International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Mathematics: applications and interpretation

Practice paper 1 HL **Total 110**

1. [Maximum mark: 5]

A school consists of 740 students divided into 5 grade levels. The numbers of students in each grade are shown in the table below.

Grade	8	9	10	11	12
Number of students	120	125	119	195	181

The Principal of the school wishes to select a sample of 25 students. She wishes to ensure that, as closely as possible, the proportion of the students from each grade in the sample is the same as the proportions in the school.

(a) Calculate the number of grade 12 students who should be in the sample. [3]

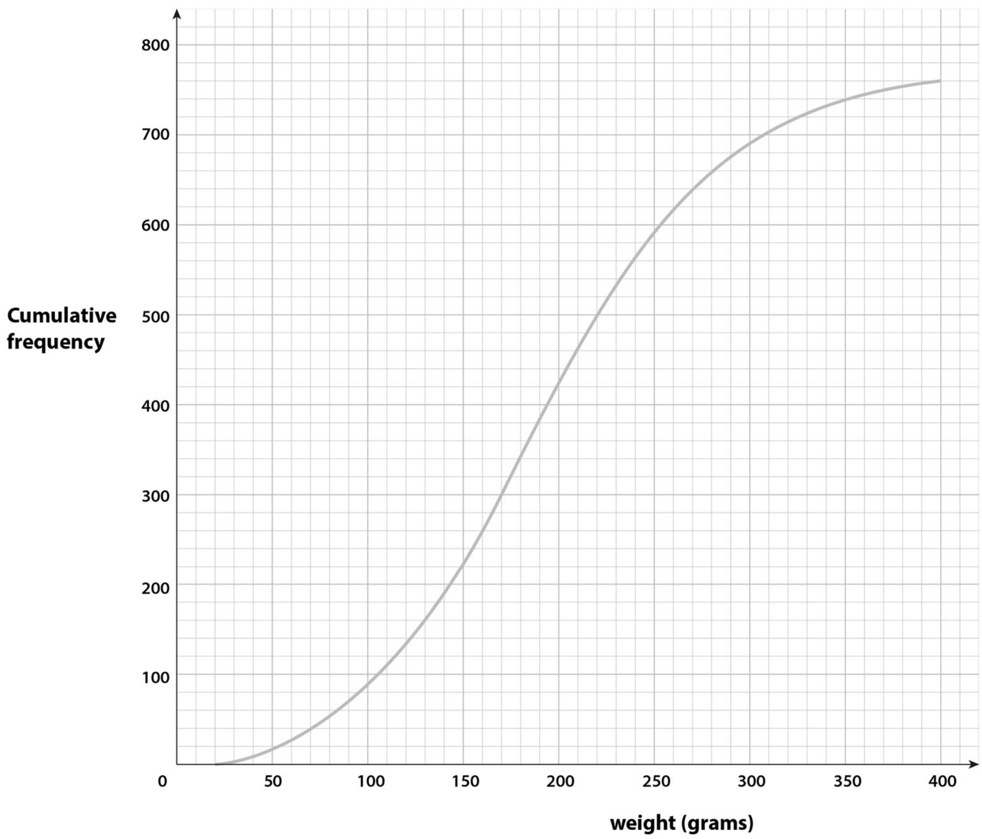
The Principal selects the students for the sample by asking those who took part in a previous survey if they would like to take part in another. She takes the first of those who reply positively, up to the maximum needed for the sample.

(b) State which two of the sampling methods listed below best describe the method used. [2]

Stratified   Quota   Convenience   Systematic   Simple random

2. [Maximum mark: 7]

A food scientist measures the weights of 760 potatoes taken from a single field and the distribution of the weights is shown by the cumulative frequency curve below.



(a) Find the number of potatoes in the sample with a weight of more than 200 grams. [2]

(b) Find

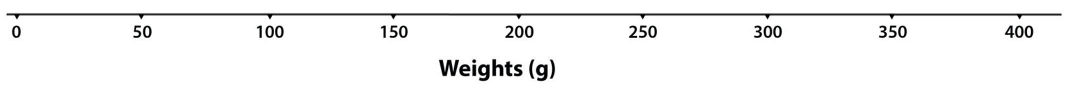
(i) the median weight

(ii) the lower quartile

(iii) the upper quartile. [3]

The weight of the smallest potato in the sample is 20 grams and the weight of the largest is 400 grams.

