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Mathematics: applications and interpretation

Practice paper 2 HL markscheme

1.

(a) Substitution of 100 or 200 into $5x-2y=615$ **(M1)**

(i) $x=203$ **A1**

(ii) $x=163$ ***A1***

[3 marks]

(b) Divides area into two appropriate shapes

For example

Area of triangle $\left(\frac{1}{2}\times 200\times 40=\right)4000$ m² **(A1)**

Area of rectangle $\left(200\times 97\right)=19400$ m² **(A1)**

23400 (m²) **A1**

Note: The area can be found using different divisions. Award **A1** for any two correct areas found and **A1** for the final answer.

[3 marks]

(c) **EITHER**

$\frac{200\times 300-23400}{2}=18300$ m² **(M1)A1**

OR

$\frac{1}{2}\times 100\times\left(203+163\right)=18300\,\text{m}^2$ **(M1)A1**

THEN

Area managed by both offices A and B is 18300 (m²) **A1**

[3 marks]

(d) Density of accommodation/students is uniform **R1**

[1 mark]

(e) $250-163=87$ (m) **(M1)A1**

Note: M1 is for an attempt to find the distance from the intersection point to one of the offices.

[2 marks]

Total [12 marks]

2.

(a) The area *A* is less than the area of the rectangle containing the cross-section which is equal to $6\times3.5=21$ **R1**

Note: $6\times3.5=21$ is not sufficient for **R1**

[1 mark]

(b) $\frac{1}{2}\times2\times(2+3.5)+\frac{1}{2}\times4\times(3.5+1)$ **(M1)(A1)**

$=14.5$ **A1**

This is an underestimate as the trapezoids are enclosed by (are under) the curve. **R1**

Note: This can be shown in a diagram.

[4 marks]

(c) $h(0)=2\Rightarrow d=2$ **A1**

[1 mark]

(d) $h'(x)=3ax^2+2bx+c$ **A1**

$h'(2)=0$ **M1**

hence $12a+4b+c=0$ **AG**

[2 marks]

(e) Substitute the points $(2,3.5)$ and $(6,1)$ **(M1)**

$8a+4b+2c+2=3.5\ (8a+4b+2c=1.5)$

and

$216a+36b+6c+2=1\ (216a+36b+6c=-1)$ **A1A1**

[3 marks]

(f) Solve on a GDC **(M1)**

$h(x) = 0.0365x^3 - 0.521x^2 + 1.65x + 2$ **A2**

$(h(x) = 0.0364583...x^3 - 0.520833...x^2 + 1.64583...x + 2)$ **[3 marks]**

(g) $\int_0^6 0.0364583...x^3 - 0.520833...x^2 + 1.64583...x + 2 \text{ d}x$

= 15.9 (15.9374...) (m^2) **(M1)A1**

Note: Accept 16.0 (16.014) from the three significant figure answer to part (g).

[2 marks]

Total [16 marks]

3.

(a) $s_{n-1} = \sqrt{\frac{50}{49}} \times 0.257$ **(M1)**

Note: M1 is for the use of the correct formula

$= 0.260$ **A1**

[2 marks]

(b) Using $\bar{x} = 0.828$ and $s_{n-1} = 0.260$ **(M1)**

$a = 7.3, b = 7.6$ **A1A1**

[3 marks]

(c)

Height (h)	$h<0.6$	$0.6 \le h < 0.7$	$0.7 \le h < 0.8$	$0.8 \le h < 0.9$	$0.9 \le h < 1.0$	$1.0 \le h < 1.1$	$h \ge 1.1$
Expected	9.5	6.1	7.3	7.6	6.8	5.3	7.4
Observed	7	4	9	10	7	7	6

$\chi^2_{calc} = 3.35$ **(M1)A1**

[2 marks]

(d) Combining columns with expected values less than 5 leaves 7 cells **(M1)**

7-1-2=4 **A1**

[2 marks]

(e) 3.35 < 9.49 **R1**

hence insufficient evidence to reject H_0 that the heights of the waves are normally distributed. **A1**

Note: The **A1** can be awarded independently of the **R1.**

[2 marks]
[Total 11 marks]

4.

(a) $\frac{\mathrm{d}N}{\mathrm{d}t} = rAe^{rt}$ **(M1)A1**

Note: M1 is for an attempt to find $\frac{\mathrm{d}N}{\mathrm{d}t}$

$= rN$ **AG**

Note: Accept solution of the differential equation by separating variables

[2 marks]

(b) $N(n+1) \approx N(n) + 1 \times N'(n) \Rightarrow N'(n) \approx N(n+1) - N(n)$ **M1**

$\Rightarrow rN(n) \approx N(n+1) - N(n)$ **M1A1**

$\Rightarrow r \approx \frac{N(n+1) - N(n)}{N(n)}$ **AG**

Note: Do not penalize the use of the = sign.

