# Practice exam papers

## Mathematics: applications and interpretation
## Higher level
## Practice set A: Paper 1

Candidate session number

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| | | | | | | | | | | |

2 hours

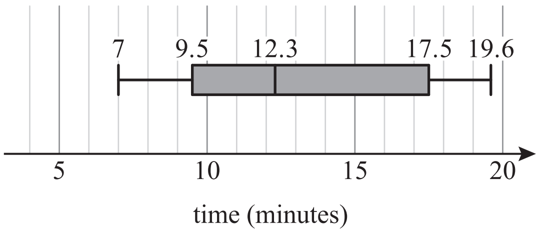
**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer **all** questions. Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1** *[Maximum mark: 6]*

The box plot summarizes the times taken by a group of 40 children to complete an obstacle course.




Two of the 40 children are selected at random. Find the probability that:

**a** both completed the course in less than 9.5 minutes *[3]*

**b** one completed the course in less than 9.5 minutes and the other in between 9.5 and 17.5 minutes. *[3]*

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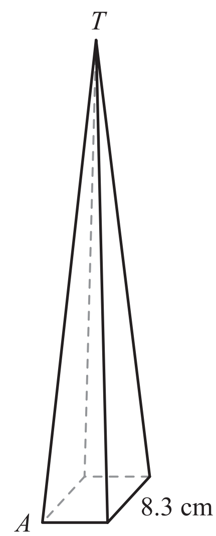
**2** *[Maximum mark: 6]*

A flag pole has the shape of a square-based pyramid shown in the diagram. The side length of the base is 8.3 cm. The edge *AT* makes an angle of 89.8° with the base.

**a** Find the height of the flag pole. Give your answer in centimetres, in standard form, correct to two significant figures. *[4]*

**b** A soldier stands 12.5 m from the pole. Find the angle of elevation of the top of the pole. *[2]*

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**3** *[Maximum mark: 6]*

Millie is investigating whether teachers and students at her college choose different food at the school cafeteria. She records the choices over several Tuesdays, each of which had the same three options. The results are show in the table.

| | Vegetable lasagne | Fishcakes | Sausages and chips |
|---|---|---|---|
| **Teachers** | 17 | 28 | 12 |
| **Students** | 87 | 74 | 92 |

**a** Is there evidence, at the 5% significance level, that meal choice depends on whether a person is a teacher or a student? State your hypotheses and justify your conclusion. *[5]*

**b** Suggest one improvement that Millie could make to her investigation. *[1]*

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**4** *[Maximum mark: 8]*

A ball is projected from a point above ground. A student measures the horizontal and vertical distances of the ball from the starting point and records the results. In the table below, $x$ m is the horizontal distance from the point of projection and $y$ m is the height of the ball above ground.

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $y$ | 7.2 | 7.4 | 6.4 |

The path of the ball is modelled by the equation $y = ax^2 + bx + c$.

**a** Use the data in the table to find the values of $a$, $b$ and $c$, giving your answers correct to two significant figures. *[3]*

**b** From what height above ground was the ball projected? *[1]*

**c** Find the value of $x$ when the ball hits the ground. *[2]*

**d** Find the straight line distance from the point of projection to the point where the ball hits the ground. *[2]*

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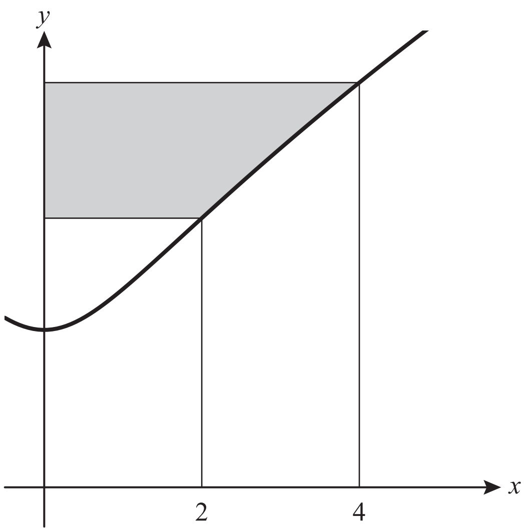
**5** *[Maximum mark: 7]*

The curve in the diagram has equation $y = \sqrt[3]{x^2 + 1}$. The shaded region is bounded by the curve, the $y$-axis and two horizontal lines.




**a** Find the area of the shaded region. *[4]*

**b** Find the volume generated when the shaded region is rotated $2\pi$ radians about the $y$-axis. *[3]*

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**6** *[Maximum mark: 6]*

Pietro is investigating the weight of loaves of bread sold by the local bakery. He knows that the weights are distributed normally with standard deviation 7.5 grams. He weighs a random sample of seven loaves and obtains the following results (in grams):

789, 812, 806, 797, 800, 802, 799

**a** Find a 90% confidence interval for the mean weight of the loaves. *[3]*

**b** Justify your choice of confidence interval. *[1]*

**c** The bread loaves are sold as weighing 800 g. A customer thinks that the average weight is around 790 g. Comment on the customer's claim in the light of your confidence interval. *[2]*

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**7** *[Maximum mark: 9]*

A boat hire company charges \$85 plus \$5 for every 30 minutes of use.

**a** Form a linear model for the cost, \$C, in terms of the hire time, $t$ hours. *[2]*

A second company charges \$60 plus \$10 for every 30 minutes of use for the first 2 hours, then \$15 for every 15 minutes of use thereafter.

**b** Form a piecewise linear model for the cost, \$C, in terms of the hire time, $t$ hours. *[4]*

John hires a boat from the first company.

**c** Find the shortest length of time for which this would prove the cheaper option for him. *[3]*

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**8** *[Maximum mark: 8]*

Ilya decorates small cakes in his bakery. The time it takes him to decorate $x$ cakes is modelled by $T(x) = 0.003x^3 + 10x + 200$ minutes.

**a** If Ilya wants to work at most 12 hours in a day, what is the largest number of cakes he can decorate? *[2]*

**b** Ilya can choose to decorate between 20 and 40 cakes per day, inclusive. Find the minimum and maximum time he can take per cake on average. *[4]*

Ilya becomes more efficient and is now able to decorate cakes at twice the rate he could before.

**c** Find the new model in the form $S(x) = ax^3 + bx + c$. *[2]*

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**9** *[Maximum mark: 5]*

The sum of the first two terms of a geometric series is 3 and its sum to infinity is 5.

Given that all terms of the series are positive, find the exact common ratio of the series. *[5]*

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**10** *[Maximum mark: 6]*

The motion of a particle attached to one end of an elastic spring is modelled by the equation $x = 0.5e^{-0.3t}\sin(2.5t)$, where $t$ seconds is the time and $x$ cm is the displacement of the particle from its initial position.

**a** In subsequent motion, find the number of times the particle will be 0.25 cm from the starting position. *[3]*

**b** Find an expression for the rate of change of the displacement at time $t$. *[3]*

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**11** *[Maximum mark: 6]*

Use the matrix form of Prim's algorithm, starting at vertex A, to find the minimum spanning tree for the graph represented by the adjacency table below. List the vertices in the order you add them and state the weight of your tree.

| | **A** | **B** | **C** | **D** | **E** | **F** |
|---|---|---|---|---|---|---|
| **A** | – | 11 | 12 | 25 | – | – |
| **B** | 11 | – | 14 | – | 22 | 18 |
| **C** | 12 | 14 | – | – | 24 | 12 |
| **D** | 25 | – | – | – | 31 | 31 |
| **E** | – | 22 | 24 | 12 | – | 35 |
| **F** | – | 18 | 20 | 31 | 35 | – |

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**12** *[Maximum mark: 8]*

The concentration at time $t$ of a reactant, $C$, in a reaction mixture at fixed temperature is given by the differential equation

$$\frac{dC}{dt} = -kC^2$$

**a** Show that

$$C = \frac{C_0}{C_0kt + 1}$$

where $C_0$ is the initial concentration of the reactant. *[6]*

**b** Hence find the long-term concentration of the reactant. *[2]*

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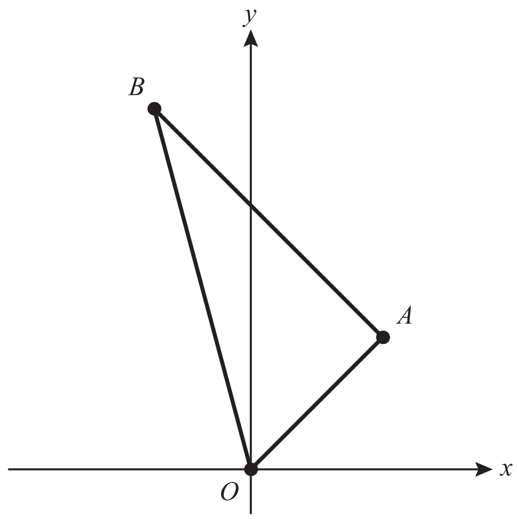
**13** *[Maximum mark: 5]*

The diagram shows triangle $OAB$, with $OB = 2OA$ and angle $AOB$ measuring 60°. Point $A$ has coordinates (3, 3).




**a** Write down, in Cartesian form, the complex number corresponding to the point $A$ in the Argand diagram. *[1]*

**b** Find the coordinates of the point $B$. *[4]*

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**14** *[Maximum mark: 6]*

Melinda has read that, 10 years ago, the proportion of households in her town which owned a pet was $\frac{1}{5}$. She suspects that the proportion is now higher. To test her belief, she conducts a survey of 70 randomly selected households and records the number of households, $X$, which own a pet.

**a** Melinda decides to knock on doors on a weekday morning and use as her sample the first 70 households who answer the door. State one criticism of this sampling method. *[1]*

**b** Melinda manages to obtain a simple random sample of 70 households and decides to conduct a test using a 5% significance level. State suitable hypotheses and find the critical region for the test. *[5]*

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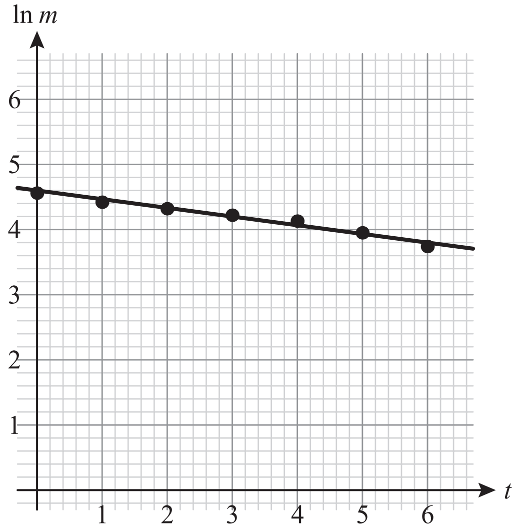
**15** *[Maximum mark: 5]*

A student is investigating the rate of decay of caffeine in the bloodstream. A subject drinks a cup of coffee and the amount of caffeine in their bloodstream, $m$ mg, is measured every hour for six hours. The graph below shows the results on a logarithmic scale, with time ($t$ hours) on the horizontal axis and $\ln m$ on the vertical axis. The graph also shows the line of best fit.




**a** Use the graph to find the equation of the line of best fit. *[3]*

**b** Hence find an expression for $m$ in terms of $t$. *[2]*

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**16** *[Maximum mark: 7]*

The position vector of an object at time $t$ is given by

$\mathbf{r} = \begin{pmatrix} a\cos kt \\ a\sin kt \end{pmatrix}$ where a > 0

**a** Show that the object is moving in a circular path. *[3]*

**b** Show that the velocity vector is perpendicular to the displacement vector. *[4]*

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**17** *[Maximum mark: 6]*

The total surface area of a cone with radius $r$ and height $h$ is given by $S = \pi r^2 + \pi r\sqrt{r^2 + h^2}$. A cone has height 5 cm and the radius increases at a rate of 2 cm per second.

Find the rate at which the surface area is increasing when the radius equals 10 cm. *[6]*

# Mathematics: applications and interpretation
# Higher level
# Practice set A: Paper 2

Candidate session number

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1 hour 30 minutes

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphical display calculator is required for this paper.
- Answer **all** questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1** *[Maximum mark: 21]*

Suresh is looking to take out a £150 000 mortgage to buy a new house.

He is considering two options:

**Mortgage A**

10% desposit

2% interest rate compounded annually

25 year repayment period

**a** **i** Find the annual repayments.

**ii** Find the total amount he would repay.

**iii** Find the total amount of interest he would pay. *[7]*

**Mortgage B**

No deposit

2.5% interest rate compounded monthly

30 year repayment period

**b** **i** Find the monthly repayments.

**ii** Find the total amount he would repay. *[5]*

**c** Explain which mortgage Suresh should choose and why. *[2]*

**d** Suresh decides to take Mortgage B and invest the money from the 10% deposit. He wants to find an account that will pay a monthly annuity of at least £50 over the lifetime of Mortgage B.

Find the minimum interest rate needed, assuming monthly compounding. *[3]*

**e** Suresh also saves £250 each month in a regular saver account paying 2% interest (compounded monthly). Show that after $n$ months the balance of the account is

$$a(b^{n-1})$$

where $a$ and $b$ are constants to be found. *[4]*

**2** *[Maximum mark: 20]*

Two of the sides of a triangle have length $x$ cm and $2x$ cm, and the angle between them is $\theta°$. The perimeter of the triangle is 10 cm.

**a** In the case $x = 2$, find the area of the triangle. *[4]*

**b** Explain why $x$ must be less than $\frac{10}{3}$. *[2]*

**c** **i** Show that $\cos\theta = \frac{15x - x^2 - 25}{x^2}$

**ii** Sketch the graph of $y = \frac{15x - x^2 - 25}{x^2}$ for $x > 0$.

**iii** Hence find the range of possible values of $x$. *[8]*

**d** Find the value of $x$ for which the triangle has the largest possible area, and state the value of that area. *[6]*

**3** *[Maximum mark: 16]*

A drone D is initially at the point (4, –2, 1) and travels with constant velocity $\begin{pmatrix}-3\\7\\2\end{pmatrix}\text{km h}^{-1}$.

A second drone E is initially at the point (–2, 1, –8) and travels with constant velocity $\begin{pmatrix}2\\-1\\3\end{pmatrix}\text{km h}^{-1}$.

**a** Find the speed of *D*. *[2]*

**b** Write down equations for position vectors of *D* and *E* at time *t* hours. *[2]*

**c** **i** Show that the paths of *D* and *E* cross.

**ii** Find the coordinates of the point at which this occurs. *[5]*

**d** Show that *D* and *E* do not collide. *[1]*

**e** **i** Find the time at which *D* and *E* are closest together.

**ii** Find the minimum distance between *D* and *E*. *[6]*

**4** *[Maximum mark: 19]*

It is proposed that the population of sheep, *P* (thousand), on a small island at time *t* years after they were introduced can be modelled by the function

$$P = \frac{5}{1 + Ce^{-kt}}$$

where *C* and *k* are constants.

**a** Find the long-term size of the population predicted by this model. *[2]*

**b** Show that $\ln\left(\frac{5}{P} - 1\right) = \ln C - kt$. *[3]*

The following data are collected:

| **t** | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| **P** | 1.9 | 2.6 | 3.1 | 3.7 |

**c** Use linear regression to estimate the values of *C* and *k*. *[5]*

**d** **i** Write down the coefficient of determination for the linear regression.

**ii** Explain what this suggests about the proposed population model. *[3]*

**e** Use the model to estimate

**i** the initial number of sheep introduced to the island

**ii** the time taken for the population to reach 4500. *[4]*

**f** Comment on the reliability of your estimates in part **e**. *[2]*

**5** *[Maximum mark: 17]*

The velocity (in $\text{m s}^{-1}$) of an object at *t* seconds is given by

$$v(t) = \frac{8 - 3t}{t^2 - 6t + 10}, \ 0 \le t \le 10.$$

Find

**a** the initial speed *[1]*

**b** the maximum speed *[2]*

**c** the length of time for which the speed is greater than $1\text{ m s}^{-1}$ *[3]*

**d** the time at which the object changes direction *[2]*

**e** the length of time for which the object is decelerating *[2]*

**f** the acceleration after 5 seconds *[2]*

**g** the distance travelled after 10 seconds *[2]*

**h** the time when the object returns to its starting position. *[3]*

**6** *[Maximum mark: 17]*

A small ball is attached to an elastic spring and placed inside a tube filled with viscous liquid. The ball oscillates, with the displacement from its equilibrium position given by the differential equation

$$\frac{d^2x}{dt^2} = -0.06\frac{dx}{dt} - 0.4x$$

The displacement is measured in centimetres and time in seconds. When $t = 2.5$, the ball passes through the equilibrium position with velocity $-3.8\,\text{cm}\,\text{s}^{-1}$.

**a** By setting $y = \frac{dx}{dt}$, write the differential equation in the form $\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix}$, where **A** is a $2 \times 2$ matrix. *[2]*

**b** Use eigenvalues of **A** to sketch the phase portrait for $x$ and $y$, justifying the direction of the trajectories. *[6]*

**c** Use Euler's method with step length 0.05 to find the distance of the ball from the equilibrium position when $t = 3$. *[4]*

The exact solution of the differential equation is $x = e^{-0.03t}(0.0667\sin(0.632t) + 6.48\cos(0.632t))$.