(c) Use the scale shown below to draw a box and whisker diagram showing the distribution of the weights of the potatoes. You may assume there are no outliers. [2]



3. [Maximum mark: 7]

The water temperature (T) in Lake Windermere is measured on the first day of eight consecutive months (m) from January to August (months 1 to 8) and the results are shown below. The value for May (month 5) has been accidently deleted.

	Jan	Feb	March	April	May	June	July	August
Month (m)	1	2	3	4	5	6	7	8
Temperature (T)(°C)	5.2	8.0	7.2	8.9		12.6	15.5	15.4

(a) Assuming the data follows a linear model for this period, find the regression line of T on m for the remaining data. [2]

(b) Use your line to find an estimate for the for the water temperature on the first day of May. [2]

(c) (i) Explain why your line should not be used to estimate the value of m at which the temperature is 10.0°C.

(ii) Explain in context why your line should not be used to predict the value for December (month 12). [2]

(d) State a more appropriate model for the water temperature in the lake over an extended period of time. You are not expected to calculate any parameters. [1]

4. [Maximum mark: 7]

Sophia pays \$200 into a bank account at the end of each month. The annual interest paid on money in the account is 3.1% which is compounded monthly.

(a) Find the value of her investment after a period of 5 years. [3]

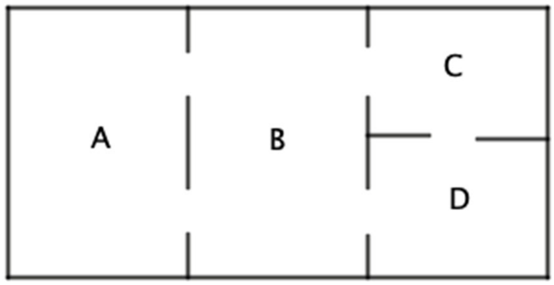
The average rate of inflation per year over a 5-year period was 1.2% per year. Ravi invested \$1 000 in an account which paid 3.6% interest per year compounded monthly at the start of this 5-year period.

(b) Find an approximation for the real interest rate for the money invested in the account. [2]

(c) Hence find the real value of the \$1 000 invested by Ravi at the end of 5 years. Give your answer to two decimal places. [2]

5. [Maximum mark: 5]

A robot moves around the maze shown below.



Whenever it leaves a room it is equally likely to take any of the exits.

The time interval between the robot entering and leaving a room is the same for all transitions.

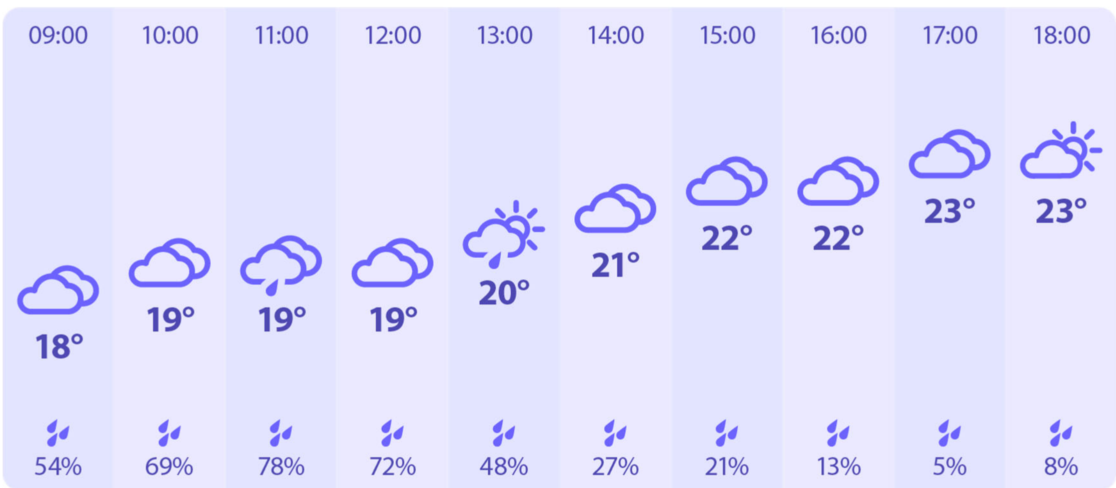
(a) Find the transition matrix for the maze. [3]

A scientist sets up the robot and then leaves it moving around the maze for a long period of time.

