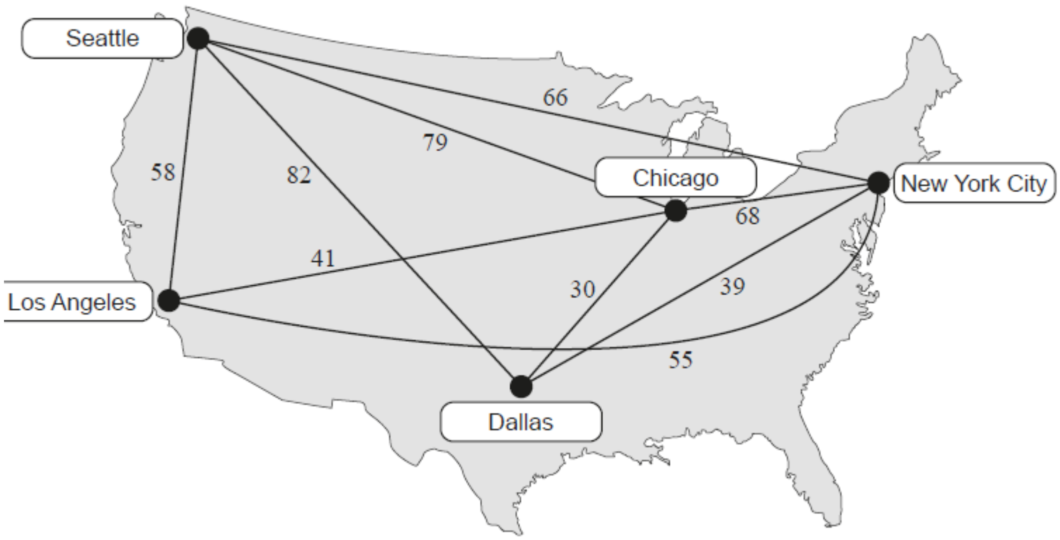
Algorithms - revision 2 [91 marks]

1. [Maximum mark: 19] 23M.2.AHL.TZ2.4

The following graph shows five cities of the USA connected by weighted edges representing the cheapest direct flights in dollars ($\$$) between cities.



(a) Explain why the graph can be described as "connected", but not "complete". [2]

(b) Find a minimum spanning tree for the graph using Kruskal's algorithm.

State clearly the order in which your edges are added, and draw the tree obtained. [3]

(c) Using only the edges obtained in your answer to part (b), find an upper bound for the travelling salesman problem. [2]

Ronald lives in New York City and wishes to fly to each of the other cities, before finally returning to New York City. After some research, he finds that there exists a direct flight between Los Angeles and Dallas costing $\$26$. He updates the graph to show this.

(d) By using the nearest neighbour algorithm and starting at Los Angeles, determine a better upper bound than that found in part (c).

State clearly the order in which you are adding the vertices. [3]

(e.i) By deleting the vertex which represents Chicago, use the deleted vertex algorithm to determine a lower bound for the travelling salesman problem. [3]

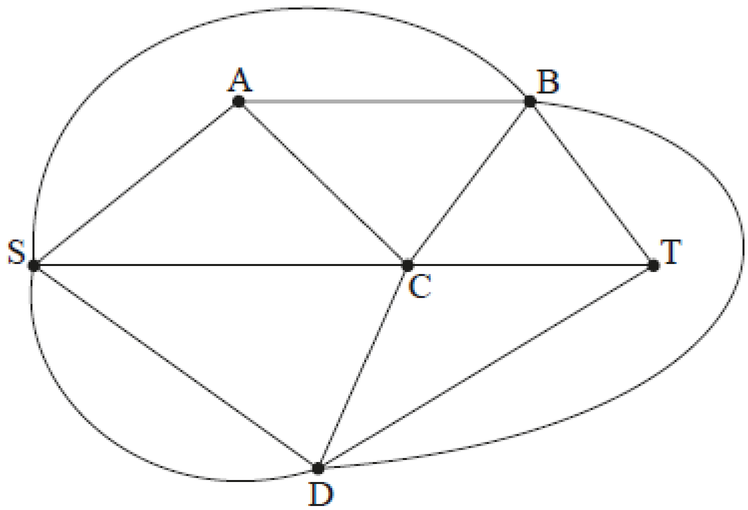
(e.ii) Similarly, by instead deleting the vertex which represents Seattle, determine another lower bound. [2]

(f) Hence, using your previous answers, write down your best inequality for the **least** expensive tour Ronald could take. Let the variable C represent the total cost, in dollars, for the tour. [2]

(g) Write down a tour that is strictly greater than your lower bound and strictly less than your upper bound. [2]

2. [Maximum mark: 7] 22N.1.AHL.TZ0.4

In a competition, a contestant has to move through a maze to find treasure. A graph of the maze is shown below, where each edge represents a corridor in the maze. The contestant starts at S and the treasure is located at T.