**d** Comment on the accuracy of your answer from part **c**. *[2]*

**e** Find the first time after $t = 2.5$ that the ball is instantaneously at rest. Find its distance from the equilibrium position at that time. *[3]*

# Mathematics: applications and interpretation
# Higher level
# Practice set A: Paper 3

Candidate session number

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1 hour

**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer **all** questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1** *[Maximum mark: 25]*

*This question is about resonance in vibrating objects.*

**a** Write down the period of the function $\cos \pi t$. *[1]*

**b** **i** Sketch the function $y = \cos \pi t + \cos 2\pi t$ for $0 \le t \le 3$.

**ii** Write down the period of the function $\cos \pi t + \cos 2\pi t$. *[2]*

**c** **i** Use technology to investigate the period of the functions given below. Write down the values of A, B and C.

| f($t$) | Period |
|---|---|
| $\cos \pi t + \cos 1.5\pi t$ | A |
| $\cos \pi t + \cos 1.25\pi t$ | B |
| $\cos \pi t + \cos 1.1\pi t$ | C |

**ii** Hence conjecture an expression for the period, $T$, of $f(t) = \cos \pi t + \cos\left(\left(1 + \frac{1}{n}\right)\pi t\right)$ where $n$ is an integer. *[4]*

**d** Prove that, for your conjectured value of $T$, $f(t + T) = f(t)$. *[3]*

**e** **i** By considering the real part of $e^{(A+B)i} + e^{(B-A)i}$, find a factorized expression for $\cos(A + B) + \cos(A - B)$.

**ii** Hence find a factorized form for the expression $\cos P + \cos Q$. *[4]*

**f** By considering the factorized form of $f(t)$ explain the shape of its graph. *[2]*

**g** A piano string oscillates when plucked. The displacement, $x$, from equilibrium as a function of time is modelled by:

$$\frac{d^2x}{dt^2} + 4x = 0$$

Show that a function of the form $x = f(t) = \cos(\omega t)$ solves this differential equation for a positive value of $\omega$ to be stated. *[4]*

**h** The piano string can be subjected to an external driving force from a tuning fork. The differential equation becomes:

$$\frac{d^2x}{dt^2} + 4x = \cos kt$$

Find a solution of the form $x = f(t) + g(k)\cos kt$ where $g(k)$ is a function to be found. *[3]*

**i** Resonance is a phenomenon in which the amplitude of the driven oscillation grows without limit. For what positive value of $k$ will resonance occur? Justify your answer. *[2]*

**2** *[Maximum mark: 30]*

*This question is about modelling the long term numbers of a badger population.*

The number of adults and juveniles in a badger population in year $n$ is modelled by:

$$\begin{pmatrix} A_{n+1} \\ J_{n+1} \end{pmatrix} = \boldsymbol{M}\begin{pmatrix} A_n \\ J_n \end{pmatrix}$$

where $\boldsymbol{M} = \begin{pmatrix} 0.5 & 0.6 \\ 2 & 0.3 \end{pmatrix}$

**a** Draw a transition diagram to represent this model. Hence describe what each number in the model represents. *[6]*

The matrix $\boldsymbol{M}$ has eigenvalues $\lambda_1$ and $\lambda_2$ where $\lambda_1 > \lambda_2$. The corresponding eigenvectors are $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.

**b** Find $\lambda_1$ and $\lambda_2$. *[4]*

**c** Find $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. *[6]*

In year 0, the initial state vector is $\boldsymbol{p}_0 = \begin{pmatrix} 100 \\ 20 \end{pmatrix}$

**d** If $\boldsymbol{p}_0 = \alpha\boldsymbol{v}_1 + \beta\boldsymbol{v}_2$, find appropriate values for $\alpha$ and $\beta$. *[3]*

**e** As $n$ gets larger, find an approximate expression for $\begin{pmatrix} A_n \\ J_n \end{pmatrix}$. Hence find the long-term growth ratio of the population. *[6]*

**f** The badgers are considered a pest, so a change is made to the habitat, which affects the model so that the new transition matrix, $\boldsymbol{N}$, is $\begin{pmatrix} 0.5 & 0.6 \\ x & 0.3 \end{pmatrix}$. Find the upper bound on the value of $x$ which will result in the long-term decline of the badger population. *[5]*

# Mathematics: applications and interpretation
# Higher level
# Practice set B: Paper 1

Candidate session number

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2 hours

**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer **all** questions. Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1** *[Maximum mark: 5]*

In this question, an *outlier* is defined as a piece of data which is more than two standard deviations above or below the mean.

The heights of eight children, in centimetres, are:

122 124 127 131 134 134 136 147

Determine whether any of the heights are outliers. *[5]*

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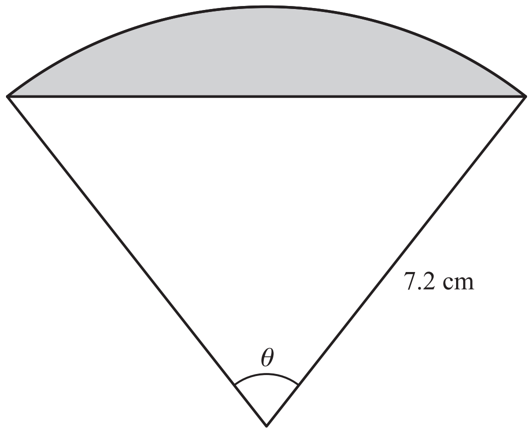
**2** *[Maximum mark: 5]*

The diagram shows a sector of a circle with radius 7.2 cm. The angle at the centre is $\theta$ radians.




The area of the shaded area is 9.7 $cm^2$. Find the value of $\theta$. *[5]*

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**3** *[Maximum mark: 7]*

**a** Use the trapezoidal rule with five strips to estimate $\int_0^5 \sin\left(\frac{x^2}{10}\right) dx$, giving your answer correct to 3 s.f. *[4]*

**b** Use your GDC to evaluate the integral correct to 5 s.f. *[1]*

**c** Using the GDC value as the exact value, find the percentage error in the approximation obtained using the trapezoidal rule. *[2]*

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**4** *[Maximum mark: 6]*

A triangle ABC has sides $AB = 8$, $BC = 6$ and angle $BAC = \frac{\pi}{6}$ radians.

Find the two possible values of angle $ABC$. *[6]*

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**5** *[Maximum mark: 5]*

On his way to school, Suresh stops for coffee with probability 0.8. If he stops for coffee, the probability that he is late for school is 0.4; otherwise, the probability that he is late is 0.1.

Given that on a particular day Suresh is late for school, what is the probability that he did not stop for coffee? *[5]*

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**6** *[Maximum mark: 7]*

A business owner finds that the rate of change of profit from the sales of a particular product is given by $-40x^3 + 60x^2 + 30$, where $x$ is the price in hundreds of dollars.

When the price is \$200 the profit is \$45.

**a** Find the profit function, $P(x)$. *[5]*

**b** Find the price that maximizes the profit. *[2]*

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**7** *[Maximum mark: 6]*

Given the functions

$f(x) = \dfrac{2 - x}{x + 3}$ $(x \neq -3)$ and $g(x) = \dfrac{2}{x - 1}$ $(x \neq 1)$,

find $(f \circ g)^{-1}$ in the form $\dfrac{ax + b}{cx + d}$. *[6]*

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**8** *[Maximum mark: 6]*

A particle is moving in a straight line with velocity given by $v = 2e^{-t^2} - 1$ for $0 \leq t \leq 4$ where $v$ is in $m\,s^{-1}$ and $t$ is in s.

**a** Find the time at which the speed is zero. *[2]*

**b** Find the displacement from the initial position when $t = 4$. *[2]*

**c** Find the total distance travelled in the first four seconds. *[2]*

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**9** *[Maximum mark: 7]*

The number of cars arriving at a multi-storey car park has a mean of 28 per hour. The cars arrive randomly.

**a** State two conditions that need to be satisfied to model the number of cars by a Poisson distribution. *[2]*

**b** Find the probability that more than 15 cars arrive in a 30-minute period. *[3]*

The number of motorbikes arriving at the car park is modelled by a Poisson distribution with mean 17 per hour.

**c** Find the probability that, in a given one-hour period, fewer than 40 vehicles (cars and motorbikes together) arrive at the car park. *[2]*

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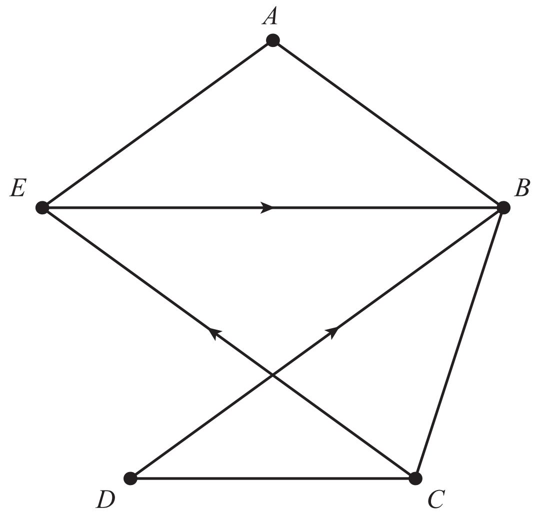
**10** *[Maximum mark: 6]*

**a** Write down the transition matrix for the following directed graph: *[2]*




**b** The graph represents links between five web pages. For example, the edge A to B means that webpage A contains a link to webpage B. Use the PageRank algorithm to rank the pages in order of importance. *[4]*

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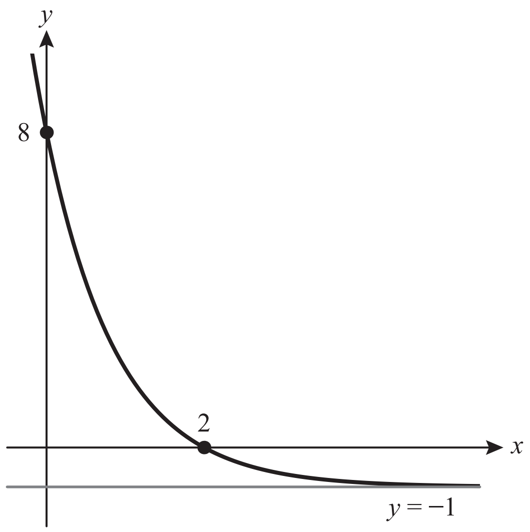
**11** *[Maximum mark: 6]*

The graph in the diagram has equation $y = A + Be^{-kx}$.




Find the values of $A$, $B$ and $k$. *[6]*

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**12** *[Maximum mark: 6]*

A doctor is investigating whether a new drug decreases mean recovery time following a particular disease. It is known that, with the old drug, the recovery times follow a normal distribution with mean 12.6 days and standard deviation 2.8 days. The doctor wishes to conduct a hypothesis test using a 2% level of significance and a random sample of 40 patients. He assumes that the standard deviation is unchanged.

**a** Find the critical region for the test. *[3]*

**b** Given that the mean recovery time with the new drug is in fact 11.3 days, find the probability of a type II error. *[3]*

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**13** *[Maximum mark: 7]*

Given the differential equation $\frac{dy}{dx} = \frac{x - y}{x + y + 1}$

**a** Sketch the slope field for $0 \leq x \leq 3$ and $0 \leq y \leq 3$. *[3]*

**b** Add the solution curve passing through (1, 1) to your diagram. *[1]*

**c** For the solution curve from part **b**, use Euler's method with step length 0.1 to estimate the value of $y$ when $x = 1.5$. Give your answer to two decimal places. *[3]*

**14** *[Maximum mark: 9]*

The populations of spiders ($x$ hundred) and flies ($y$ hundred) are modelled by a system of differential equations.

In a simple model, the system has the form $\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \mathbf{A}\begin{pmatrix}x-2\\ y-3\end{pmatrix}$. The equilibrium point of the system is (2, 3).

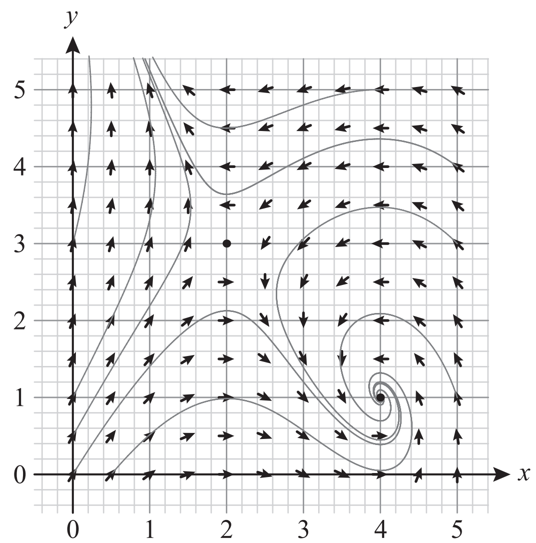
The matrix **A** has eigenvalues 0.6 and –0.4, and the corresponding eigenvectors $\begin{pmatrix}-1\\ 3\end{pmatrix}$ and $\begin{pmatrix}1\\ 1\end{pmatrix}$.

**a** Sketch the phase portrait for the system. *[2]*

**b** If initially there are 200 spiders and 300 flies, find the long-term relationship between the number of spiders and the number of flies, in the form $ax + by = c$. *[3]*

**c** Suggest why this model is not appropriate in the long term. *[1]*

In a refined model, there are two equilibrium points, (2, 3) and (4, 1). The phase portrait for the system is shown below. The horizontal axis shows the number of spiders and the vertical axis the number of flies.




**d** If initially there are 100 spiders and 200 flies, describe how the number of flies changes over time. *[1]*

**e** If initially there are 200 spiders and 100 flies, describe how the numbers of spiders and flies change in the long term. *[2]*

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**15** *[Maximum mark: 6]*

Katie wants to buy a large bag of sugar. The weights of large bags of sugar follow a normal distribution with mean 2.01 kg and standard deviation 0.06 kg. The shop is out of large bags of sugar, so she buys two small bags instead. The weights of small bags of sugar follow a normal distribution with mean 1.08 kg and standard deviation 0.04 kg.

**a** Find the probability that a large bag of sugar weighs more than twice as much as a small bag. *[3]*

**b** Find the probability that a large bag of sugar weighs more than two small bags. *[3]*

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**16** *[Maximum mark: 6]*

The height of a wave, $h_1$ cm, at time $t$ seconds is given by the equation $h_1 = 12.3 \sin(3.2t)$. The height of another wave is given by $h_2 = 11.6 \sin(0.8 + 3.2t)$.

Find an expression for the height of the combined wave, $h = h_1 + h_2$, in the form $h = A \sin(k + bt)$. *[6]*

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**17** *[Maximum mark: 10]*

The growth of a population of bacteria in a petri dish is modelled by the function

$$f(t) = \frac{L}{1 + Ce^{-kt}}$$

where $L, C, k > 0$.

**a** Show that $f'(t) = \dfrac{LCke^{-kt}}{(1 + Ce^{-kt})^2}$ *[2]*

The function has a point of inflection.

**b** Find, in terms of $k$ and $C$, the time at which this occurs. *[6]*

**c** Find the range of possible values of $C$. *[2]*

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# Mathematics: applications and interpretation
# Higher level
# Practice set B: Paper 2

Candidate session number

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| | | | | | | | | | | |

1 hour 30 minutes

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are permitted access to a graphical display calculator for this paper.
- Answer **all** questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1** *[Maximum mark: 20]*

Stella is planning to start a small business selling cosmetics gift boxes. She plans to start by selling 30 boxes in the first month. In each subsequent month she plans to sell 10 more boxes than in the previous month.

**a** **i** According to Stella's plan, how many boxes will she sell in the 12th month?

**ii** How many boxes will she sell in the first year?

**iii** In which month will she sell her 2000th box? *[8]*

Giulio also sells cosmetics gift boxes. He also sells 30 boxes in the first month, but expects to increase his sales by 10% each month.

**b** **i** How many boxes will Giulio sell in the first year?

**ii** In which month will Giulio first sell more than 1000 boxes per month? *[6]*

**c** Stella makes a profit of £2.20 per box and Guilio makes a profit of £3.10 per box.

**i** Find the profit each person makes in the first year.

**ii** In which month will Giulio's **total** profit first overtake Stella's? *[6]*

**2** *[Maximum mark: 19]*

Laura is investigating whether a certain drug is effective in reducing cholesterol. She measures the cholesterol level (in $mg\,dL^{-1}$) of ten volunteers before and after a course of the drug:

| **Volunteer** | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| **Before** | 187 | 135 | 219 | 149 | 203 | 156 | 129 | 180 | 212 | 166 |
| **After** | 179 | 138 | 204 | 151 | 197 | 154 | 128 | 173 | 198 | 166 |

She carries out a paired test on the data at a 5% significance level.

**a** State two advantages of using a paired test over a two-sample test. *[2]*

**b** **i** Write down the hypotheses for the test.

**ii** Calculate the $p$-value for the test.

**iii** State the conclusion of the test. Give a reason for your answer. *[5]*

Laura decides she needs to assess the statistical validity of the test used in part **b**. She collects more data and summarizes the difference, $d$, between each participant's cholesterol level before and after the course of medication:

| **Difference** | $-10 \le d < -5$ | $-5 \le d < 0$ | $0 \le d < 5$ | $5 \le d < 10$ | $10 \le d < 15$ | $15 \le d < 20$ |
|---|---|---|---|---|---|---|
| **Observed frequency** | 3 | 14 | 30 | 24 | 17 | 5 |

**c** For these data find unbiased estimates of:

**i** the mean

**ii** the variance. *[2]*

Laura conducts a chi-squared goodness of fit test to determine whether these data are consistent with being from a normal distribution. The test is carried out at a 10% significance level.