(b) Find the probability that the robot is in room B when the scientist returns. [2]

6. [Maximum mark: 8]

The diagram below shows part of the screen from a weather forecasting website showing the data for town A. The percentages on the bottom row represent the likelihood of some rain during the hour leading up to the time given. For example, there is a 69% chance (a probability of 0.69) of rain falling on any point in town A between 0900 and 1000.



Paula works at a building site in the area covered by this page of the website from 0900 to 1700. She has lunch from 1300 to 1400.

(a) Write down the probability it rains during Paula's lunch break. [1]

In the following parts you may assume all probabilities are independent.

Paula needs to work outside between 1000 and 1300 and will also spend her lunchtime outside.

(b) Find the probability it will not rain while Paula is outside. [2]

(c) Find the probability it will rain at least once while Paula is outside. [2]

(d) Given it rains at least once while Paula is outside find the probability that it rains during her lunch hour. [3]

7. [Maximum mark: 6]

The position of a helicopter relative to a communications tower at the top of a mountain at time *t* (hours) can be described by the vector equation below.

$$\boldsymbol{r} = \begin{pmatrix} 20 \\ -25 \\ 0 \end{pmatrix} + t\begin{pmatrix} 4.2 \\ 5.8 \\ -0.5 \end{pmatrix}$$

The entries in the column vector give the displacements east and north from the communications tower and above/below the top of the mountain respectively, all measured in kilometres.

(a) Find the speed of the helicopter. [2]

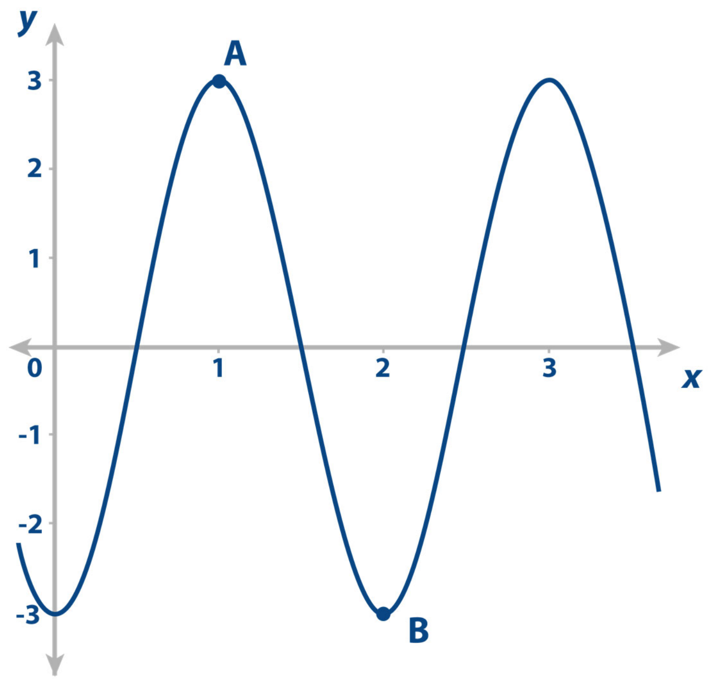
(b) Find the distance of the helicopter from the communications tower at $t = 0$. [2]

(c) Find the bearing on which the helicopter is travelling. [2]

8. [Maximum mark: 7]

Let $f(x) = a\cos(b(x-c))$, $a,b,c \in \mathbb{R}^{+}$.

Part of the graph of $y = f(x)$ is shown below. Point A is a local maximum and has coordinates $(1, 3)$ and point B is a local minimum with coordinates $(2, -3)$.