(a) Complete the adjacency matrix, $\boldsymbol{M}$, for the graph.

	S	A	B	C	D	T
S	0	1	1	1	☐	0
A	1	0	1	1	☐	0
B	1	1	0	1	1	1
C	1	1	1	0	1	1
D	☐	☐	1	1	0	1
T	0	0	1	1	1	0

[2]

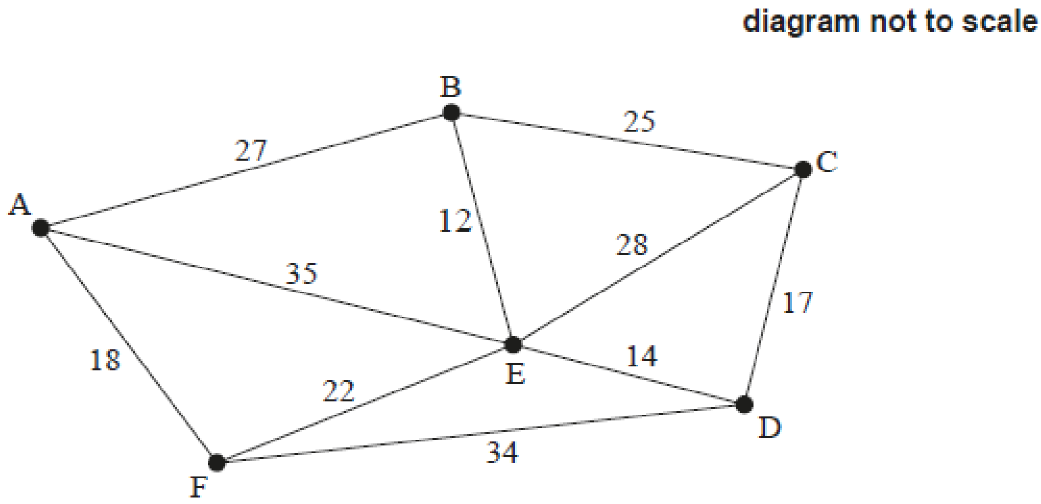
The competition rules state that the contestant can walk along a maximum of four corridors.

(b) Find the number of walks from S to T with a maximum of 4 edges. [4]

(c) Explain why the number of ways the contestant can reach the treasure is less than the answer to part (b). [1]

3. [Maximum mark: 14] 22N.2.AHL.TZ0.4

A company has six offices, A, B, C, D, E and F. One of the company managers, Nanako, needs to visit the offices. She creates the following graph that shows the distances, in kilometres, between some of the offices.



(a) Write down a Hamiltonian cycle for this graph. [1]

(b) State, with a reason, whether the graph contains an Eulerian circuit. [1]

Nanako wishes to find the shortest cycle to visit all the offices. She decides to complete a weighted adjacency table, showing the least distance between each pair of offices.

	A	B	C	D	E	F
A		27	52	p	35	18
B			25	26	12	q
C				17	28	r
D					14	34
E						22
F						

Write down the value of

(c.i) p. [1]

(c.ii) q. [1]

(c.iii) r. [1]

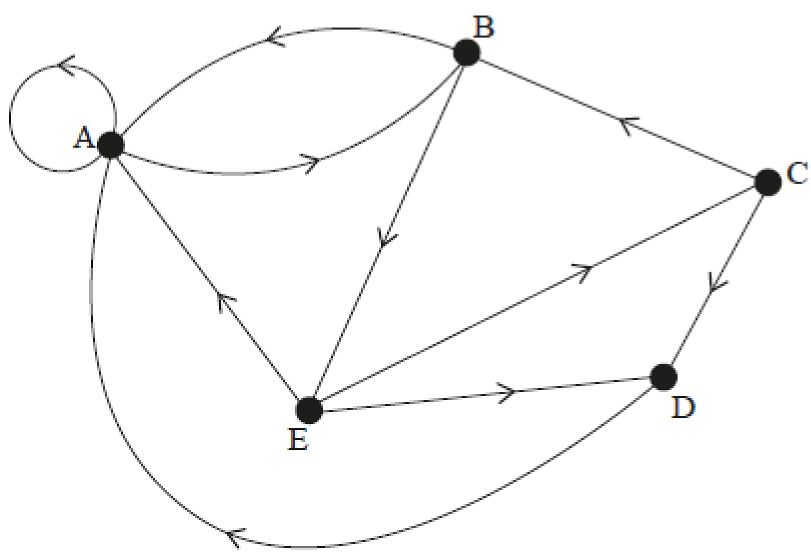
(d) Starting at vertex E, use the nearest neighbour algorithm to find an upper bound for Nanako's cycle. [3]

(e) By deleting vertex F, find a lower bound for Nanako's cycle. [4]

(f) Explain, with a reason, why the answer to part (e) might not be the best lower bound. [2]

4. [Maximum mark: 5] 22M.1.AHL.TZ2.6

Consider the following directed network.