**d** **i** Write down the hypotheses for this test.

**ii** Copy and complete the following table.

| **Difference** | $d < -5$ | $-5 \le d < 0$ | $0 \le d < 5$ | $5 \le d < 10$ | $10 \le d < 15$ | $d \ge 15$ |
|---|---|---|---|---|---|---|
| **Expected frequency** | | | | | | |

**iii** Write down the number of degrees of freedom.

**iv** Find the $p$-value for the test.

**v** State the conclusion of the test. Give a reason for your answer. *[9]*

**e** Explain whether the result of this test supports the validity of the test in part **b**. *[1]*

**3** *[Maximum mark: 15]*

The table shows the lengths (in km) of roads connecting seven villages. The information can also be represented on a weighted undirected graph.

| | **B** | **C** | **D** | **E** | **F** | **G** |
|---|---|---|---|---|---|---|
| **A** | 7 | – | 12 | 10 | – | 8 |
| **B** | – | – | – | 10 | – | – |
| **C** | | – | 8 | 12 | 17 | 9 |
| **D** | | | – | – | – | – |
| **E** | | | | – | 14 | – |
| **F** | | | | | – | – |

**a** Write down the degree of each vertex of the graph. *[2]*

**b** Draw the graph. *[2]*

**c** A road inspector would like to drive along each road exactly once and return to the starting point. Explain why it is possible to do this. Find the length of his route. *[3]*

**d** The road between A and G is closed. Find the length of the shortest route the inspector can take in order to drive along each road exactly once and return to the starting point. State which, if any, roads need to be used twice. *[4]*

**e** Internet cables are to be placed under the roads so that each village is connected, directly or indirectly, to village A. Use Kruskal's algorithm to find the minimum length of cable required. *[4]*

**4** *[Maximum mark: 21]*

In a simple predator–prey model, the numbers of predators ($x$ hundred) and prey ($y$ thousand), at time $t$ years, are modelled by the system of equations

$$\begin{cases} \dot{x} = -5x + 2y \\ \dot{y} = -6x + 3y \end{cases}$$

**a** Show that $\begin{pmatrix}1\\1\end{pmatrix}$ and $\begin{pmatrix}1\\3\end{pmatrix}$ are eigenvectors of the matrix $\begin{pmatrix}-5 & 2\\-6 & 3\end{pmatrix}$ and find the corresponding eigenvalues. *[3]*

**b** Sketch the phase portrait for the system, showing the directions of the eigenvectors and the direction of the trajectories. *[3]*

**c** Write down the general solution of the system. *[2]*

**d** In the case where initially there are 100 predators and 2000 prey, find expressions for $x$ and $y$ in terms of $t$, and then describe the long-term behaviour of the two populations. *[5]*

**e** In the case where initially there are 200 predators and 1000 prey, the prey population dies out when $t = T_0$.

**i** Find the value of $T_0$, and find the number of predators at that time.

**ii** After there are no more prey, the number of predators satisfies the equation $\dot{x} = -5x$. Find a new expression for the number of predators for $t > T_0$. *[8]*

**5** *[Maximum mark: 18]*

A ball is thrown from the origin $O$ with velocity $(8\mathbf{i} + 14\mathbf{j})\,\text{m s}^{-1}$. It moves freely under gravity so has acceleration $-9.8\mathbf{j}\,\text{m s}^{-2}$.

**a** Find the ball's velocity at time $t$. *[3]*

The ball passes through the point $P$ $t$ seconds after being thrown. The point $Q$ is vertically below $P$ on the same horizontal plane as $O$ and $OQ = 2PQ$.

**b** Find the value of $t$. *[5]*

**c** Find the speed of the ball at $P$. *[2]*

The ball has the same speed at another point $R$ as it does at $P$.

**d** Find the time taken for the ball to travel from $O$ to $R$. *[2]*

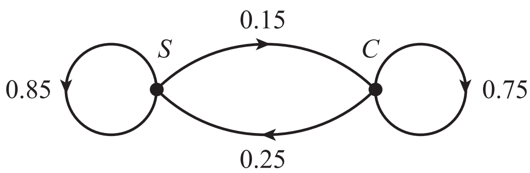
One second after the ball is thrown from $O$ a second ball is thrown from $O$ with velocity $(12\mathbf{i} + k\mathbf{j})\,\text{m s}^{-1}$.

**e** Given that the balls collide, find the value of $k$. *[6]*

**6** *[Maximum mark: 17]*

There are two subscription television providers in the market, a satellite television company, $S$, and a cable television company, $C$.

The following graph shows the probabilities of customers changing between the two providers:




**a** Write down the transition matrix, $\mathbf{T}$, for this graph. *[2]*

Initially, $S$ has 9000 subscribers in a particular town and $C$ has 7000.

**b** Find the number of subscribers each provider has after 4 years. *[2]*

**c** Find the eigenvalues and corresponding eigenvectors of $\mathbf{T}$. *[4]*

**d** Hence write down matrices $\mathbf{P}$ and $\mathbf{D}$ such that $\mathbf{T} = \mathbf{PDP}^{-1}$. *[2]*

**e** Find an expression for the number of customers $S$ has after $n$ years. *[5]*

**f** Hence state the number of customers $S$ can expect to have in the long term. *[1]*

**g** Give one reason why this model is unlikely to be accurate. *[1]*

# Mathematics: applications and interpretation
# Higher level
# Practice set B: Paper 3

Candidate session number

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| | | | | | | | | | | |

1 hour

**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphical calculator is required for this paper.
- Answer **all** questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1** *[Maximum mark: 25]*

*This question is about estimating parameters from data.*

Let $X_1$ and $X_2$ both be random variables representing independent observations from a population with mean $\mu$ and variance $\sigma^2$.

In this question you may use without proof the fact that

$\mathrm{Var}(X) = \mathrm{E}(X^2) - \mathrm{E}(X)^2$

**a** Find an expression for $\overline{X}$, the random variable representing the sample mean of the two observed values. *[1]*

**b** Show that $\mathrm{E}(\overline{X}) = \mu$ and find an expression for $\mathrm{Var}(\overline{X})$ in terms of $\sigma$. *[4]*

The sample variance is defined as

$$S^2 = \frac{X_1^2 + X_2^2}{2} - \overline{X}^2$$

**c** **i** Find $\mathrm{E}(X^2)$ in terms of $\mathrm{Var}(X)$ and $\mathrm{E}(X)$.

**ii** Show that $\mathrm{E}(S^2) = \frac{1}{2}\sigma^2$. *[4]*

An unbiased estimator of a population parameter is one whose expected value equals the population parameter.

**d** **i** Show that $M = \dfrac{2X_1 + 3X_2}{5}$ is an unbiased estimator of $\mu$.

**ii** When comparing two unbiased estimators, the one with a lower variance is said to be more efficient. Determine whether $M$ or $\overline{X}$ is a more efficient unbiased estimator of $\mu$. *[5]*

In a promotion, tokens are placed at random in boxes of cereal. $Y$ is the random variable describing the number of boxes of cereal that need to be opened up to and including the one where a token is found. Two independent investigations were conducted.

**e** The tokens are placed in cereal boxes with probability $p$. The presence of a token in a cereal box is independent of other boxes.

**i** Find an expression for $L$, the probability of observing $Y_1 = a$ and $Y_2 = b$ in terms of $a$, $b$ and $p$.

**ii** Find the value of $p$ which maximizes $L$. This is called the maximum likelihood estimator of $p$. *[8]*

In the first observation, $Y$ was found to be 4. In the second observation, $Y$ was found to be 8.

**f** **i** Find an unbiased estimate for the variance of $Y$.

**ii** Find a maximum likelihood estimate for $p$. *[3]*

**2** *[Maximum mark: 30]*

*This question is about the path of three snails chasing after each other.*

**a** Find $\left|e^{\frac{2i\pi}{3}} - 1\right|$. *[3]*

Three snails – Alf, Bill and Charlotte – are positioned on the vertices of an equilateral triangle whose centre of rotational symmetry is the origin of the Argand plane. Alf is positioned at the point $z = 1$ and Bill is above the real axis.

**b** Find the positions of the other two snails. *[2]*

**c** If Bill is stationary and Alf moves towards him at speed 1 unit per second, how far does Alf travel until he reaches Bill? How long does it take Alf to get there? *[2]*

Alf chases Bill, Bill chases Charlotte and Charlotte chases Alf. They all travel with speed 1 unit per second. The position of Alf at time $t$ is denoted by $z_A$ and the position of Bill is denoted by $z_B$.

**d** Explain why $\frac{dz_A}{dt} = \frac{z_B - z_A}{|z_B - z_A|}$ *[2]*

**e** Write $z_B$ in terms of $z_A$. *[1]*

**f** If $z_A = re^{i\theta}$, find an expression for $\frac{dz_A}{dt}$ in terms of $r$, $\theta$, $\frac{dr}{dt}$ and $\frac{d\theta}{dt}$. *[2]*

**g** Hence, by comparing real and imaginary parts, find differential equations for $\frac{dr}{dt}$ and $\frac{d\theta}{dt}$. *[7]*

**h** Solve the differential equations you found in part **g**. *[7]*

**i** How long does it take Alf to reach Bill? How far has Alf travelled until he reaches Bill? How many rotations does he make around the origin? *[4]*

# Mathematics: applications and interpretation
# Higher level
# Practice set C: Paper 1

Candidate session number

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2 hours

**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer **all** questions. Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1** *[Maximum mark: 5]*

In an arithmetic sequence, the fifth term is 7 and the tenth term is 81. Find the sum of the first 20 terms. *[5]*

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**2** *[Maximum mark: 5]*

Sacha is investigating the relationship between time spent doing homework and time spent on social media. In her year at school, at the time of the survey, 60% of students are aged 17 and the rest are aged 18. Sacha wants to represent both age groups fairly, so she takes a random sample of six 17-year-olds and four 18-year-olds.

**a** State the name of this sampling technique. *[1]*

The results are shown in the table, showing the number of hours per day spent on each activity.

| **Student** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Time spent on social media ($x$)** | 1.7 | 3.5 | 2.6 | 1.7 | 2.1 | 3.2 | 3.8 | 2.5 | 3.1 | 3.6 |
| **Time spent on homework ($y$)** | 4.2 | 2.1 | 3.2 | 3.5 | 4.2 | 2.5 | 0.6 | 2.5 | 2.7 | 1.5 |

Sacha finds that there is a strong negative correlation between the two variables, and decides to use a linear regression line to model the relationship between them.

**b** Find the equation of the regression line in the form $y = ax + b$. *[2]*

**c** Interpret, in the context of this question, the meaning of the coefficients $a$ and $b$. *[2]*

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**3** *[Maximum mark: 6]*

A child makes a caterpillar out of modelling clay. The density of the clay is $1.45\,\text{g}\,\text{cm}^{-3}$.

She starts by making a sphere of radius 3 cm.

**a** Find the mass of this sphere. *[2]*

She then adds more spheres, each with radius half the previous one.

**b** Show that the mass of the caterpillar can never reach 200 g. *[4]*

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**4** *[Maximum mark: 6]*

Data collected over the course of recent seasons shows that the probability distribution of goals scored each game by Athletic Town is as follows:

| **Goals per game** | 0 | 1 | 2 | 3 | 4 | >4 |
|---|---|---|---|---|---|---|
| **Probability** | $\frac{1}{4}$ | $k$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{16}$ | 0 |

**a** Find the value of $k$. *[2]*

**b** Find the expected number of goals scored by Athletic Town in a 38 game season. *[4]*

**5** *[Maximum mark: 6]*

$z$ is the complex number which satisfies the equation $3z - 4z^* = 18 + 21i$.

Find $\left|\frac{z}{3}\right|$. *[6]*

**6** *[Maximum mark: 8]*

Let $f(x) = \dfrac{1}{x+2}$ and $g(x) = \dfrac{1}{x}$

**a** Write down the transformation which maps the graph of $y = g(x)$ to $y = f(x)$. *[1]*

**b** Hence or otherwise write down the equations of the asymptotes of $y = f(x)$. *[2]*

**c** Find $f^{-1}(x)$. *[3]*

**d** Write down the equations of the asymptotes of $y = f^{-1}(x)$. *[2]*

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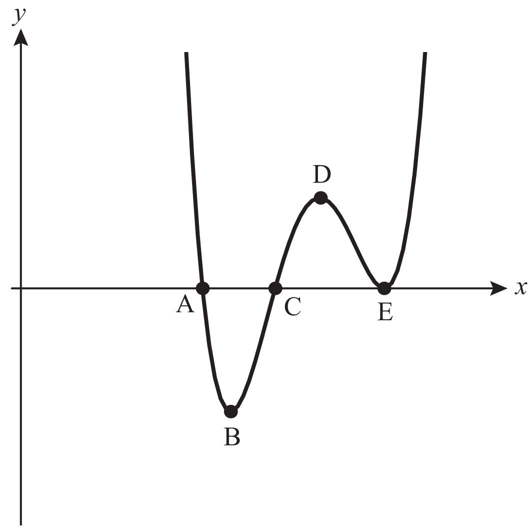
**7** *[Maximum mark: 5]*

The graph of $y = f'(x)$ is shown in the diagram.




Write down the labels of the following points, justifying your choice in each case:

**a** local maximum point(s) of $f(x)$ *[2]*

**b** point(s) of inflection of $f(x)$. *[3]*

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**8** *[Maximum mark: 5]*

Vectors **a** and **b** satisfy

$\mathbf{a} \cdot \mathbf{b} = 17$ and $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$.

Find, in degrees, the size of the angle between the two vectors. *[5]*

**9** *[Maximum mark: 6]*

A particle moves in a straight line, with the velocity at time $t$ seconds given by $v = \frac{\sin t}{\sqrt{t+1}}\,\mathrm{m\,s^{-1}}$.

**a** Find the distance travelled by the particle in the first five seconds of motion, giving your answer to one decimal place. *[3]*

**b** Find the first two times when the magnitude of acceleration is $0.3\,\mathrm{m\,s^{-2}}$. *[3]*

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**10** *[Maximum mark: 6]*

Kishan is comparing the battery life of two brands of smartphone. He collects a random sample of eight phones of Brand A and ten phones of Brand B. The results, in hours, are as follows:

Brand A: 62, 56, 72, 81, 67, 66, 73, 78

Brand B: 83, 91, 76, 67, 73, 88, 75, 77, 81, 73

**a** State an assumption necessary in order to use a $t$-test. *[1]*

**b** Test, using a 5% significance level, whether there is evidence that the mean lifetimes for the two brands are different. State the hypotheses and conclusion clearly. *[5]*

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**11** *[Maximum mark: 9]*

Two large rocks are submerged in the sea at points $A$ and $B$ with position vectors $(8\mathbf{i} - 3\mathbf{j})$ km and $(5\mathbf{i} + \mathbf{j})$ km, respectively.

A ship's captain wants to chart a course that keeps the ship equidistant from both rocks.

**a** Find the midpoint, $M$, of $\overrightarrow{AB}$. *[2]*

**b** Hence find the vector equation of the path the ship should follow. *[3]*

The ship is travelling at $20\,\text{km}\,\text{h}^{-1}$ and the ship starts on its course at 09:00.

**c** Given that the ship reaches $M$ at 09:12, find the position vector of the ship at time $t$ hours after 09:00. *[4]*

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**12** *[Maximum mark: 5]*

In a large population of children, the times taken to read a page of text have mean 7.6 minutes and variance 3.7 minutes$^2$.

**a** Estimate the probability that the mean time for a sample of 40 children is more than 8 minutes. *[4]*

**b** Explain why it was necessary to use the Central Limit Theorem in your calculation in part **a**. *[1]*

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**13** *[Maximum mark: 6]*

The force between two electrical charges is inversely proportional to the square of the distance between them: $F = \frac{k}{r^2}$, where $F$ newtons is the force and $r$ metres is the distance. When the two charges are 2 mm apart, the force between them is $3.6 \times 10^{-6}$ newtons.

**a** Find the value of $k$, giving your answer in standard form. *[2]*

**b** The charges are moving apart from each other at a speed of $0.07\,\text{m}\,\text{s}^{-1}$.
Find the rate at which the force between them is decreasing when they are 2 mm apart. *[4]*

**14** *[Maximum mark: 7]*

An ecologist wishes to develop a natural forest on a recently cultivated piece of land. There are three states: grass, $g$, shrubs, $s$, and trees, $t$, that the vegetation can be in. The forest is subdivided into a number of regions and each year these regions are observed and categorized as being in one of the three states.

From one year to the next, there is a 45% probability of a grass region becoming a shrubs region and a 5% chance of a grass region becoming a trees region.

There is a 10% chance of a shrubs region becoming a grass region and a 40% chance of it becoming a trees region.

There is a 0% chance of a trees region becoming a grass region and a 10% chance of it becoming a shrubs region.

**a** Write down the transition matrix for this system. *[2]*

The steady state of the system is $G, S, T$.