[3 marks]

(c) Correct method **(M1)**

$r \approx \frac{52-40}{40} = 0.3$

$r \approx \frac{70-52}{52} = 0.346$

$r \approx \frac{98-70}{70} = 0.4$ **A2**

Note: A1 for a single error **A0** for two or more errors.

[3 marks]

(d) $r = 0.349\ (0.34871...)$ or $\frac{68}{195}$ **A1**

$52 = Ae^{0.34871...\times 2}$ **(M1)**

$A = 25.8887...$

$N = 25.9e^{0.349t}$ **A1**

[3 marks]

(e) $(36.6904... - 40)^2 + 0 + (73.6951... - 70)^2 + (104.4435... - 98)^2$ **(M1)**

= $66.1\ (66.126...)$ **A1**

[2 marks]

(f) (i) The sum of the square residuals is approximately 10 times as large as the minimum possible, so Jorge's model is unlikely to fit the data exactly **R1**

(ii) For example
Selecting a single point for the curve to pass through

Approximating the gradient of the curve by the gradient of a chord **R1R1**

[3 marks]
[Total 16 marks]

5.

(a) (i) $\begin{vmatrix} 0.3-\lambda & -0.1 \\ -0.2 & 0.4-\lambda \end{vmatrix} = 0$ **(M1)(A1)**

$\lambda =$ 0.5 and 0.2 **A1**

(ii) Attempt to solve either

$$\begin{pmatrix} -0.2 & -0.1 \\ -0.2 & -0.1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix} \textbf{ or } \begin{pmatrix} 0.1 & -0.1 \\ -0.2 & 0.2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

or equivalent **(M1)**

$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ **A1A1**

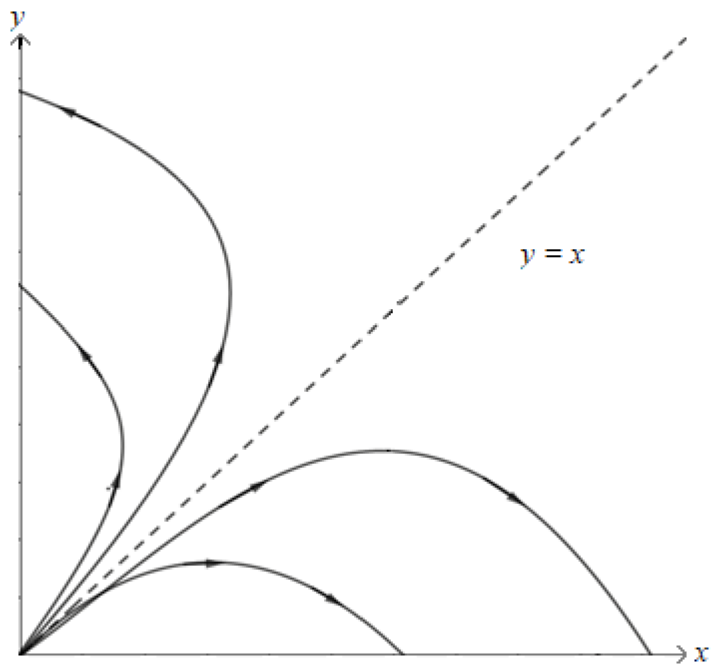
Note: accept equivalent forms

[6 marks]

(b) $\begin{pmatrix} x \\ y \end{pmatrix} = Ae^{0.5t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + Be^{0.2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ **A1**

[1 mark]

(c)



A1A1A1

Note: A1 for $y = x$ correctly labelled, **A1** for at least two trajectories above $y = x$ and **A1** for at least two trajectories below $y = x$, including arrows.

[3 marks]

(d) $y > 2000$ **A1**

[1 mark]

(e) (i) $\begin{pmatrix} x \\ y \end{pmatrix} = Ae^{0.5t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + Be^{0.2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

At $t = 0$ $2000 = A + B$, $2900 = -2A + B$ **M1A1**

Note: Award **M1** for the substitution of 2000 and 2900

Hence $A = -300$, $B = 2300$ **A1A1**

$0 = -300e^{0.5t} + 2300e^{0.2t}$ **M1**

$t = 6.79\,(6.7896...)$ (years) **A1**

(ii) $y = 600e^{0.5\times 6.79} + 2300e^{0.2\times 6.79}$ **(M1)**

$= 26827.9...$

= 26830 (to the nearest 10 animals) **A1**

[8 marks]

[Total 19 marks]

6.