(a) Find

(i) the value of *a*

(ii) the value of *b*

(iii) the least value of *c*. [4]

(b) Write down a sequence of transformations that will transform the graph of $y = \cos x$ onto the graph of $y = f(x)$. [3]

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9. [Maximum mark: 9]

A farmer owns a triangular field ABC. The length of side [AB] is 85 m and side [AC] is 110 m. The angle between these two sides is 55°.

(a) Find the area of the field. [3]

The farmer would like to divide the field into two equal parts by constructing a straight fence from A to a point D on [BC].

(b) Find BD. Fully justify any assumptions you make. [6]

10. [Maximum mark: 8]

The production of oil (P), in barrels per day, from an oil field satisfies the differential equation $\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{1000}{2+t}$ where t is measured in days from the start of production.

(a) (i) Find $\int_0^5 \frac{1000}{2+t}\mathrm{d}t$.

(ii) State in context what this value represents. [2]

The production of oil at $t=0$ is 20,000 barrels per day.

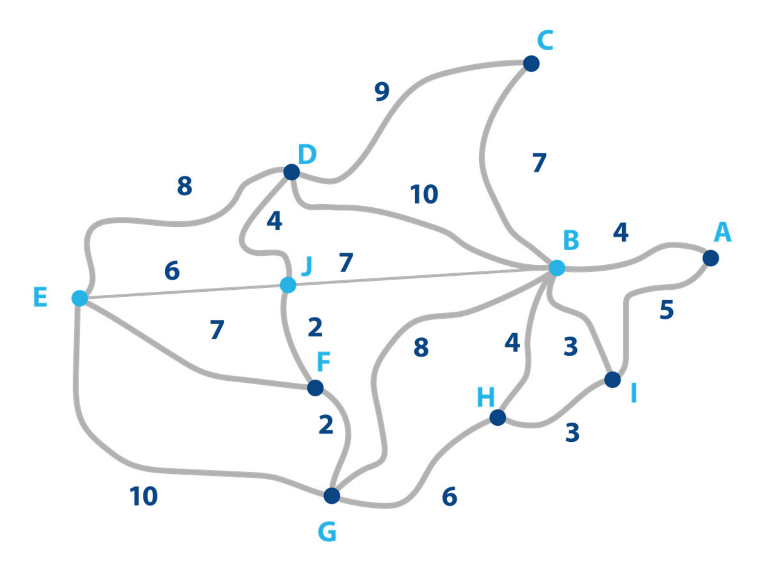
(b) Find an expression for P in terms of t. [4]

(c) Determine $\int_0^{365} P(t)\,\mathrm{d}t$ and state what it represents. [2]

11. [Maximum mark: 7]

Nymphenburg Palace in Munich has extensive grounds with 9 points of interest (stations) within them.

These nine points, along with the palace, are shown as the vertices in the graph below. The weights on the edges are the walking times in minutes between each of the stations and the total of all the weights is 105 minutes.



A	Nymphenburg Palace
B	Fountain
C	Magdalenenklause
D	Pagodenburg
E	Large Cascade
F	Pan with Spring
G	Badenburg
H	Amalienburg
I	Summerhouse
J	Bridge

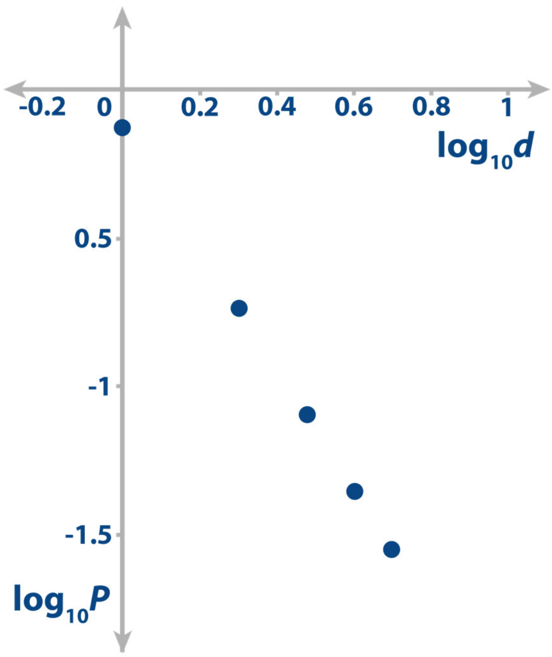
Anders decides he would like to walk along all the paths shown beginning and ending at the Palace (vertex A).