**b** Form a system of equations in $G, S$ and $T$. *[3]*

**c** Hence find the exact steady state proportion of grass, shrubs and trees in the forest. *[2]*

**15** *[Maximum mark: 6]*

Find the particular solution of the differential equation

$$(3x^2 + 1)\frac{dy}{dx} = 4x$$

for which $y = 1$ when $x = 0$. *[6]*

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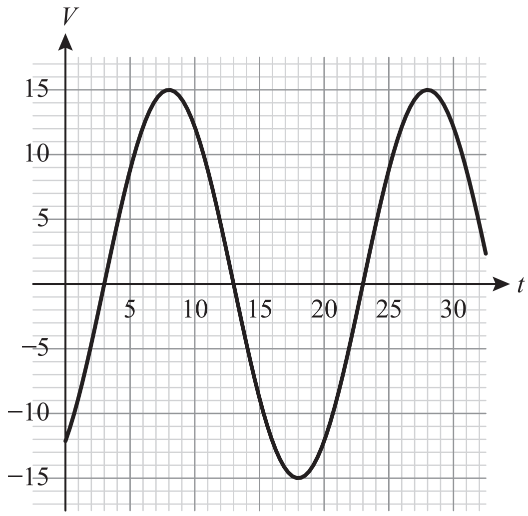
**16** *[Maximum mark: 5]*

The voltage in an electric circuit is modelled by the function $V = a \sin(b(t - c))$. The graph of $y = V(t)$ is shown.




**a** Find the values of $a$, $b$ and $c$. *[4]*

**b** On the diagram above, sketch the graph of $y = |V(t)|$. *[1]*

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**17** *[Maximum mark: 8]*

The differential equation $\frac{d^2x}{dt^2} + 0.49x = 0$ is used to model the population of fish in a lake. The variable $x$ is the number of fish above 400 at time $t$ months.

**a** By writing $y = \frac{dx}{dt}$, express the equation as a system of two first order equations. *[2]*

**b** Find the eigenvalues of the corresponding matrix. Hence sketch the phase portrait for the system. *[4]*

**c** Describe how the number of fish in the lake changes over time. *[2]*

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**18** *[Maximum mark: 6]*

The number of fish Bill catches per day at a particular lake follows a Poisson distribution with mean 8.7.

He thinks a new fishing rod will increase the number of fish he catches, so he conducts a hypothesis test by recording the number of fish caught the next day with his new rod.

**a** State the null and alternative hypotheses for this test. *[1]*

Bill wants the probability of making a Type I error to be less than 10%.

**b** Given that the mean number of fish caught with his new rod is actually 9.6, find the smallest possible probability of making a type II error. *[5]*

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# Mathematics: applications and interpretation
# Higher level
# Practice set C: Paper 2

Candidate session number

| | | | | | | | | | | |
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| | | | | | | | | | | |

1 hour 30 minutes

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphical display calculator is needed for this paper.
- Answer **all** questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1** *[Maximum mark: 21]*

The marks of Miss Rahman's class of 12 students on Mathematics Paper 1 and Paper 2 are given in the table.

| **Student** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Paper 1 | 72 | 105 | 98 | 106 | 63 | 58 | 52 | 87 | 75 | 72 | 91 | 68 |
| Paper 2 | 72 | 87 | 91 | 98 | 68 | 56 | 61 | 72 | 73 | 61 | 97 | 52 |

**a** Find the mean and standard deviation of each set of marks. Hence write two comments comparing the marks on the two papers. *[4]*

**b** The critical value of the Pearson's correlation coefficient for twelve pieces of data is 0.532. Determine whether there is significant positive correlation between the two sets of marks. *[3]*

**c** Two students did not sit Paper 2.

**i** Student 13 scored 95 marks on Paper 1. Use a regression line to estimate what mark he would have got on Paper 2.

**ii** Student 14 scored 45 marks on Paper 1. Can your regression line be used to estimate her mark for Paper 2? Justify your answer. *[5]*

**d** It is known that, in the population of all the students in the world who took Paper 1, the marks followed the distribution $N(68,11^2)$. It is also known that 12% of all students achieved Grade 7 in this paper.

**i** How many of the 12 students in Miss Rahman's class achieved Grade 7 in Paper 1?

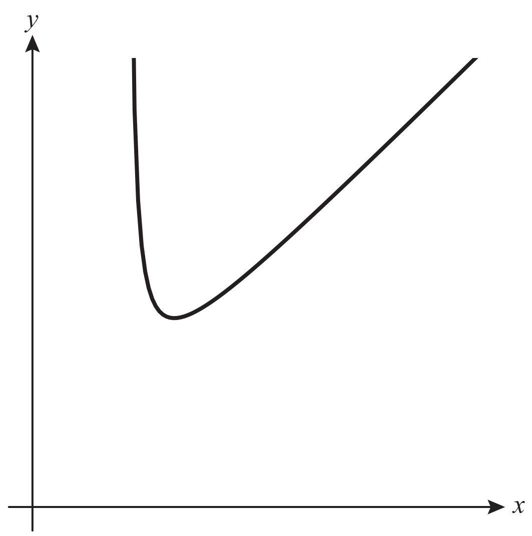
**ii** Find the probability that, in a randomly selected group of 12 students, there are more Grade 7s than in Miss Rahman's class. *[6]*

**e** Paper 1 is marked out of 110. To compare the results to another paper, Miss Rahman rescales the marks so that the maximum mark is 80.

Find the mean and standard deviation of the rescaled Paper 1 marks for the 12 students in the class. *[3]*

**2** *[Maximum mark: 19]*

The graph below shows the function $f(x) = x + \frac{1}{x-a}$ for $x > a$, where $a > 0$.




**a** Find $f'(x)$. *[2]*

**b** The function $f(x)$ is increasing for $x > 3$. Find the value of $a$. *[3]*

**c** The normal to the graph is drawn at the point P where $x = 4$. The normal crosses the graph again at the point Q.

Find the coordinates of Q. *[5]*

**d** Find the area enclosed by the curve and the $x$-axis between the points P and Q. *[2]*

**e** Find the volume generated when the region from part **d** is rotated fully around the $x$-axis. *[3]*

**f** The value of $a$ is now increased so that the area enclosed by the new curve, the $x$-axis and the lines $x = 4$ and $x = 5$ equals 5.
Find the new value of $a$. *[4]*

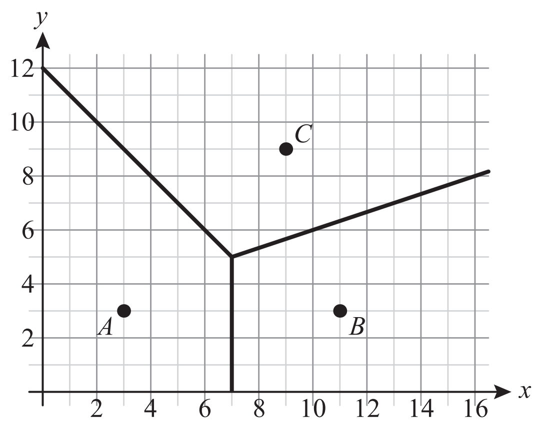
**3** *[Maximum mark: 21]*

The diagram below shows a map of a forest, with one unit representing one kilometre. The boundary of the forest is determined by the lines with equations $x = 0$, $x = 16$, $y = 0$ and $y = 12$. Three log cabins are located at A(3, 3), B(11, 3) and C(9, 9).

**a** Find the distance from B to C. *[2]*

**b** Find the bearing of C from B. *[3]*

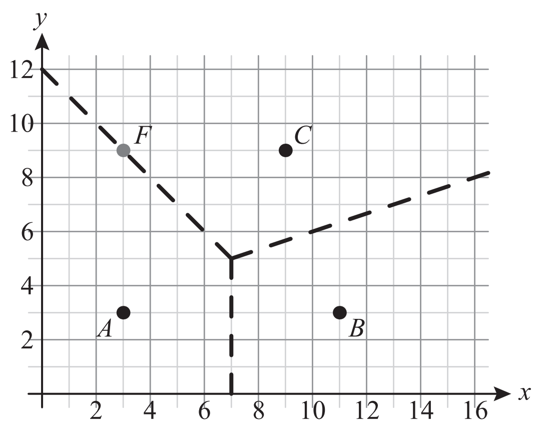
Each cabin is responsible for looking after a part of the forest. The areas of responsibility are the cells of the Voronoi diagram with sites A, B and C, as shown here.




**c** Write down the equation of the perpendicular bisector of AB and find the equation of the perpendicular bisector of BC.
Hence show that the vertex of the Voronoi diagram is located at the point with coordinates (7, 5). *[6]*

**d** Cabin A is responsible for an area of 59.5 km$^2$. Find the size of the areas of responsibility of cabins B and C. *[4]*

**e** Another cabin is built at the point F(3, 9). Draw the new Voronoi diagram, with sites A, B, C and F, on the diagram shown below. Show the edges of the new diagram as solid lines. (You do not need to show any equations of perpendicular bisectors, or calculations for the coordinates of the new vertex.) *[3]*




**f** A restaurant is to be built in the forest, inside the quadrilateral ABCF. The owners of the cabins want it to be as far as possible from any cabin. Find the coordinates of the location of the restaurant, showing all your working. *[3]*

**4** *[Maximum mark: 17]*

The transformation $T$ is represented by the matrix $\mathbf{M} = \begin{pmatrix} 5 & a \\ -1 & 7 \end{pmatrix}$

Find, in terms of $a$:

**a** the image of the point (–1, 2) under the transformation $T$. *[2]*

**b** the inverse matrix $\mathbf{M}^{-1}$. *[2]*

The transformation $T$ maps the point $P$ to the point with coordinates $(a - 20, 11)$.

**c** Find the coordinates of $P$. *[3]*

For the rest of this question, $a = -3$.

**d** Find the eigenvalues and eigenvectors of **M**. *[4]*

**e** Hence write down the equation of the invariant lines of the transformation. *[2]*

The triangle $S$ has vertices at ($k$, 2), (0, 3) and (0, 9), where $k$ is a constant.

Triangle $S$ is transformed to $S'$ by the transformation $T$.

**f** Given that the area of $S'$ is 720, find the value of $k$. *[4]*

**5** *[Maximum mark: 16]*

The table shows the prices, in GBP, of flights between six cities available on a certain day.

| | | **To** | | | | | |
|---|---|---|---|---|---|---|---|
| | | A | B | C | D | E | F |
| **From** | A | – | 128 | – | 263 | – | – |
| | B | – | – | – | 112 | – | – |
| | C | – | 96 | – | – | 206 | – |
| | D | 312 | – | – | – | 108 | – |
| | E | – | – | 217 | – | – | 89 |
| | F | – | – | – | – | 72 | – |

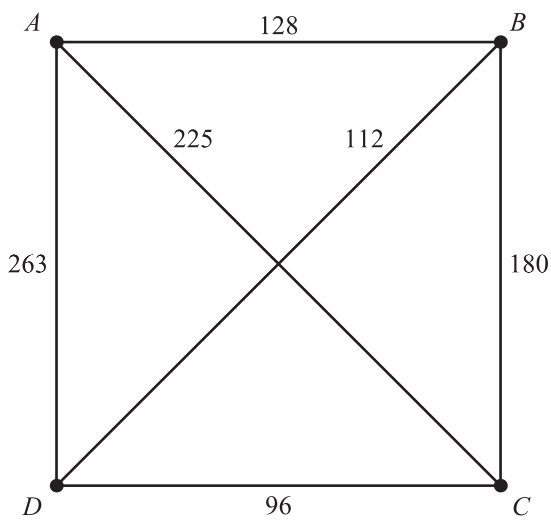
**a** What is the cheapest way to fly from A to D? State the route and the associated cost. *[2]*

**b** The flights are shown on a directed graph. Construct an adjacency matrix for this graph. *[2]*

**c** How many ways are there to fly from C to E using exactly three flights? *[2]*

**d** Jason is in city F. To which cities can he *not* fly using at most three flights? *[3]*

On a different day, the prices of direct flights between A, B, C, D are shown in the diagram below. The prices of flights between two cities are the same in both directions.



**e** Complete the table showing the cheapest way to fly (not necessarily directly) between each pair of cities. *[3]*

| | B | C | D |
|---|---|---|---|
| **A** | 128 | 225 | |
| **B** | – | 180 | |
| **C** | – | – | |

**f** Priya is a sales representative who needs to visit each of the four cities at least once and return to the starting point.

**i** Use the nearest neighbour algorithm to show that she can do this for a cost of at most 561 GBP.

**ii** By removing vertex A, find a lower bound for the cost of her trip.

**iii** Hence explain why the cheapest possible route for Priya costs exactly 561 GBP. *[4]*

**6** *[Maximum mark: 16]*

The potential energy, $V$, between two helium atoms separated by a distance $r$ is given by

$$V = \frac{A}{r^{12}} - \frac{B}{r^6}$$

where $A$, $B$ are positive constants.

**a** Find the potential energy as the separation between the atoms becomes:

**i** very small

**ii** very large. *[2]*

**b** Find, in terms of $A$ and $B$, the separation of the atoms when the potential energy is zero. *[2]*

The two particles are at their equilibrium separation when the potential energy between them is minimized.

**c** Find the equilibrium separation $r_0$. *[4]*

**d** **i** Find $\frac{d^2v}{dr^2}$

**ii** Hence justify that the value found in part **c** does give the minimum potential energy. *[5]*

The maximum binding energy of the atoms is the minimum potential energy.

**e** Show that the maximum binding energy is given by $-\frac{B^2}{2A}$. *[3]*

# Mathematics: applications and interpretation
# Higher level
# Practice set C: Paper 3

Candidate session number

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | |

1 hour

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphical display calculator is required for this paper.
- Answer **all** questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1** *[Maximum mark: 25]*

*This question is about an air traffic controller modelling the flight paths of various aircraft.*

An air traffic controller has information about the trajectories of aircrafts relative to the base of his tower. The information is in terms of vectors:

$$\begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$$

where:

$x(t)$ is the displacement east in km

$y(t)$ is the displacement north in km

$z(t)$ is the vertical displacement from sea level in km

$t$ is the time in hours after midnight.

**a** Helicopter A takes off at 9am from the point with position vector $\begin{pmatrix} 6500 \\ -4400 \\ 0 \end{pmatrix}$. It rises vertically at a rate of 3 km per hour.

**i** Find the position of helicopter A at time $t$ hours after midnight for $t \geq 9$.

**ii** State one assumption of this model. *[3]*

**b** Plane B has trajectory $\begin{pmatrix} 600t \\ 1000 - 500t \\ 10 \end{pmatrix}$ for $t > 6$.

**i** Find the position of plane B at 9am.

**ii** Find the speed of plane B.

**iii** Show that A and B do not hit each other.

**iv** The air traffic controller must provide an alert if any two aircraft are on course to come within 10 km of each other. Determine whether the air traffic controller must provide an alert for helicopter A and plane B. *[9]*

**c** Just after take-off, the position of plane C is modelled by:

$$\begin{pmatrix} -100 + 100t + 5000t^2 \\ -200 + 200t + 10\,000t^2 \\ 10t + 500t^2 \end{pmatrix}$$

**i** Show that during this phase the plane is travelling in a straight line.

**ii** Find the angle of elevation of the plane's trajectory.

**iii** Find the magnitude of the acceleration of plane C during this phase. *[8]*

**d** Plane D is in a holding pattern with position modelled by

$$\boldsymbol{d} = \begin{pmatrix} 100 \sin 3\pi t \\ 100 \cos 3\pi t \\ t \end{pmatrix}$$

The vector **v** has components $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

**i** What is the geometric interpretation of the vector $\mathbf{q} = \mathbf{d} - (\mathbf{d} \cdot \mathbf{v})\mathbf{v}$?

**ii** The velocity of plane D is $\boldsymbol{v}_d$. Evaluate $\boldsymbol{v}_d \cdot \mathbf{q}$ and hence describe the trajectory of plane D. *[5]*

**2** *[Maximum mark: 30]*

*This question is about methods for studying the rate of a chemical reaction.*

Juanita is a chemist, studying the rate of reaction of an enzyme catalyzed process. She expects a model of the form:

$$v = \frac{\alpha S}{\beta + S}$$

This is called the Michaelis–Menten model.

The dependent variable is $v$, the rate of the reaction. The independent variable is $S$, the concentration of the reactant.

There are two parameters of the system: $\alpha$ and $\beta$. Both are positive numbers.

**a** **i** Find $\frac{dv}{ds}$. Hence explain why $v$ is an increasing function of $S$.

**ii** Find the limit of $v$ as $S \to \infty$. Hence provide an interpretation for $\alpha$.

**iii** By finding $v$ when $S = \beta$ write down an interpretation of $\beta$. *[8]*

**b** Show that, if the Michaelis–Menten model is true, then the graph of $\frac{1}{v}$ against $\frac{1}{s}$ is expected to be a straight line.