(a) Let X represent the mass of a melon

$\mathrm{P}(X > 3.0) = 0.212\,(0.2118...)$ **(M1)A1**

[2 marks]

(b) (i) $0.2118... \times 0.2118...$ **(M1)**

$= 0.0449\,(0.04488...)$ **A1**

(ii) Let T represent the total mass

$\mathrm{E}(T) = 5.2$ **A1**

$\mathrm{Var}(T) = 0.5^2 + 0.5^2 = 0.5$ **(M1)A1**

$T \sim \mathrm{N}(5.2, 0.5)$

$\mathrm{P}(T > 6.0) = 0.129\ (0.1289...)$ ***A1***

[6 marks]

(c) Let μ be the mean mass of the melons produced by the farm.

H_0: μ = 2.6 kg, H_1: μ > 2.6 kg only **A1**

Note: Accept H_0: The mean mass of melons produced by the farm is equal to 2.6 kg

H_1: The mean mass of melons produced by the farm is greater than 2.6 kg

Note: Award **A0** if 2.6 kg does not appear in the hypothesis.

[1 mark]

(d) Under H_0 $\bar{X} \sim \mathrm{N}\left(2.6, \frac{0.5^2}{16}\right)$ **A1**

$\mathrm{P}(\bar{X} > a) = 0.05$ **(M1)**

$a = 2.81\ (2.805606...)$ **(A1)**

Critical region is $\bar{X} > 2.81$ **A1**

[4 marks]

(e) Let W represent the new mass of the melons

$\mathrm{E}(W) = 1.1 \times 2.6 = 2.86$ **A1**

Standard deviation of $W = 1.1 \times 0.5$ **(M1)**

$= 0.55$ **A1**

Note: award **M1A0** for $\mathrm{Var}(W) = 1.1^2 \times 0.5^2 = 0.3025$

[3 marks]

(f) P(Type II error)=

$$\mathrm{P}\left(\bar{X} < 2.81 \mid \mu = 2.86, \sigma = \frac{0.55}{4}\right)$$ **(M1)**

= $0.346\ (0.346204...)$ **A1**

[2 marks]

Note: Accept 0.358 from use of the three-figure answer to part (d)

Total [18 marks]

7.

(a) $|\boldsymbol{r}| = \sqrt{1.5^2\cos^2\left(0.1t^2\right) + 1.5^2\sin^2\left(0.1t^2\right)}$ **M1**

=1.5 as $\sin^2\theta + \cos^2\theta = 1$ **R1**

Note: use of the identity needs to be explicitly stated.

Hence moves in a circle as displacement from a fixed point is constant. **R1**

Radius = 1.5 m **A1**

[4 marks]

(b) (i) $\boldsymbol{v} = \begin{pmatrix} -0.3t\sin\left(0.1t^2\right) \\ 0.3t\cos\left(0.1t^2\right) \end{pmatrix}$ **M1A1**

Note: M1 is for an attempt to differentiate each term

(ii) $\boldsymbol{v} \bullet \boldsymbol{r} = \begin{pmatrix} 1.5\cos\left(0.1t^2\right) \\ 1.5\sin\left(0.1t^2\right) \end{pmatrix} \bullet \begin{pmatrix} -0.3t\sin\left(0.1t^2\right) \\ 0.3t\cos\left(0.1t^2\right) \end{pmatrix}$ **M1**

Note: M1 is for an attempt to find $\boldsymbol{v} \bullet \boldsymbol{r}$

$$= 1.5\cos\left(0.1t^2\right) \times \left(-0.3t\sin\left(0.1t^2\right)\right) + 1.5\sin\left(0.1t^2\right) \times 0.3t\sin\left(0.1t^2\right) = 0$$

A1

Hence velocity and position vector are perpendicular. **AG**

[4 marks]

(c) (i) $\boldsymbol{a} = \begin{pmatrix} -0.3\sin\left(0.1t^2\right) - 0.06t^2\cos\left(0.1t^2\right) \\ 0.3\cos\left(0.1t^2\right) - 0.06t^2\sin\left(0.1t^2\right) \end{pmatrix}$ **M1A1A1**

(ii)

$$\left(-0.3\sin\left(0.1t^2\right) - 0.06t^2\cos\left(0.1t^2\right)\right)^2 + \left(0.3\cos\left(0.1t^2\right) - 0.06t^2\sin\left(0.1t^2\right)\right)^2 = 400$$

(M1)(A1)

Note: M1 is for an attempt to equate the magnitude of the acceleration to 20.

$t = 18.3\ \left(18.256...\right)$ (s) **A1**

(iii) Angle turned through is $0.1 \times 18.256^2 =$ **M1**

$= 33.329...$ **A1**

$\frac{33.329}{2\pi}$ **M1**

$\frac{33.329}{2\pi} = 5.30...$

5 complete revolutions **A1**

[10 marks]

[Total marks 18]