(a) Write down the adjacency matrix for this network. [2]

(b) Determine the number of different walks of length 5 that start and end at the same vertex. [3]

5. [Maximum mark: 7] 21M.1.AHL.TZ1.10

An engineer plans to visit six oil rigs $(\mathrm{A}–\mathrm{F})$ in the Gulf of Mexico, starting and finishing at A. The travelling time, in minutes, between each of the rigs is shown in the table.

	A	B	C	D	E	F
A		55	63	79	87	93
B	55		46	58	88	92
C	63	46		87	77	66
D	79	58	87		23	70
E	87	88	77	23		47
F	93	92	66	70	47	

The data above can be represented by a graph G.

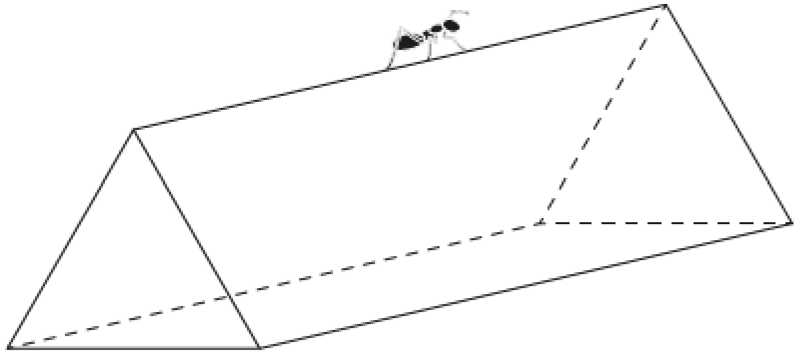
(a.i) Use Prim's algorithm to find the weight of the minimum spanning tree of the subgraph of G obtained by deleting A and starting at B. List the order in which the edges are selected. [4]

(a.ii) Hence find a lower bound for the travelling time needed to visit all the oil rigs. [2]

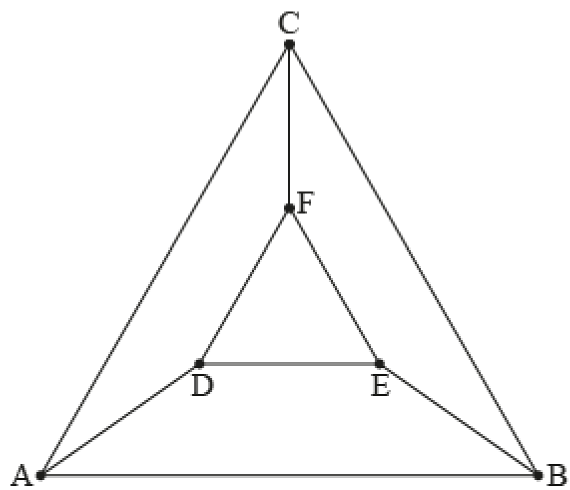
(b) Describe how an improved lower bound might be found. [1]

6. [Maximum mark: 5] 21M.1.AHL.TZ1.16

An ant is walking along the edges of a wire frame in the shape of a triangular prism.



The vertices and edges of this frame can be represented by the graph below.

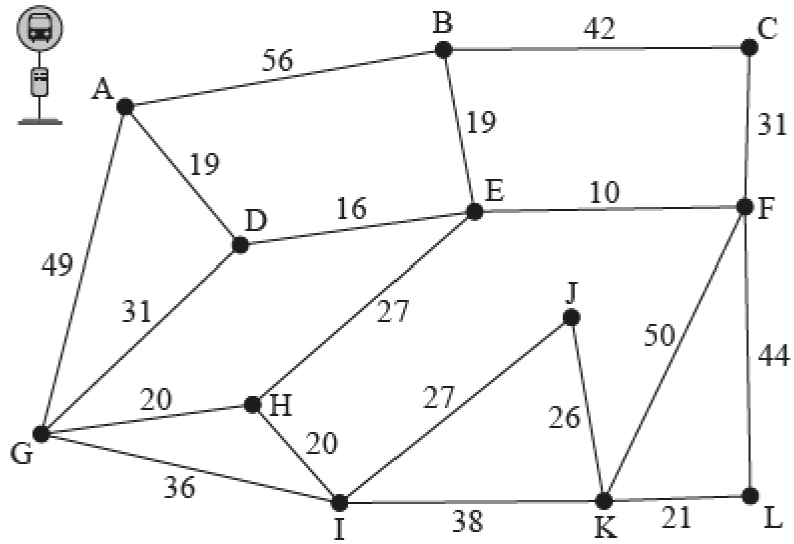


(a) Write down the adjacency matrix, $\boldsymbol{M}$, for this graph. [3]