Write down the gradient and the intercept of this line in terms of $\alpha$ and $\beta$. *[3]*

**c** Juanita makes five observations as shown in the table:

| Observation | $S$ | $v$ |
|---|---|---|
| $A$ | 1 | 18 |
| $B$ | 5 | 44 |
| $C$ | 10 | 62 |
| $D$ | 20 | 78 |
| $E$ | 30 | 81 |

**i** Find the equation of the line of best fit of $\frac{1}{v}$ against $\frac{1}{s}$.

**ii** Hence estimate the values of $\alpha$ and $\beta$. *[5]*

**d** For the data from part **c**, a 5% significance test on the correlation coefficient is conducted.

**i** Find the value of the sample correlation coefficient for $\frac{1}{v}$ against $\frac{1}{s}$.

**ii** State the appropriate null and alternative hypotheses for the test.

**iii** State the $p$-value of the test, and hence the conclusion. *[5]*

**e** The instruments used to measure the rate of reaction have an error quoted as 10%. Find the percentage error in $\beta$ if the true value of the rate of reaction is 10% higher than the value found in observation A. Comment on your answer. *[5]*

**f** Juanita reads in a book that a 95% confidence interval for the intercept in a regression model is

$$b_0 \pm 3.18\sqrt{MS_E\left(\frac{1}{n} + \frac{\bar{x}^2}{SS_x}\right)}$$

where:

$b_0$ is the value of the intercept found

$MS_E = \frac{\Sigma(y_i - \hat{y}_i)^2}{n-2}$

$\hat{y}_i$ is the value of $y$ predicted by the regression line for the $i$th data item

$SS_x = \Sigma(x_i - \bar{x})^2$

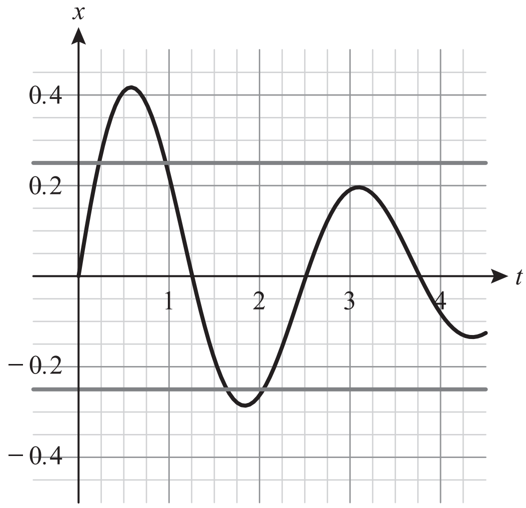
Use Juanita's formula to find a 95% confidence interval for the value of $\alpha$. *[4]*

# Practice Set A: Paper 1 Mark scheme

| | | | |
|---|---|---|---|
| **1** | **a** | $\frac{1}{4}$ or 10 seen | A1 |
| | | $\frac{10}{40} \times \frac{9}{39}$ | (M1) |
| | | $= \frac{3}{52}$ | A1 |
| | | | *[3 marks]* |
| | **b** | $\frac{10}{40} \times \frac{20}{39}$ | (M1) |
| | | $\times 2$ | (M1) |
| | | $= \frac{10}{39}$ | A1 |
| | | | *[3 marks]* |
| | | | *Total [6 marks]* |
| **2** | **a** | Find the diagonal of the square base: $\sqrt{8.3^2 + 8.3^2}$ [= 11.738] | M1 |
| | | Height $= \frac{11.738}{2}\tan(89.8°)$ | M1 |
| | | $= 1681$ | A1 |
| | | $= 1.7 \times 10^3$ cm | A1 |
| | | | *[4 marks]* |
| | **b** | $\tan^{-1}\left(\frac{\text{height}}{\text{distance}}\right)$ | |
| | | Allow this mark even if the units are incorrectly converted. | M1 |
| | | $= 53.7°$ | A1 |
| | | | *[2 marks]* |
| | | | *Total [6 marks]* |
| **3** | **a** | $H_0$: The menu choice is independent of whether the person is a teacher or a student | |
| | | $H_1$: The two factors are dependent | A1 |
| | | Attempt to calculate a chi-squared value or a p-value | M1 |
| | | $p = 0.0104$ or $\chi^2 = 9.12$ | A1 |
| | | Comparison: $p < 0.05$ | M1 |
| | | There is sufficient evidence that the menu choice is dependent on whether the person is a teacher or a student. | A1 |
| | | | *[5 marks]* |
| | **b** | e.g. She could choose another day with a different menu. | A1 |
| | | | *[1 mark]* |
| | | | *Total [6 marks]* |
| **4** | **a** | Use $x$ and $y$ values to set up three equations | M1 |
| | | $a + b + c = 7.2$ | |
| | | $4a + 2b + c = 7.4$ | |
| | | $9a + 3b + c = 6.4$ | A1 |
| | | $a = -0.60$, $b = 2.0$, $c = 5.8$ | A1 |
| | | | *[3 marks]* |
| | **b** | 5.8 m | A1**ft** |
| | | | *[1 mark]* |
| | **c** | Use GDC to solve $-0.60x^2 + 2.0x + 5.8 = 0$ | M1 |
| | | $x = 5.2$ (m) | A1 |
| | | | *[2 marks]* |
| | **d** | $\sqrt{5.2^2 + 5.8^2}$ | (M1) |
| | | $= 7.8$ m | A1 |
| | | | *[2 marks]* |
| | | | *Total [8 marks]* |
| **5** | **a** | Limits $\sqrt[3]{5}$, $\sqrt[3]{17}$ (seen in either part) | A1 |
| | | $x = \sqrt{y^3 - 1}$ | A1 |
| | | $\int \sqrt{y^3 - 1}\,dy$ | M1 |
| | | $= 2.57$ | A1 |
| | | | *[4 marks]* |
| | **b** | Using $x^2$ | M1 |
| | | $\int \pi (y^3 - 1)\,dy$ | M1 |
| | | $= 24.9$ | A1 |
| | | | *[3 marks]* |
| | | | *Total [7 marks]* |

| | | | |
|---|---|---|---|
| **6** | **a** | Using z-interval | M1 |
| | | $\bar{x} = 800.7$ | (M1) |
| | | [796, 805] | A1 |
| | | | *[3 marks]* |
| | **b** | z-interval because the underlying population distribution is normal with a known standard deviation. | A1 |
| | | | *[1 mark]* |
| | **c** | 790 is outside the confidence interval | M1 |
| | | So the customer's claim does not seem justified. | A1 |
| | | | *[2 marks]* |
| | | | *Total [6 marks]* |
| **7** | **a** | $C = 10t + 85$ | (M1)(A1) |
| | | **Note:** Award M1 for linear model with correct $y$-intercept. | |
| | | | *[2 marks]* |
| | **b** | For $0 < t < 2$: $C = 20t + 60$ | A1 |
| | | For $t \geq 2$: $C = 30t + C$ | (M1) |
| | | $20(2) + 60 = 60(2) + C$ | M1 |
| | | $C = -20$ | |
| | | $C = 60t - 20$ | A1 |
| | | Or can be given as $C = \begin{cases} 20t + 60, & 0 < t < 2 \\ 60t - 20, & t \geq 2 \end{cases}$ | |
| | | | *[4 marks]* |
| | **c** | $10t + 85 = 20t + 60$ | M1 |
| | | $t = 2.5$, which is not in domain $0 < t < 2$ | |
| | | $10t + 85 = 60t - 20$ | M1 |
| | | $t = 2.1$ | |
| | | So minimum hire time is 2.1 hours | A1 |
| | | | *[3 marks]* |
| | | | *Total [9 marks]* |
| **8** | **a** | Solve $0.003x^3 + 10x + 200 = 720$ using GDC | M1 |
| | | 36 cakes | A1 |
| | | | *[2 marks]* |
| | **b** | Sketch graph of $y = \frac{T(x)}{x}$ | M1 |
| | | Minimum point marked at $x = 32.2$ | M1 |
| | | Min = 19.3 minutes | A1 |
| | | Max = 21.2 minutes | A1 |
| | | | *[4 marks]* |
| | **c** | $S(x) = T\left(\frac{1}{2}x\right)$ | M1 |
| | | $= 0.003\left(\frac{1}{2}x\right)^3 + 10\left(\frac{1}{2}x\right) + 200$ | |
| | | $= 0.000375x^3 + 5x + 200$ | A1 |
| | | | *[2 marks]* |
| | | | *Total [8 marks]* |
| **9** | Use $\frac{u_1}{(1 - r)} = 5$ | | M1 |
| | Use $u_1 + u_1 r = 3$ | | M1 |
| | Express $u_1$ from both equations and equate: | | |
| | $5(1 - r) = \frac{3}{(1 + r)}$ | | M1 |
| | $1 - r^2 = \frac{3}{5}$ | | A1 |
| | $r = \sqrt{\frac{2}{5}}$ | | A1 |
| | | | *Total [5 marks]* |

**10 a** Use GDC to draw the graph with lines at $x = 0.25$ and $x = -0.25$ A1




Award M1A0 if only $x = 0.25$ considered. M1
4 times. A1

*[3 marks]*

**b** Attempt to use the product rule. M1
$-0.15e^{-0.3t}$ or $1.25 \cos(2.5t)$ seen M1
$\frac{dx}{dt} = -0.15e^{-0.3t} \sin(2.5t) + 1.25e^{-0.3t} \cos(2.5t)$ A1

*[3 marks]*
*Total [6 marks]*

**11** The shortest edge from A is AB, so add B next. M1
The shortest edge from A or B is AC, so add C next M1
The next shortest edge is BC, but both B and C have already been added, so skip. A1
The shortest edge between a selected and an unselected vertex is BF, so add F next. These are the edges selected so far: M1

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| **A** | – | 11 | 12 | 25 | – | – |
| **B** | **11** | – | 14 | – | 22 | 18 |
| **C** | **12** | 14 | – | – | 24 | 12 |
| **D** | 25 | – | – | – | 31 | 31 |
| **E** | – | 22 | 24 | 12 | – | 35 |
| **F** | – | **18** | 20 | 31 | 35 | – |

Next add E (BE = 22) and finally D (ED = 12) A1
The weight of the tree is (11 + 23 + 18 + 22 + 12) = 86 A1

*Total [6 marks]*

**12 a** $\int \frac{1}{C^2} dC = \int -k \, dt$ M1

$-\frac{1}{C} = -kt + c$ A1A1

**Note:** award A1 for LHS and A1 for RHS
Substitutes in : $t = 0$, $C = C_0$: M1

$-\frac{1}{C_0} = c$ A1

$\frac{1}{C} = kt + \frac{1}{C_0}$ M1

$\frac{C_0}{C} = C_0 kt + 1$ AG

$C = \frac{C_0}{C_0 kt + 1}$

*[6 marks]*

| | | | |
|---|---|---|---|
| | **b** | As $t \to \infty$, $\dfrac{C_0}{C_0kt+1} \to 0$ | M1 |
| | | So long-term concentration is zero. | A1 |
| | | | *[2 marks]* |
| | | | *Total [8 marks]* |
| **13** | **a** | $3 + 3\text{i}$ | A1 |
| | | | *[1 mark]* |
| | **b** | Multiply by $re^{\text{i}\theta}$ where $r = 2$ | M1 |
| | | and $\theta = \dfrac{\pi}{3}$ | A1 |
| | | $b = -2.20 + 8.20\text{i}$ or $b = (3 - 3\sqrt{3}) + (3 + 3\sqrt{3}\ \text{i})$ | (A1) |
| | | So the coordinates are (–2.20, 8.20) | A1 |
| | | | *[4 marks]* |
| | | | *Total [5 marks]* |
| **14** | **a** | e.g. The sample would exclude households where everyone is at work or school. | A1 |
| | | | *[1 mark]* |
| | **b** | $H_0$: $p = \dfrac{1}{5}$, $H_1$: $p > \dfrac{1}{5}$ | A1 |
| | | Using $X \sim B\left(70, \dfrac{1}{5}\right)$ | M1 |
| | | Probability calculations shown (looking for $P(X \geq k) < 0.05$ or $P(X \leq k-1) > 0.95$) | M1 |
| | | $P(X \leq 19) = 0.945 < 0.95$ | |
| | | $P(X \leq 20) = 0.970 > 0.95$ | A1 |
| | | The critical region is $X \geq 21$ | A1 |
| | | | *[5 marks]* |
| | | | *Total [6 marks]* |
| **15** | **a** | gradient $= \dfrac{-0.8}{6}\left(= -\dfrac{4}{30}\right)$ | (M1) |
| | | intercept = 4.6 | (M1) |
| | | $\ln m = 4.6 - \dfrac{4}{30}t$ | A1 |
| | | | *[3 marks]* |
| | **b** | $m = e^{4.6 - \frac{4}{30}t}$ | M1 |
| | | $= 99.5e^{-\frac{4}{30}t}$ | A1 |
| | | | *[2 marks]* |
| | | | *Total [5 marks]* |
| **16** | **a** | $\lvert\mathbf{r}\rvert = \sqrt{a^2\cos^2 kt + a^2\sin^2 kt}$ | M1 |
| | | $= \sqrt{a^2(\cos^2 kt + \sin^2 kt)}$ | |
| | | $= a$ | A1 |
| | | Object is at a fixed distance $a$ from the origin so is moving in a circle | R1 |
| | | | *[3 marks]* |
| | **b** | $\mathbf{v} = \begin{pmatrix} -ka\sin kt \\ ka\cos kt \end{pmatrix}$ | M1A1 |
| | | **Note:** Award M1 for attempt to differentiate | |
| | | $\mathbf{r}\cdot\mathbf{v} = (a\cos kt)(-ka\sin kt) + (a\sin kt)(ka\cos kt)$ | M1 |
| | | $= 0$ | A1 |
| | | So the vectors **r** and **v** are perpendicular | |
| | | | *[4 marks]* |
| | | | *Total [7 marks]* |
| **17** | | $\dfrac{dS}{dt} = 2\pi r\dfrac{dr}{dt}\ldots$ | A1 |
| | | $\ldots + \pi\dfrac{dr}{dt}\sqrt{r^2+25}\ldots$ | A1 |
| | | $\ldots + \pi r\dfrac{2r\frac{dr}{dt}}{2\sqrt{r^2+25}}$ | M1A1 |
| | | Substitute $r = 10$, $\dfrac{dr}{dt} = 2$ into their expression | M1 |
| | | $\dfrac{dS}{dt} = 252\ \text{cm}^2\,\text{s}^{-1}$ | A1 |
| | | | *Total [6 marks]* |

# Practice Set A Paper 2: Mark scheme

**1 a i** $N = 25$
$I\% = 2$
$PV = -135\,000$
$FV = 0$
$P/Y = 1$
$C/Y = 1$ (M1)(A1)
**Note:** Award M1 for an attempt to use financial package on GDC; award A1 for all entries correct.
Payment per year = £6914.76 A1

**ii** $6914.76 \times 25$ (M1)
= £172 869 A1

**iii** $172\,869 - 150\,000$ (M1)
= £22 869 A1

*[7 marks]*

**b i** $N = 360$
$I\% = 2.5$
$PV = -150\,000$
$FV = 0$
$P/Y = 12$
$C/Y = 12$ (M1)(A1)
**Note:** Award M1 for an attempt to use financial package on GDC; award A1 for all entries correct.
Payment per month = £592.68 A1

**ii** $592.68 \times 360$ (M1)
= £213 364.80 A1

*[5 marks]*

**c** EITHER
Suresh should choose Mortgage A... A1
...because total repayment is lower than Mortgage B R1
OR
Suresh should choose Mortgage B... A1
...because there is no deposit/he could invest the £15 000 R1
**Note:** Do not award R0A1.

