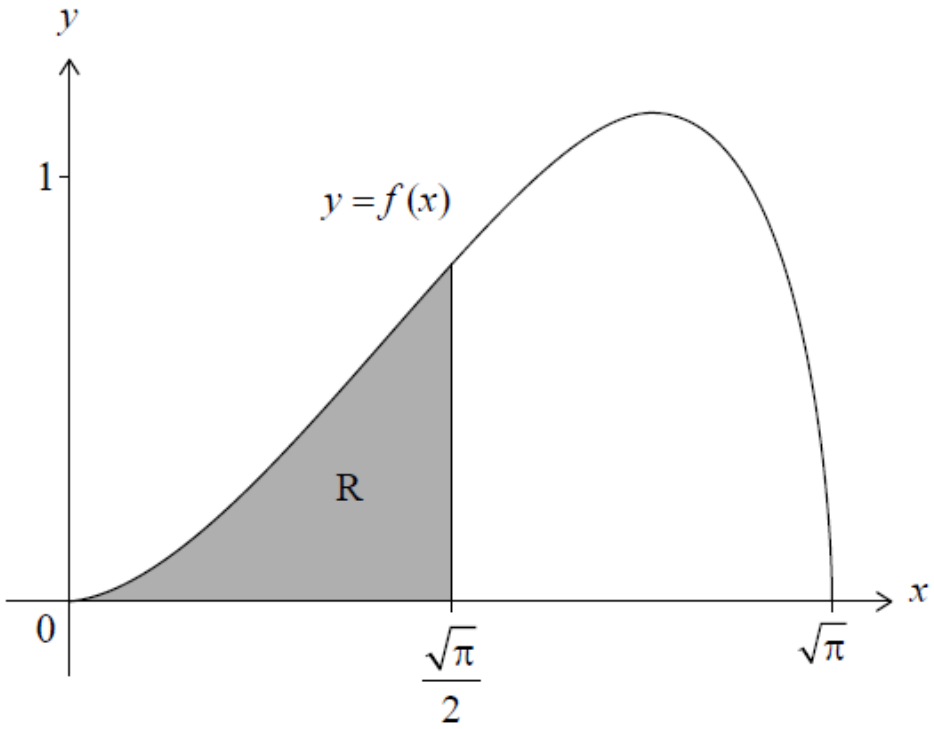
Definite integrals [47 marks]

1. [Maximum mark: 6] 24M.1.AHL.TZ1.6

The function f is defined as $f(x) = \sqrt{x \sin(x^2)}$, where $0 \leq x \leq \sqrt{\pi}$.

Consider the shaded region R enclosed by the graph of f, the x-axis and the line $x = \frac{\sqrt{\pi}}{2}$, as shown in the following diagram.



The shaded region R is rotated by 2π radians about the x-axis to form a solid.

Show that the volume of the solid is $\frac{\pi\left(2-\sqrt{2}\right)}{4}$. [6]

2. [Maximum mark: 7] 23M.1.AHL.TZ1.9

The function f is defined by $f\left(y\right) = \sqrt{r^2 - y^2}$ for $-r \leq y \leq r$

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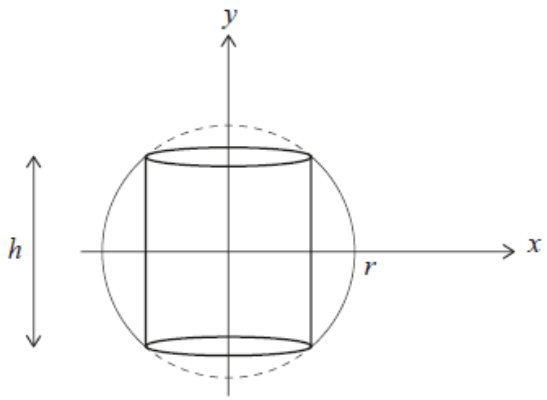
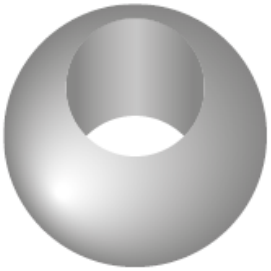
The region enclosed by the graph of $x = f(y)$ and the y-axis is rotated by $360\ ^\circ$ about the y-axis to form a solid sphere. The sphere is drilled through along the y-axis, creating a cylindrical hole. The resulting spherical ring has height, h.

This information is shown in the following diagrams.

diagram not to scale





The spherical ring has a volume of π cubic units. Find the value of h. [7]

3. [Maximum mark: 5] 22M.1.AHL.TZ1.1

Find the value of $\int_1^9 \left(\frac{3\sqrt{x}-5}{\sqrt{x}} \right) \mathrm{d}\ x$. [5]

4. [Maximum mark: 6] 22M.1.AHL.TZ2.7

By using the substitution $u = \sec x$ or otherwise, find an expression for $\int_0^{\frac{\pi}{3}} \sec^n x \tan x \, \mathrm{d}x$ in terms of n, where n is a non-zero real number. [6]

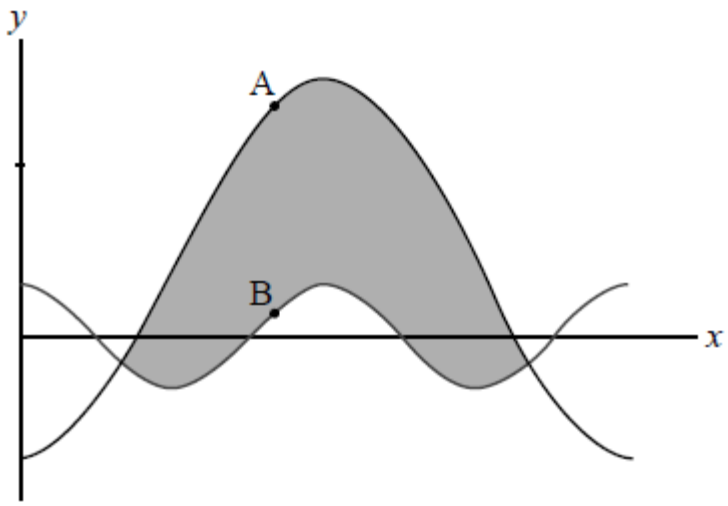
5. [Maximum mark: 6] 19N.1.AHL.TZ0.H_2

Given that $\int_0^{\ln k} \mathrm{e}^{2x} \mathrm{d}x = 12$, find the value of k. [6]

6. [Maximum mark: 17] 19M.1.AHL.TZ2.H_9

Consider the functions f and g defined on the domain $0 < x < 2\pi$ by $f(x) = 3\cos 2x$ and $g(x) = 4 - 11\cos x$.

The following diagram shows the graphs of $y = f(x)$ and $y = g(x)$



(a) Find the x-coordinates of the points of intersection of the two graphs. [6]

(b) Find the exact area of the shaded region, giving your answer in the form $p\pi + q\sqrt{3}$, where $p, q \in \mathbb{Q}$. [5]

(c) At the points A and B on the diagram, the gradients of the two graphs are equal.

Determine the y-coordinate of A on the graph of g. [6]