

(ii) $P = 10^{-0.126...}d^{-2}$ **(M1)**

$\approx 0.748d^{-2}$ **A1**
[3 marks]
[Total 7 marks]

13.
(a) $\dot{x} = y$ **M1**
$\dot{y} = 2t - 4y^2$ **A1**
[2 marks]

(b) $t_{n+1} = t_n + 0.1$

$x_{n+1} = x_n + 0.1y_n$

$y_{n+1} = y_n + 0.1\left(2t_n - 4y_n^2\right)$ **(M1)(A1)**

Note: Award **M1** for a correct attempt to substitute the functions in part (a) into the formula for Euler's method for coupled systems.

When $t = 1$

$x = 0.202\ (0.20201...)$ **A1**

$\dot{x} = 0.598\ (0.59822...)$ **A1**

Note: Accept $y = 0.598$.

[4 marks]

Total [6 marks]

14.

(a) $5.385...\mathrm{e}^{1.1902...\mathrm{i}} \approx 5.39\mathrm{e}^{1.19\mathrm{i}}$ **A1A1**

Note: Accept equivalent answers: $5.39\mathrm{e}^{-5.09\mathrm{i}}$

[2 marks]

(b) multiply by $\mathrm{e}^{\frac{2\pi}{3}\mathrm{i}}$ **(M1)**

$-5.33 - 0.77\mathrm{i},\ 3.33 - 4.23\mathrm{i}$ **A1A1**

[3 marks]

Total [5 marks]

15.

(a) $f'(x) = \left(-2ax + 1\right) \times \frac{1}{2} \times \left(-ax^2 + x + a\right)^{-\frac{1}{2}}$

Note: M1 is for use of the chain rule.

$= \dfrac{-2ax + 1}{2\sqrt{-ax^2 + x + a}}$ **M1A1**

[2 marks]

(b) $-2ax+1=0$ **(M1)**

$x=\frac{1}{2a}$ **A1**

[2 marks]

(c) Value of local maximum = $\sqrt{-a\times\frac{1}{4a^2}+\frac{1}{2a}+a}$ **M1A1**

$=\sqrt{\frac{1}{4a}+a}$

This has a minimum value when $a=0.5$ **(M1)A1**

[4 marks]

Total [8 marks]

16.

(a) Let X be the number of people who arrive between 9.00 am and 9.01 am

$X\sim$ Po(9)

$\mathrm{P}(X>7)=\mathrm{P}(X\geq 8)$ **(M1)**

$0.676\ (0.67610...)$ **A1**

[2 marks]

(b) Mean number of people arriving each 30 seconds is 4.5 **(M1)**

Let X_1 be the number who arrive in the first 30 seconds and X_2 the number who arrive in the second 30 seconds.

P(Shunsuke will be able to get on the ride)

$$=\mathrm{P}(X_1\leq 4)\times\mathrm{P}(X_2\leq 3)+\mathrm{P}(X_1=5)\times\mathrm{P}(X_2\leq 2)+$$
$$\mathrm{P}(X_1=6)\times\mathrm{P}(X_2\leq 1)+\mathrm{P}(X_1=7)\times\mathrm{P}(X_2=0)$$

M1M1

Note: M1 for first term, **M1** for any of the other terms

= $0.1821...+0.02965...+0.007828...+0.0009149...$ **(A1)(A1)**

Note: (A1) for one correct value, **(A1)(A1)** for four correct values.

= $0.221\ (0.220531...)$ **A1**

[6 marks]

Total [8 marks]