*[2 marks]*

**d** $N = 360$
$I\% = 0$
$PV = -15\,000$
$FV = 0$
$P/Y = 12$
$C/Y = 12$ (M1)(A1)
**Note:** Award M1 for an attempt to use financial package on GDC; award A1 for all entries correct.
Interest rate > 1.25% A1

*[3 marks]*

**e** $250 \times 1.02 + 250 \times 1.02^2 + \cdots + 250 \times 1.02^n$ M1A1
$= 250 \times 1.02\left(\frac{1 - 1.02^n}{1 - 1.02}\right)$ M1
$= 12\,750(1.02^n - 1)$ A1

*[4 marks]*

*Total [21 marks]*

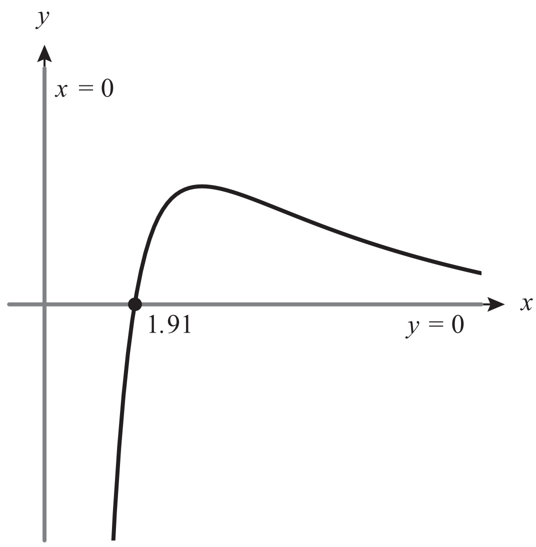
**2 a** $\cos\theta = \frac{2^2 + 4^2 - 4^2}{2 \times 2 \times 4}$ [= 0.25] (M1)
$\sin\theta = \sqrt{\frac{15}{16}}$ [= 0.968] (M1)
Area $= \frac{1}{2}(2 \times 4) \times$ their $\sin\theta$ M1
= 3.87 [cm$^2$] A1

*[4 marks]*

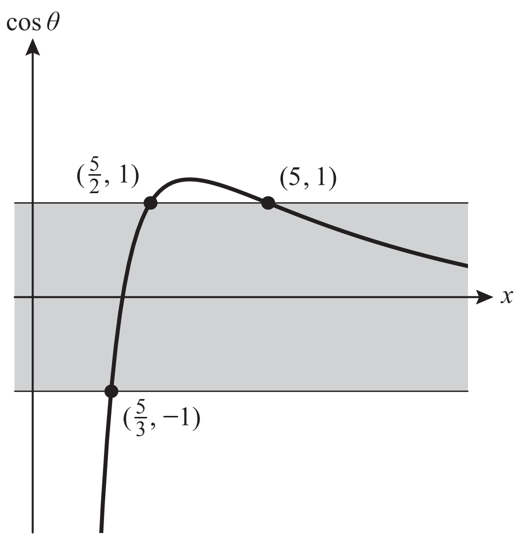
**b** The third side is $10 - 3x$ … M1
… which must be positive A1
*[2 marks]*

**c i** $(10 - 3x)^2 = x^2 + (2x)^2 - 2x(2x)\cos\theta$ M1
$100 - 60x + 9x^2 = 5x^2 - 4x^2\cos\theta$ A1
$\cos\theta = \dfrac{60x - 4x^2 - 100}{4x^2}$ M1
$= \dfrac{15x - x^2 - 25}{x^2}$ (AG)

**ii** A2




**iii** Need $-1 < \cos\theta < 1$ (allow ≤ here) M1




Intersections at $x = \dfrac{5}{3}, \dfrac{5}{2}, 5$ A1

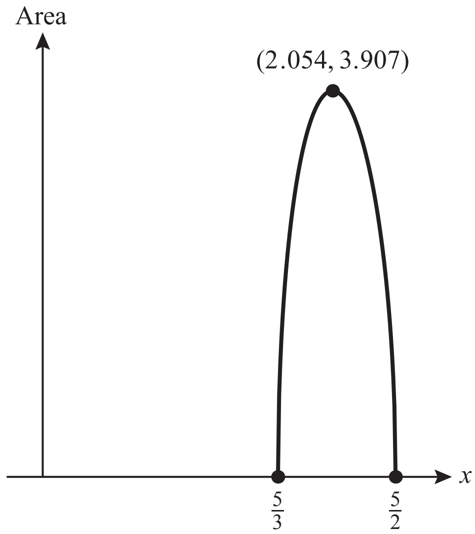
So $\dfrac{5}{3} < x < \dfrac{5}{2}$ A1
*[8 marks]*

**d** State or use $\sin\theta = \sqrt{1 - \cos^2\theta}$ M1

State or use Area $= \dfrac{1}{2}x(2x)\sin\theta$ M1

Use $\cos\theta = \dfrac{15x - x^2 - 25}{x^2}$ M1

Sketch area as a function of $x$: M1




Max area for $x = 2.05$ A1
Max area = 3.91 [cm$^2$] A1

*[6 marks]*
*Total [20 marks]*

**3 a** speed $= \sqrt{(-3)^2 + 7^2 + 2^2}$ M1
$= 7.87\,\text{km h}^{-1}$ A1

*[2 marks]*

**b** $\mathbf{r}_D = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} + t\begin{pmatrix} -3 \\ 7 \\ 2 \end{pmatrix}$ A1

$\mathbf{r}_E = \begin{pmatrix} -2 \\ 1 \\ -8 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ A1

*[2 marks]*

**c i** $\begin{cases} 4 - 3\lambda = -2 + 2\mu \ (1) \\ -2 + 7\lambda = 1 - \mu \ (2) \\ 1 + 2\lambda = -8 + 3\mu \ (3) \end{cases}$ M1

Solve any two simultaneously: $\lambda = 0$, $\mu = 3$ M1A1
Check these values in the third equation M1
So the paths cross.

**ii** (4, −2, 1) A1

*[5 marks]*

**d** D is at (4, −2, 1) when $t = 0$ but E is at (4, −2, 1) when $t = 3$ so they don't collide R1

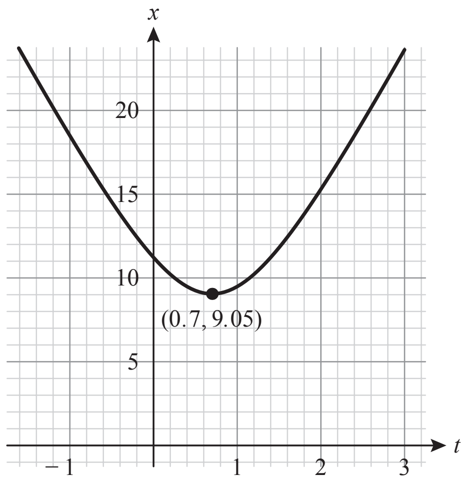
*[1 mark]*

**e i** $\overrightarrow{DE} = \begin{pmatrix} -2 + 2t \\ 1 - t \\ -8 + 3t \end{pmatrix} - \begin{pmatrix} 4 - 3t \\ -2 + 7t \\ 1 + 2t \end{pmatrix}$ M1

$= \begin{pmatrix} -6 + 5t \\ 3 - 8t \\ -9 + t \end{pmatrix}$ A1

$|\overrightarrow{DE}| = \sqrt{(-6 + 5t)^2 + (3 - 8t)^2 + (-9 + t)^2}$ M1

Sketch distance as a function of $t$: M1




$t = 0.7$ hours A1

**ii** $d_{min} = 9.05$ km A1

*[6 marks]*

*Total [16 marks]*

**4** **a** As $t \to \infty$, $P \to \dfrac{5}{1+0} = 5$ (M1)

So long term population is 5000 A1

*[2 marks]*

**b** $\dfrac{5}{P} - 1 = Ce^{-kt}$ A1

$\ln\left(\dfrac{5}{P} - 1\right) = \ln Ce^{-kt}$ M1

$\ln\left(\dfrac{5}{P} - 1\right) = \ln C + \ln e^{-kt}$ M1

$\ln\left(\dfrac{5}{P} - 1\right) = \ln C - kt$ AG

*[3 marks]*

**c** $\ln\left(\dfrac{5}{P} - 1\right) = -0.251t + 0.973$ M1A1

$k = 0.251$ A1

$C = e^{0.973}$ M1

$C = 2.65$ A1

*[5 marks]*

**d** **i** $R^2 = 0.996$ A1

**ii** Since 0.996 is very close to 1... R1

...this suggests a very good fit for the model A1

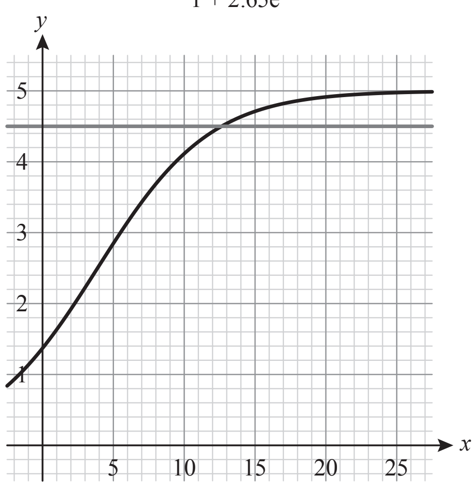
**Note:** do not award R0A1

*[3 marks]*

**e** **i** $P = \dfrac{5}{1 + 2.65e^{0}}$ M1

$P = 1370$ A1

**ii** Intersection of $y = \dfrac{5}{1 + 2.65e^{-0.251x}}$ and $y = 4.5$ M1




$t = 12.6$ years A1

*[4 marks]*

**f** Interpolation is required as both are outside the range of the data... R1
... so both could be unreliable/should be treated cautiously A1

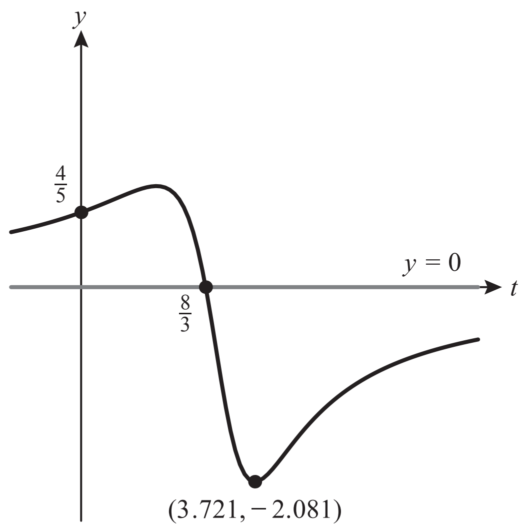
*[2 marks]*

*Total [19 marks]*

**5** **a** $v(0) = \frac{8}{10} = 0.8\,\text{m s}^{-1}$ A1

*[1 mark]*

**b** Sketch graph $y = v(t)$ and identify minimum point (M1)




Max speed = $|-2.08| = 2.08\,\text{m s}^{-1}$

**Note**: Award M1A0 for $-2.08\,\text{m s}^{-1}$ A1

*[2 marks]*

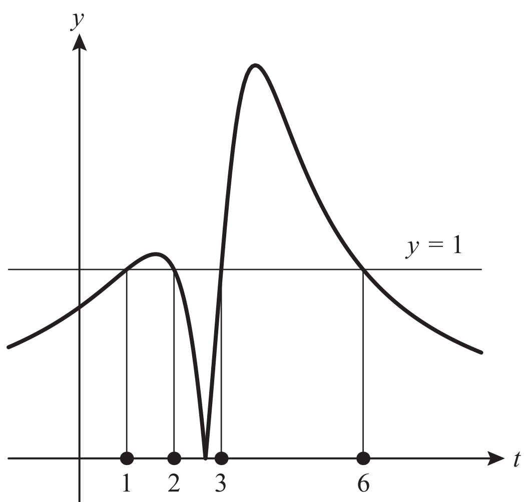
**c** EITHER

$v > 1$ for $1 < t < 2$ M1

$v < 1$ for $3 < t < 6$ M1

OR

Graph $y = |v(t)|$ M1




$|v| > 1$ for $1 < t < 2$ or $3 < t < 6$ M1

So speed > 1 for 4 seconds A1

*[3 marks]*

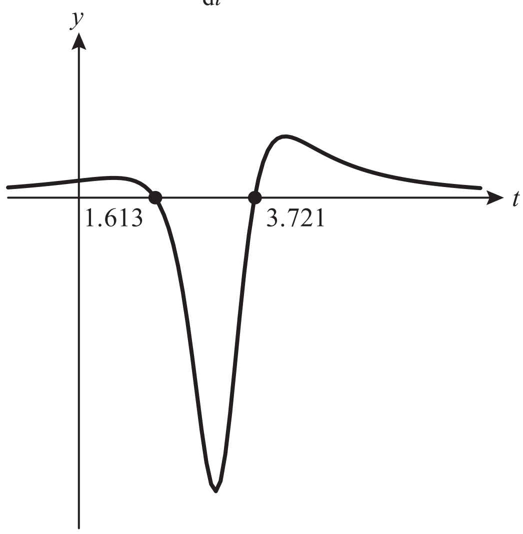
**d** Object changes direction when $v = 0$ (M1)

$t = \frac{8}{3} = 2.67$ s A1

*[2 marks]*

**e** EITHER

Sketch graph of $y = \frac{dv}{dt}$: $y < 0$ for $1.61 < t < 3.72$ (M1)




OR

Use graph of $y = v(t)$: gradient negative for $1.61 < t < 3.72$

(between turning points) (M1)

So $a < 0$ for 2.11 seconds A1

*[2 marks]*

**f** From GDC, $\frac{dv}{dt}$ at $t = 5$... (M1)

...gives $a = 0.52\,\text{m s}^{-2}$ A1

*[2 marks]*

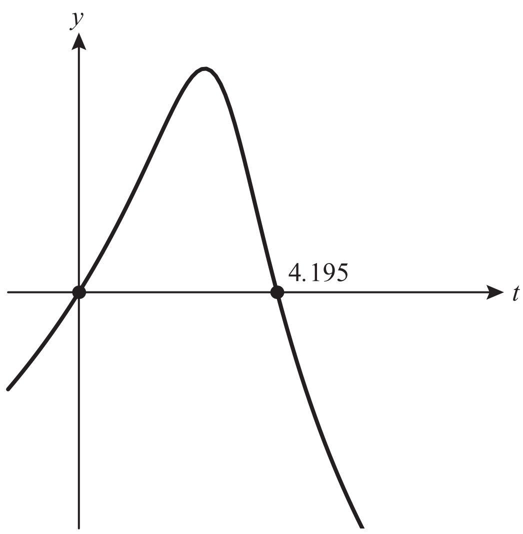
**g** From GDC:

distance $= \int_0^{10} \left| \frac{8 - 3t}{t^2 - 6t + 10} \right| dt$ M1

$= 9.83\,\text{m}$ A1

*[2 marks]*

**h** Sketch graph of $y = \int_0^x v\ dt$ (M1)




Identify $x$-intercept as being point at which object back at start (M1)

$t = 4.20$ seconds A1

*[3 marks]*

*Total [17 marks]*

**6 a** Use $\frac{dy}{dt} = -0.4x - 0.06y$ (M1)

So $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -0.4 & -0.06 \end{pmatrix}$ A1

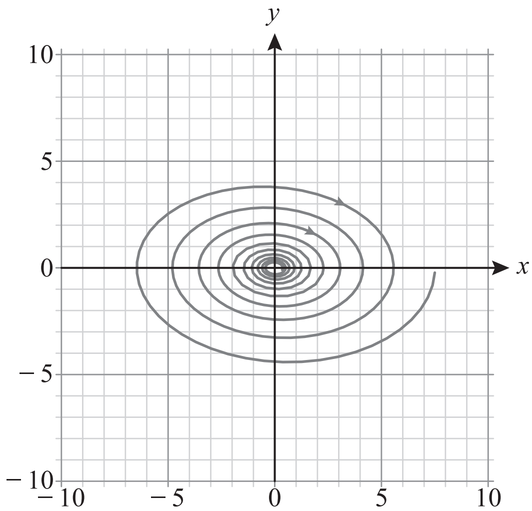
*[2 marks]*

**b** Eigenvalues: $\begin{vmatrix} -\lambda & 1 \\ -0.4 & -0.06-\lambda \end{vmatrix} = 0$ (M1)

$\lambda^2 + 0.06\lambda + 0.4 = 0$ (M1)

$\lambda = -0.03 \pm 0.632\,\text{i}$ A1

Sketch: The trajectories spiral towards the origin (because the real part is negative) M1




Direction of spiral: When $x = 0$ and $y > 0$, $\frac{dx}{dt} = y > 0$ ($x$ increases) M1

So spiral is clockwise A1

*[6 marks]*

**c** Use

$x_{n+1} = x_n + 0.05\,y_n$

$y_{n+1} = y_n + 0.05(-0.4x_n - 0.06y_n)$ (M1)

Initial values: $t = 2.5$, $x_0 = 0$, $y_0 = -3.8$ A1

Construct a table: M1

| t | $x$ | $y$ |
|---|---|---|
| 2.50 | 0.00 | −3.80 |
| 2.55 | −0.19 | −3.79 |
| 2.60 | −0.38 | −3.77 |
| 2.65 | −0.57 | −3.75 |
| 2.70 | −0.76 | −3.73 |
| 2.75 | −0.94 | −3.71 |
| 2.80 | −1.13 | −3.68 |
| 2.85 | −1.31 | −3.64 |
| 2.90 | −1.49 | −3.60 |
| 2.95 | −1.67 | −3.56 |
| 3.00 | −1.85 | −3.52 |

The distance is 1.85 cm. A1

*[4 marks]*

**d** The exact solution gives $x = -1.83$ when $t = 3$ M1

So the Euler method is quite accurate. A1

*[2 marks]*

**e** Use GDC to find stationary point or solve $\frac{dx}{dt} = 0$ [or (4.91, −5.59) seen] M1

$t = 4.91$ A1

The distance is 5.59 cm A1

*[3 marks]*

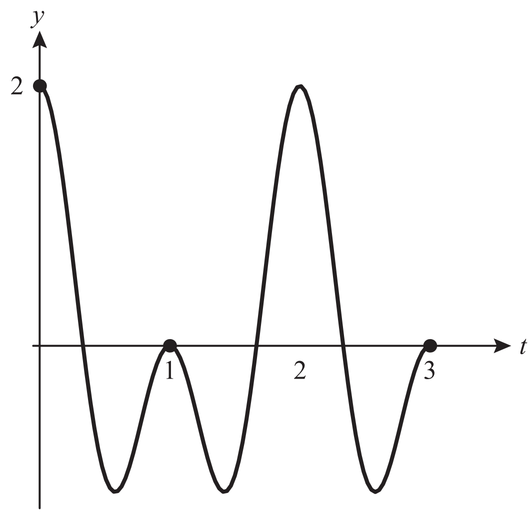
*Total [17 marks]*

# Practice Set A: Paper 3 Mark scheme

**1** **a** 2 A1

*[1 mark]*

**b** **i** A1




**ii** 2 A1

*[2 marks]*

**c** **i** $A = 4$ A1

$B = 8$ A1

$C = 20$ A1

**ii** $T = 2n$ A1

*[4 marks]*

**d** $f(t + 2n) = \cos(\pi(t + 2n)) + \cos\left(\pi\left(1 + \frac{1}{n}\right)(t + 2n)\right)$ M1

$= \cos(\pi t + 2n\pi)) + \cos\left(\left(1 + \frac{1}{n}\right)\pi t + 2\pi(n + 1)\right)$ A1