Use the Chinese Postman algorithm, clearly showing all the stages, to find the shortest time to walk along all the paths. [7]

12. [Maximum mark: 7]

It is believed that the power *P* of a signal at a point *d* km from an antenna is inversely proportional to d^n where $n \in \mathbb{Z}^+$.

The value of *P* is recorded at distances of 1 m to 5 m and the values of $\log_{10} d$ and $\log_{10} P$ are plotted on the graph below.



(a) Explain why this graph indicates that *P* is inversely proportional to d^n. [2]

The values of $\log_{10} d$ and $\log_{10} P$ are shown in the table below.

$\log_{10} d$	0	0.301	0.477	0.602	0.699
$\log_{10} P$	-0.127	-0.740	-1.10	-1.36	-1.55

(b) Find the equation of the least squares regression line of $\log_{10} P$ against $\log_{10} d$. [2]

(c) (i) Use your answer to part (b) to write down the value of *n* to the nearest integer.

(ii) Find an expression for *P* in terms of *d*. [3]

13. [Maximum mark: 6]

Consider the second order differential equation

$$\ddot{x} + 4(\dot{x})^2 - 2t = 0$$

where x is the displacement of a particle for $t \geq 0$.

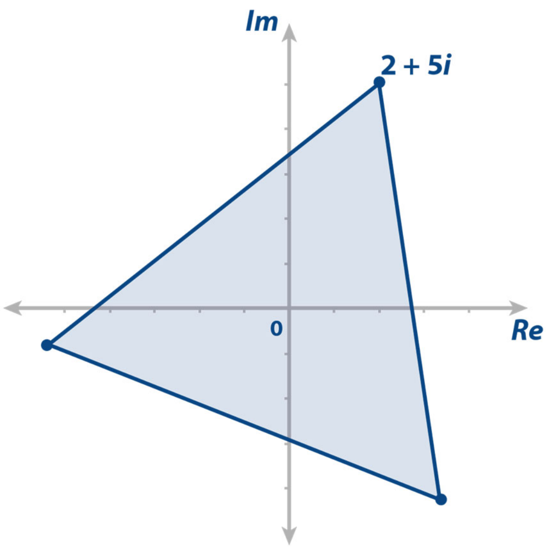
(a) Write the differential equation as a system of coupled first order differential equations. [2]

When $t = 0$, $x = \dot{x} = 0$

(b) Use Euler's method with a step length of 0.1 to find an estimate for the value of the displacement and velocity of the particle when $t = 1$. [4]

14. [Maximum mark: 5]

(a) Write down $2 + 5\mathrm{i}$ in exponential form. [2]



An equilateral triangle is to be drawn on the Argand plane with one of the vertices at the point corresponding to $2 + 5\mathrm{i}$ and all the vertices equidistant from 0.

(b) Find the points that correspond to the other two vertices. Give your answers in Cartesian form. [3]

15. [Maximum mark: 8]

Consider the function $f(x)=\sqrt{-ax^2+x+a},\ a\in\mathbb{R}^+$.

(a) Find $f'(x)$. [2]

For $a>0$ the curve $y=f(x)$ has a single local maximum.

(b) Find in terms of *a* the value of *x* at which the maximum occurs. [2]

(c) Hence find the value of *a* for which *y* has the smallest possible maximum value. [4]

16. [Maximum mark: 8]

The cars for a fairground ride hold four people. They arrive at the platform for loading and unloading every 30 seconds.

During the hour from 9 am the arrival of people at the ride in any interval of *t* minutes can be modelled by a Poisson distribution with a mean of $9t\ (0<t<60)$.

When the 9 am car leaves there is no one in the queue to get on the ride.

Shunsuke arrives at 9.01 am.

(a) Find the probability that more than 7 people arrive at the ride before Shunsuke. [2]

(b) Find the probability there will be space for him on the 9.01 car. [6]