(b) Find the number of ways that the ant can start at the vertex A, and walk along exactly 6 edges to return to A. [2]

7. [Maximum mark: 7] 21M.1.AHL.TZ2.11

The diagram below shows a network of roads in a small village with the weights indicating the distance of each road, in metres, and junctions indicated with letters.



Musab is required to deliver leaflets to every house on each road. He wishes to minimize his total distance.

(a) Musab starts and finishes from the village bus-stop at A. Determine the total distance Musab will need to walk. [5]

(b) Instead of having to catch the bus to the village, Musab's sister offers to drop him off at any junction and pick him up at any other junction of his choice.

Explain which junctions Musab should choose as his starting and finishing points. [2]

8. [Maximum mark: 10] 19N.3.AHL.TZ0.Hdm_1

A driver needs to make deliveries to five shops A, B, C, D and E. The driver starts and finishes his journey at the warehouse W. The driver wants to find the shortest route to visit all the shops and return to the warehouse. The distances, in kilometres, between the locations are given in the following table.

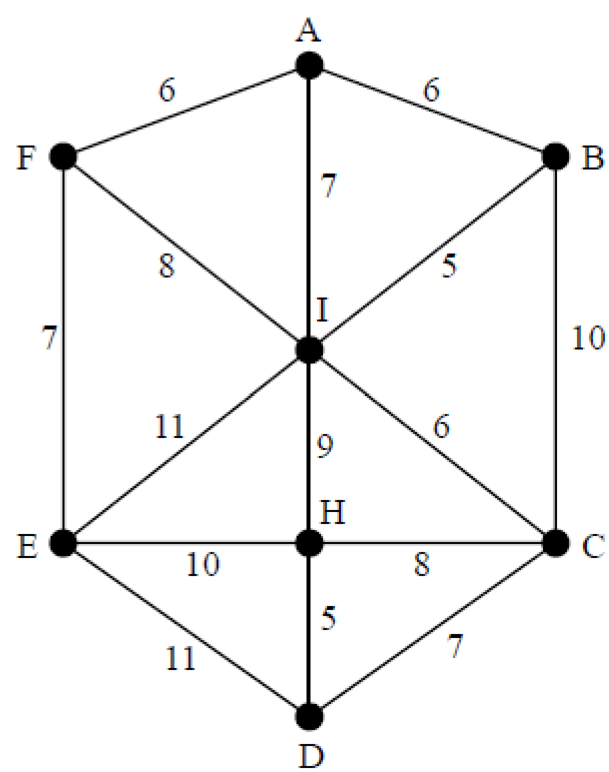
	A	*B*	*C*	*D*	*E*	*W*
A	-	11	28	15	20	40
B	11	-	25	20	32	36
C	28	25	-	16	22	39
D	15	20	16	-	12	42
E	20	32	22	12	-	41
W	40	36	39	42	41	-

(a) By deleting W, use the deleted vertex algorithm to find a lower bound for the length of a route that visits every shop, starting and finishing at W. [6]

(b) Starting from W, use the nearest-neighbour algorithm to find a route which gives an upper bound for this problem and calculate its length. [4]

9. [Maximum mark: 17] 18N.3.AHL.TZ0.Hdm_4

Consider the graph G represented in the following diagram.



(a) State, with a reason, whether or not G has an Eulerian circuit. [1]

(b) Use Kruskal's algorithm to find a minimum spanning tree for G, stating its total weight. Indicate clearly the order in which the edges are added. [4]

The graph G is a plan of a holiday resort where each vertex represents a villa and the edges represent the roads between villas. The weights of the edges are the times, in minutes, Mr José, the security guard, takes to walk along each of the roads. Mr José is based at villa A.

(c) Use a suitable algorithm to show that the minimum time in which Mr José can get from A to E is 13 minutes. [5]

(d) Find the minimum time it takes Mr José to patrol the resort if he has to walk along every road at least once, starting and ending at A. State clearly which roads need to be repeated. [7]