$= \cos(\pi t) + \cos\left(\left(1 + \frac{1}{n}\right)\pi t\right) = f(t)$

Since $\cos(x + 2\pi k) = \cos x$ if $k$ is an integer. R1

*[3 marks]*

**e** **i** $\text{Re}\,(e^{(A+B)i} + e^{(A-B)i}) = \text{Re}\left(e^{Ai}(e^{Bi} + e^{-Bi})\right)$

$= \text{Re}((\cos A + i \sin A)(\cos B + i \sin B + \cos B - i \sin B))$ M1

$= \text{Re}((\cos A + i \sin A)(2 \cos B))$

$= 2 \cos A \cos B$ A1

**ii** If $P = A + B$ and $Q = A - B$ then

$A = \frac{P+Q}{2}, B = \frac{P-Q}{2}$ M1

$\cos P + \cos Q = 2 \cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$ A1

*[4 marks]*

**f** $f(t) = 2 \cos\left(\pi\left(1 + \frac{1}{2n}\right)t\right)\cos\left(\frac{\pi}{2n}t\right)$

A1

The graph of $\cos\left(\pi\left(1 + \frac{1}{2n}\right)t\right)$ provides the high frequency oscillations.
Their amplitude is determined / enveloped by the lower frequency curve
$\cos\left(\frac{\pi}{2n}t\right)$ R1

*[2 marks]*

**g** $\frac{d^2x}{dt^2} = -\omega^2 \cos \omega t$ M1A1

The DE becomes:

$-\omega^2 \cos \omega t + 4 \cos \omega t = 0$ M1

This is solved when $\omega^2 = 4$ so $\omega = 2$ A1

*[4 marks]*

**h** $\frac{d^2x}{dt^2} = -4\cos 2t - k^2 g(k)\cos kt$ M1

The DE becomes:

$-4\cos 2t - k^2 g(k)\cos kt + 4\cos 2t + 4g(k)\cos kt = \cos kt$ M1

$(4g(k) - k^2 g(k))\cos kt = \cos kt$

This is true for all $t$ when $g(k)(4 - k^2) = 1$

$g(k) = \frac{1}{4 - k^2}$ A1

*[3 marks]*

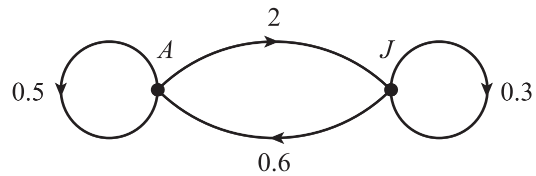
**i** When $k = 2$ A1

Since $\frac{1}{4 - k^2} \to \infty$ as $k \to 2$ R1

*[2 marks]*

*Total [25 marks]*

**2 a** A2




0.5 is a measure of the survival rate of adult badgers. A1

0.6 is a measure of the rate at which juveniles mature into adults. A1

2 is the (average) number of juveniles each adult produces. A1

0.3 is a measure of the survival rate of juveniles. A1

**Note:** Allow some leeway in the descriptions here – for example, do not worry about people confusing rates with relative rates.

*[6 marks]*

**b** The characteristic equation is

$(0.5 - \lambda)(0.3 - \lambda) - 2 \times 0.6 = 0$ (M1)

$\lambda^2 - 0.8\lambda - 1.05 = 0$ (A1)

$\lambda_1 = \frac{3}{2}, \lambda_2 = -\frac{7}{10}$ A1A1

*[4 marks]*

**c** When $\lambda = \frac{3}{2}$

$\begin{pmatrix} 0.5 & 0.6 \\ 2 & 0.3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 1.5\begin{pmatrix} x \\ y \end{pmatrix}$ (M1)

$0.5x + 0.6y = 1.5x$

$2x + 0.3y = 1.5y$ (M1)

Both equations are equivalent to $2x - 1.2y = 0$

$\boldsymbol{v}_1$ is therefore (anything parallel to) $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ A1

When $\lambda = -\frac{7}{10}$

$\begin{pmatrix} 0.5 & 0.6 \\ 2 & 0.3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -0.7\begin{pmatrix} x \\ y \end{pmatrix}$ (M1)

$0.5x + 0.6y = -0.7x$

$2x + 0.3y = -0.7y$ (M1)

Both equations are equivalent to $2x + y = 0$

$\boldsymbol{v}_2$ is therefore (anything parallel to) $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ A1

*[6 marks]*

**d** **Note:** These answers will depend on the eigenvalues quoted in part **c**.

$$\begin{pmatrix} 100 \\ 20 \end{pmatrix} = \alpha\begin{pmatrix} 3 \\ 5 \end{pmatrix} + \beta\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

So $100 = 3\alpha + \beta$

$20 = 5\alpha - 2\beta$ M1

$\alpha = 20, \beta = 40$ A1A1

*[3 marks]*

**e** $\begin{pmatrix} A_n \\ J_n \end{pmatrix} = \boldsymbol{M}^n\begin{pmatrix} 100 \\ 20 \end{pmatrix}$ (M1)

$= \boldsymbol{M}^n(20\boldsymbol{v}_1 + 40\boldsymbol{v}_2)$ (M1)

$= 20 \times (1.5)^n \boldsymbol{v}_1 + 40 \times (-0.7)^n \boldsymbol{v}_2$ (M1)

As $n \to \infty$, $(-0.7)^n \to 0$ so (R1)

$\begin{pmatrix} A_n \\ J_n \end{pmatrix} \approx 20 \times (1.5)^n \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ A1

So the long-term growth ratio is 1.5 A1

*[6 marks]*

**f** The eigenvalues are found using
$(0.5 - \lambda)(0.3 - \lambda) - x \times 0.6 = 0$ M1
$\lambda^2 - 0.8\lambda + 0.15 - 0.6x = 0$
$\lambda = \frac{0.8 \pm \sqrt{0.64 - 4(0.15 - 0.6x)}}{2} = \frac{0.8 \pm \sqrt{0.04 + 2.4x}}{2}$ A1

As seen in part **e**, the long-term growth ratio is given by the larger of the two eigenvalues. To result in decline, this must be less than 1. R1

$\frac{0.8 + \sqrt{0.04 + 2.4x}}{2} < 1$ M1
$0.8 + \sqrt{0.04 + 2.4x} < 2$
$\sqrt{0.04 + 2.4x} < 1.2$
$0.04 + 2.4x < 1.44$
$2.4x < 1.4$
$x < \frac{1.4}{2.4} = \frac{7}{12} \approx 0.583$ A1

*[5 marks]*
*Total [30 marks]*

# Practice Set B Paper 1: Mark scheme

**1** mean = 131.9, SD = 7.41 — A1
Boundaries for outliers: mean ± 2 × SD — (M1)
= 117.1, 146.7 — A1A1**ft**
147 is an outlier — A1

*[5 marks]*
*Total [5 marks]*

**2** Sector area $\frac{1}{2}(7.2)^2\theta$ (= 25.92 $\theta$) — M1
Triangle area $\frac{1}{2}(7.2)^2 \sin\theta$ (= 25.92 $\sin\theta$) — M1
$\frac{1}{2}(7.2)^2\theta - \frac{1}{2}(7.2)^2\sin\theta = 9.7$ or equivalent (e.g. $\theta - \sin\theta = 0.3742$) — A1
Solve their equation using GDC — M1
$\theta = 1.35$ — A1

*[5 marks]*
*Total [5 marks]*

**3 a** Five strips give $h = 1$ — A1
Table of values:

| $x$ | f($x$) |
|---|---|
| 0 | 0 |
| 1 | 0.09983 |
| 2 | 0.3894 |
| 3 | 0.7833 |
| 4 | 0.9996 |
| 5 | 0.5985 |

M1

0.5[0 + 0.5985 + 2(0.09983 + 0.3894 + 0.7833 + 0.9996)] — M1
= 2.57 — A1

*[4 marks]*

**b** 2.6387 — A1

*[1 mark]*

**c** $\frac{2.6387 - 2.57}{2.6387} \times 100$ — M1
= 2.60% — A1**ft**

*[2 marks]*
*Total [7 marks]*

**4** $\frac{6}{\sin(\frac{\pi}{6})} = \frac{8}{\sin(ACB)}$ — (M1)
$\sin(ACB) = \frac{2}{3}$ — A1
$ACB = 0.730$ or $2.41$ (41.8° or 138°) — A1A1
$ABC = \pi - ACB - BAC$ — (M1)
= 1.89 or 0.206 (108° or 11.8°) — A1

*Total [6 marks]*

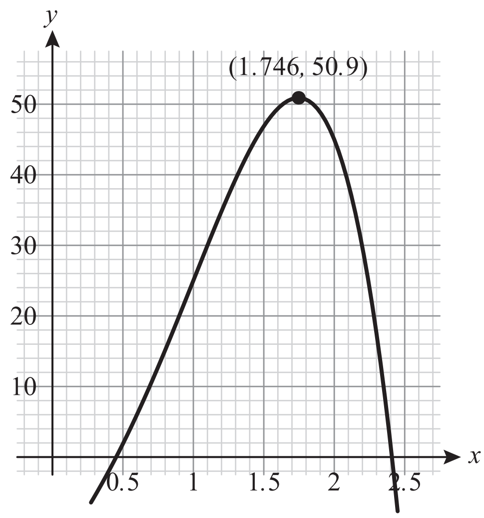
**5** P(late) = 0.8 × 0.4 + 0.2 × 0.1 (= 0.34) — (M1)
P(late and not coffee) = 0.2 × 0.1 (= 0.02) — (M1)
P(not coffee|late) — M1
$= \frac{0.02}{0.34}$ — A1
$= \frac{1}{17}$ — A1

*Total [5 marks]*

**6 a** $P(x) = \int -40x^3 + 60x^2 + 30 \, dx$ — M1
**Note:** Award M1 for evidence of integration
$= -10x^4 + 20x^3 + 30x + c$ — A1A1
**Note:** Award A1 for any two correct terms in $x$; award A1 for all terms correct and constant of integration
$45 = -10(2)^4 + 20(2)^3 + 30(2) + c$ — M1
$c = -15$
$P(x) = -10x^4 + 20x^3 + 30x - 15$ — A1

*[5 marks]*

**b** Sketch of graph (M1)




£175 A1

*[2 marks]*

*Total [7 marks]*

**7** $f \circ g(x) = \dfrac{2 - \dfrac{2}{x-1}}{\dfrac{2}{x-1} + 3}$ M1

$= \dfrac{2(x-1) - 2}{2 + 3(x-1)}$ (M1)

$= \dfrac{2x-4}{3x-1}$ A1

$x = \dfrac{2y-4}{3y-1}$

$3xy - x = 2y - 4$ (M1)

$3xy - 2y = x - 4$ M1

$y = \dfrac{x-4}{3x-2}$ A1

*Total [6 marks]*

**8** **a** Attempt to solve $2e^{-t^2} - 1 = 0$ graphically or otherwise (M1)

$t = 0.833$ s A1

*[2 marks]*

**b** $\int_0^4 2e^{-t^2} - 1 \, dt$ M1

$= -2.23$ m A1

*[2 marks]*

**c** $\int_0^4 \left|2e^{-t^2} - 1\right| dt$ M1

$= 3.26$ m A1

*[2 marks]*

*Total [6 marks]*

**9** **a** Cars arrive independently of each other A1

Cars arrive at a constant average rate A1

*[2 marks]*

**b** $X \sim Po(14)$ (M1)

$P(X > 15) = 1 - P(X \leq 15)$ (M1)

$= 0.331$ A1

*[3 marks]*

**c** $Y \sim Po(45)$ (M1)

$P(Y \leq 39) = 0.208$ A1

*[2 marks]*

*Total [7 marks]*

**10 a**

| | **A** | **B** | **C** | **D** | **E** |
|---|---|---|---|---|---|
| **A** | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ |
| **B** | $\frac{1}{2}$ | 0 | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| **C** | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 |
| **D** | 0 | 0 | $\frac{1}{3}$ | 0 | 0 |
| **E** | $\frac{1}{2}$ | 0 | $\frac{1}{3}$ | 0 | 0 |

M1A1

*[2 marks]*

**b** Raise the matrix to a large power (M1)

Each column is

$$\begin{pmatrix} 0.25 \\ 0.31 \\ 0.19 \\ 0.06 \\ 0.19 \end{pmatrix}$$ A1

The rank order is: B A1**ft**

A, C & E, D A1

*[4 marks]*

*Total [6 marks]*

**11** A = –1 A1

$x = 0$: $A + B = 8$ M1

$\Rightarrow B = 9$ A1

$-1 + 9e^{-2k} = 0 \Rightarrow e^{-2k} = \frac{1}{9}$ M1

Attempt taking logarithm of both sides, e.g. $2k = -\ln\left(\frac{1}{9}\right)$ M1

$k = \ln 3$ A1

*Total [6 marks]*

**12 a** $\bar{X} \sim \mathrm{N}\left(12.6, \frac{2.8^2}{40}\right)$ M1

$P(\bar{X} \leq k) = 0.02$ (M1)

The critical region is $\bar{X} \leq 11.7$ A1

*[3 marks]*

**b** $\bar{X} \sim \mathrm{N}\left(11.3, \frac{2.8^2}{40}\right)$ M1

$P(\bar{X} > 11.7)$ M1

$= 0.183$ A1

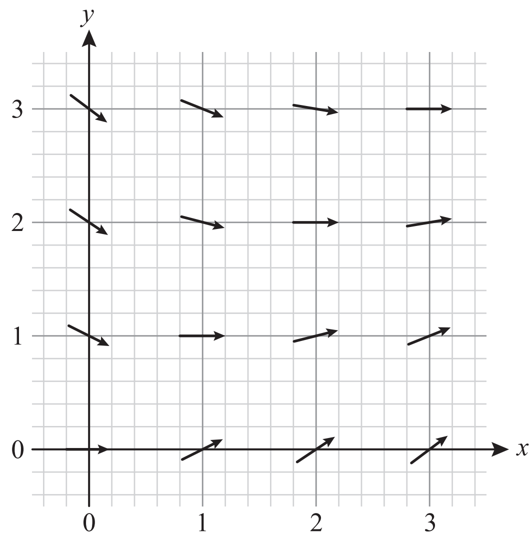
*[3 marks]*

*Total [6 marks]*

**13 a**

Table of values: M1

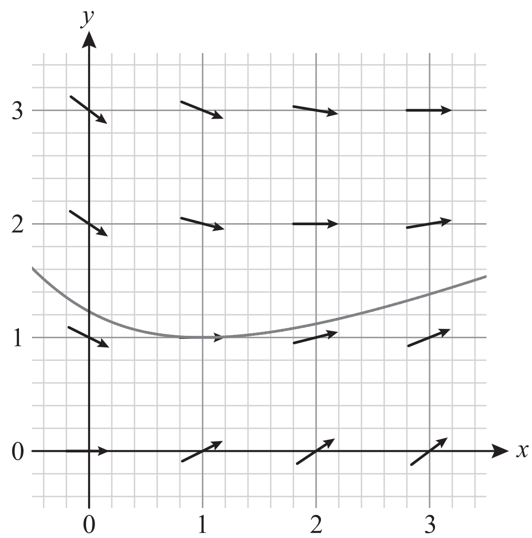
| **(x, y)** | **0** | **1** | **2** | **3** |
|---|---|---|---|---|
| **0** | 0.00 | –0.50 | –0.67 | –0.75 |
| **1** | 0.50 | 0.00 | –0.25 | –0.40 |
| **2** | 0.67 | 0.25 | 0.00 | –0.17 |
| **3** | 0.75 | 0.40 | 0.17 | 0.00 |




A2

*[3 marks]*

**b**




A1

*[1 mark]*

**c** Use Euler's method:

$x_{n+1} = x_n + 0.1, y_{n+1} = y_n + 0.1\left(\frac{x_n - y_n}{x_n + y_n + 1}\right)$ (M1)

| $x$ | $y$ |
|---|---|
| 1 | 1 |
| 1.1 | 1.000 |
| 1.2 | 1.003 |
| 1.3 | 1.009 |
| 1.4 | 1.018 |
| 1.5 | 1.029 |

A1

$y(1.5) \approx 1.03$ A1

*[3 marks]*

*Total [7 marks]*

| | | | |
|---|---|---|---|
| **14** | **a** | Saddle point at (2, 3) with at least one trajectory in each quadrant | M1 |
| | | Correct direction of arrows | A1 |
| | | | *[2 marks]* |
| | **b** | Eigenvector $\begin{pmatrix}-1\\3\end{pmatrix}$ considered or gradient –3 | M1 |
| | | $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}2\\3\end{pmatrix}+t\begin{pmatrix}-1\\3\end{pmatrix}$ or $(y-3)=-3(x-2)$ | M1 |
| | | $3x+y=9$ | A1 |
| | | | *[3 marks]* |
| | **c** | e.g. It predicts that the number of flies becomes negative. | A1 |
| | | | *[1 mark]* |
| | **d** | The number of flies increases. | A1 |
| | | | *[1 mark]* |
| | **e** | They vary periodically / oscillate | A1 |
| | | and approach (the stable population of) 400 spiders and 100 flies. | A1 |
| | | | *[2 marks]* |
| | | | *Total [9 marks]* |
| **15** | **a** | $L>2S \Leftrightarrow L-2S\geq 0$ | (M1) |
| | | $L-2S\sim N(-0.15, 0.1^2)$ | A1 |
| | | $P(L-2S\geq 0)=0.0668$ | A1 |
| | | | *[3 marks]* |
| | **b** | $L>S_1+S_2 \Leftrightarrow L-S_1-S_2\geq 0$ | (M1) |
| | | $L-S_1-S_2\sim N(-0.15, 0.0825^2)$ | A1 |
| | | $P(L-S_1-S_2\geq 0)=0.0345$ | A1 |
| | | | *[3 marks]* |
| | | | *Total [6 marks]* |
| **16** | | $h_1+h_2=\text{Im}\,(12.3e^{3.2it}+11.6e^{i(0.8+3.2t)})$ | M1 |
| | | $=\text{Im}\,[e^{3.2it}(12.3+11.6e^{0.8i})]$ | M1 |
| | | $=\text{Im}\,[e^{3.2it}\times 22.0e^{0.388i}]$ | |
| | | (convert to exponential form using GDC) | M1 |
| | | $=\text{Im}\,[22.0e^{i(3.2t+0.388)}]$ | A1 |
| | | $=22.0\sin(3.2t+0.388)$ | |
| | | Correct amplitude | A1 |
| | | Correct $(3.2t+0.388)$ | A1 |
| | | | *Total [6 marks]* |
| **17** | **a** | $f'(t)=-L(1+Ce^{-kt})^{-2}(-Cke^{-kt})$ | M1A1 |
| | | **Note:** Award M1 for attempt at chain rule | |
| | | $=\dfrac{LCke^{-kt}}{(1+Ce^{-kt})^2}$ | AG |
| | | | *[2 marks]* |
| | **b** | $f''(t)=LCk\,\dfrac{-ke^{-kt}(1+Ce^{-kt})^2-e^{-kt}2(1+Ce^{-kt})(-Cke^{-kt})}{(1+Ce^{-kt})^4}$ | M1A1 |
| | | **Note:** Award M1 for attempt at quotient/product rule | |
| | | Sets $f''(t)=0$ | M1 |
| | | $LCk\,\dfrac{-ke^{-kt}(1+Ce^{-kt})+2Cke^{-2kt}}{(1+Ce^{-kt})^3}=0$ | |
| | | $-e^{-kt}(1+Ce^{-kt})+2Ce^{-2kt}=0$ | A1 |
| | | $kt=\ln C$ | M1 |
| | | $t=\dfrac{1}{k}\ln C$ | A1 |
| | | | *[6 marks]* |
| | **c** | $\dfrac{1}{k}\ln C\geq 0$ | M1 |
| | | $C\geq 1$ | A1 |
| | | | *[2 marks]* |
| | | | *Total [10 marks]* |

# Practice Set B: Paper 2 Mark scheme

| | | | | |
|---|---|---|---|---|
| **1** | **a** | **i** | Arithmetic sequence, $u_1 = 30$, $d = 10$ | (M1) |
| | | | $u_{12} = 30 + 11 \times 10$ | M1 |
| | | | $= 140$ | A1 |
| | | **ii** | $S_{12} = 6(60 + 11 \times 10)$ or $\frac{12(30 + 140)}{2}$ | M1 |
| | | | $= 1020$ | A1 |
| | | **iii** | $\frac{N}{2}(60 + 10(N - 1)) = 2000$ | |
| | | | OR | |
| | | | Create table of values | M1 |
| | | | $N = 17.7$ | |
| | | | OR | |
| | | | $S_{17} = 1870$, $S_{18} = 2070$ | A1 |
| | | | In the 18th month | A1 |
| | | | | *[8 marks]* |
| | **b** | **i** | Geometric sequence, $u_1 = 30$, $r = 1.1$ | (M1) |
| | | | $S_{12} = \frac{30(1.1^{12} - 1)}{1.1 - 1}$ | M1 |
| | | | $= 642$ | A1 |
| | | **ii** | $30 \times 1.1^{N-1} > 1000$ | M1 |
| | | | $N = 37.8$ | (M1) |
| | | | In the 38th month | A1 |
| | | | | *[6 marks]* |
| | **c** | **i** | Multiply answer to **aii** or **bi** by the profit at least once | M1 |
| | | | Stella: 1020 × 2.20 = £2244 | A1 |
| | | | Giulio: 642 × 3.10 = £1990 | A1 |
| | | **ii** | $\frac{30(1.1^N - 1)}{0.1} \times 3.10 > \frac{N}{2}(60 + 10(N - 1)) \times 2.20$ | M1 |
| | | | $N = 21.4$ | (M1) |
| | | | In the 22nd month | A1 |
| | | | | *[6 marks]* |
| | | | | *Total [20 marks]* |
| **2** | **a** | | There are fewer assumptions in using a paired test | A1 |
| | | | A paired test eliminates variation between individuals | A1 |
| | | | | *[2 marks]* |
| | **b** | **i** | $H_0: \mu_B = \mu_A$ | |
| | | | $H_1: \mu_B > \mu_A$ | A1 |
| | | **ii** | Finds differences between values before and after | (M1) |
| | | | $p = 0.0197$ | A1 |
| | | **iii** | $0.0197 < 0.05$ | R1 |
| | | | Reject the null hypothesis; there is sufficient evidence at the 5% level of a decrease in the mean level of cholesterol | A1 |
| | | | **Note:** Award R1 for a correct comparison of their correct $p$-value to the test level, award A1 for the correct result from that comparison. Do not award R0A1. | |
| | | | | *[5 marks]* |
| | **c** | **i** | 5.35 | A1 |
| | | **ii** | 36.1 | A1 |
| | | | | *[2 marks]* |
| | **d** | **i** | $H_0$: The data follow a normal distribution | |
| | | | $H_1$: The data do not follow a normal distribution | A1 |

**ii** A2

| Difference | $d < -5$ | $-5 \le d < 0$ | $0 \le d < 5$ | $5 \le d < 10$ | $10 \le d < 15$ | $d \ge 15$ |
|---|---|---|---|---|---|---|
| **Expected frequency** | 3.95 | 13.41 | 26.98 | 28.24 | 15.38 | 5.04 |

**Note:** Award A2 for all 6 correct expected values, A1 for 4 or 5 correct values, A0 otherwise.

| | | |
|---|---|---|
| **iii** | Combining first two columns | (M1) |
| | Degrees of freedom = 5 – 2 – 1 = 2 | A1 |

**iv** $p = 0.562$ A2

**v** $0.562 > 0.1$ R1

Do not reject the null hypothesis; there is insufficient evidence at the 10% level that the data do not follow a normal distribution A1

**Note:** Award R1 for a correct comparison of their correct $p$-value to the test level, award A1 for the correct result from that comparison. Do not award R0A1.

*[9 marks]*

**e** Yes, since the differences need to be normally distributed for the paired $t$-test to be valid R1

*[1 mark]*

*Total [19 marks]*

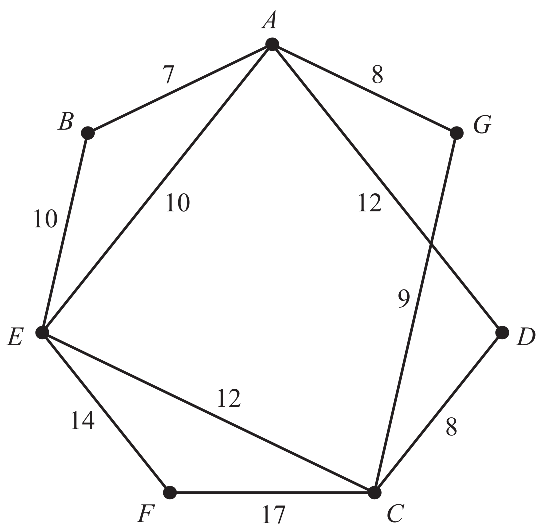
**3 a** Award 1 mark for at least two correct entries

| **Vertex** | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| **Degree** | 4 | 2 | 4 | 2 | 4 | 2 | 2 |

A1A1

*[2 marks]*

**b**




Award 1 mark for correct connections and 1 mark for correct numbers. A1A1

*[2 marks]*

**c** Every vertex has even degree R1
Attempt to add the weights of all the edges (M1)
107 km A1

*[3 marks]*

**d** Include GC (9) M1
CD(8) + DA(12) = 20
CE(12) + EA(10) = 22 M1
Repeat edges GC, CD and DA A1
Length = [(107 − 8) + (9 + 20)] = 128 km A1

*[4 marks]*

**e** Add edges, starting with AB(7), AG(8), CD(8) and CG(9) M1
Add AE or BE (10) M1
Skip CE; add EF(14) A1
The length of the cable required is 56 km A1

*[4 marks]*

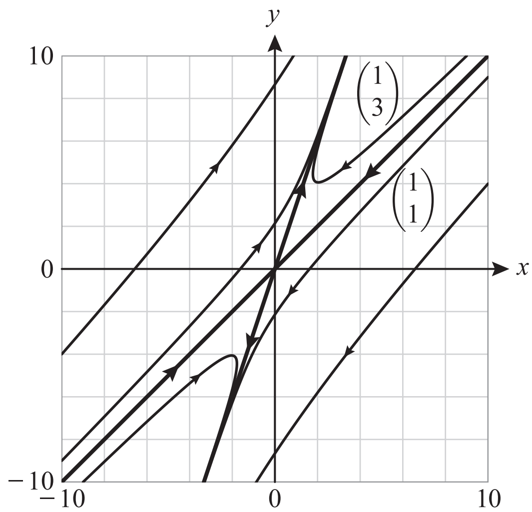
*Total [15 marks]*

**4 a** $\begin{pmatrix}-5 & 2\\-6 & 3\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}-3\\-3\end{pmatrix}, \begin{pmatrix}-5 & 2\\-6 & 3\end{pmatrix}\begin{pmatrix}1\\3\end{pmatrix}=\begin{pmatrix}1\\3\end{pmatrix}$ M1A1

The eigenvalues are −3 and 1 A1

*[3 marks]*

**b**




| | |
|---|---|
| Eigenvector directions shown | A1 |
| Trajectories OR stating that the origin is a saddle point | A1 |
| Correct directions of trajectories | A1 |
| | *[3 marks]* |

**c** $\begin{pmatrix} x \\ y \end{pmatrix} = Ae^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + Be^{t}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

| | |
|---|---|
| $e^{-3t}$ and $e^t$ terms | M1 |
| Eigenvectors and constants in correct place | A1 |
| | *[2 marks]* |

**d**

| | |
|---|---|
| Using $x = 1, y = 2, t = 0$: | |
| $(A + B = 1, A + 3B = 2)$ | (M1) |
| Solve the equations $\left(A = \frac{1}{2}, B = \frac{1}{2}\right)$ | M1 |
| $x = \frac{1}{2}e^{-3t} + \frac{1}{2}e^{t}, y = \frac{1}{2}e^{-3t} + \frac{3}{2}e^{t}$ | A1A1 |
| Both increase without a limit. | A1 |
| | *[5 marks]* |

**e**

| | |
|---|---|
| **i** Using $x = 2, y = 1, t = 0$: | |
| $(A + B = 2, A + 3B = 1)$ | (M1) |
| Solve the equations $\left(A = \frac{5}{2}, B = -\frac{1}{2}\right)$ | M1 |
| $\left[x = \frac{5}{2}e^{-3t} - \frac{1}{2}e^{t}, y = \frac{5}{2}e^{-3t} - \frac{3}{2}e^{t}\right]$ | |
| Set $y = 0$: $y = \frac{5}{2}e^{-3t} - \frac{3}{2}e^{t}$ | M1 |
| $t = 0.128$ | A1 |
| (When $t = 0.128, x = 1.14$) So 114 predators at that time. | A1 |
| **ii** Attempt to solve $\frac{dx}{dt} = -5x$ (e.g. separate variables or state exponential decay) | (M1) |
| Use $t = 0.128, x = 1.14$ | M1 |
| $x = 2.16e^{-5t}$ | A1 |
| | *[8 marks]* |
| | *Total [21 marks]* |

**5 a**

| | |
|---|---|
| $\mathbf{v} = c_1\mathbf{i} + (c_2 - 9.8t)\mathbf{j}$ | M1 |
| When $t = 0$, $\mathbf{v} = 8\mathbf{i} + 14\mathbf{j}$: $8\mathbf{i} + 14\mathbf{j} = c_1\mathbf{i} + c_2\mathbf{j}$ | M1 |
| $\mathbf{v} = 8\mathbf{i} + (14 - 9.8t)\mathbf{j}$ | A1 |
| | *[3 marks]* |

**b**

| | |
|---|---|
| $\mathbf{r} = (8t + c_1)\mathbf{i} + (14t - 4.9t^2 + c_2)\mathbf{j}$ | M1 |
| When $t = 0$, $\mathbf{r} = 0\mathbf{i} + 0\mathbf{j}$: $0\mathbf{i} + 0\mathbf{j} = c_1\mathbf{i} + c_2\mathbf{j}$ | M1 |
| $\mathbf{r} = 8t\mathbf{i} + (14t - 4.9t^2)\mathbf{j}$ | A1 |
| $OQ = 2PQ$: $8t = 2(14t - 4.9t^2)$ | M1 |
| $t = 2.04$ s | A1 |
| | *[5 marks]* |

**c** When $t = 2.04$, $\mathbf{v} = 8\mathbf{i} - 6\mathbf{j}$ — M1
speed $= \sqrt{8^2 + (-6)^2} = 10\,\text{m s}^{-1}$ — A1
*[2 marks]*

**d** $14 - 9.8t = 6$ — M1
$t = 0.816\,\text{s}$ — A1
*[2 marks]*

**e** $\mathbf{r} = 12(t-1)\mathbf{i} + (k(t-1) - 4.9(t-1)^2)\mathbf{j}$ — M1A1
For collision:
$\begin{cases} 8t = 12(t-1) \; (1) \\ 14t - 4.9t^2 = k(t-1) - 4.9(t-1)^2 \; (2) \end{cases}$ — M1
From (1): $t = 3$ — A1
Substitute their value of $t$ into (2): — M1
$k = 8.75$ — A1
*[6 marks]*
*Total [18 marks]*

**6 a** $\begin{pmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{pmatrix}$ — M1A1
*[2 marks]*

**b** $\begin{pmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{pmatrix}^4 \begin{pmatrix} 9000 \\ 7000 \end{pmatrix} = \begin{pmatrix} 9870 \\ 6130 \end{pmatrix}$ — M1A1
*[2 marks]*

**c** $\begin{vmatrix} 0.85 - \lambda & 0.25 \\ 0.15 & 0.75 - \lambda \end{vmatrix} = 0$ — M1
$\lambda = 1, 0.6$ — A1
$\begin{pmatrix} 5 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ — (M1)A1
*[4 marks]*

**d** EITHER
$\mathbf{P} = \begin{pmatrix} 5 & 1 \\ 3 & -1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 1 & 0 \\ 0 & 0.6 \end{pmatrix}$ — A1A1
OR
$\mathbf{P} = \begin{pmatrix} 1 & 5 \\ -1 & 3 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 0.6 & 0 \\ 0 & 1 \end{pmatrix}$ — A1A1
*[2 marks]*

**e** $\mathbf{P}^{-1} = \frac{1}{8}\begin{pmatrix} 1 & 1 \\ 3 & -5 \end{pmatrix}$ — A1
$\begin{pmatrix} 5 & 1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0.6^n \end{pmatrix}\frac{1}{8}\begin{pmatrix} 1 & 1 \\ 3 & -5 \end{pmatrix}\begin{pmatrix} 9000 \\ 7000 \end{pmatrix}$ — M1A1
Number of subscribers for $S = 10\,000 - 1000 \times 0.6^n$ — M1A1
*[5 marks]*

**f** 10 000 — A1
*[1 mark]*

**g** Does not take into account people who might not have subscription television to start with or who want to revert to not having it. — A1
*[1 mark]*
*Total [17 marks]*

# Practice Set B Paper 3: Mark scheme

**1 a** $\bar{X} = \frac{X_1 + X_2}{2}$ A1

*[1 mark]*

**b** $E(\bar{X}) = E\left(\frac{1}{2}X_1 + \frac{1}{2}X_2\right) = \frac{1}{2}E(X_1) + \frac{1}{2}E(X_2)$ M1

$= \frac{1}{2}\mu + \frac{1}{2}\mu$ A1

$= \mu$ AG

$\text{Var}(\bar{X}) = \text{Var}\left(\frac{1}{2}X_1 + \frac{1}{2}X_2\right) = \frac{1}{4}\text{Var}(X_1) + \frac{1}{4}\text{Var}(X_2)$ M1

$= \frac{1}{4}\sigma^2 + \frac{1}{4}\sigma^2$

$= \frac{1}{2}\sigma^2$ A1

*[4 marks]*

**c i** $E(X^2) = \text{Var}(X) + E(X)^2$ A1

**ii** $E(S^2) = E\left(\frac{X_1^2 + X_2^2}{2} - \bar{X}^2\right) = \frac{1}{2}E(X_1^2) + \frac{1}{2}E(X_2^2) - E(\bar{X}^2)$ M1

$= \frac{1}{2}(\text{Var}(X_1) + E(X_1)^2) + \frac{1}{2}(\text{Var}(X_1) + E(X_1)^2) - (\text{Var}(\bar{X}) + E(\bar{X})^2)$ M1

$= \frac{1}{2}(\sigma^2 + \mu^2) + \frac{1}{2}(\sigma^2 + \mu^2) - \left(\frac{1}{2}\sigma^2 + \mu^2\right)$ A1

$= \frac{1}{2}\sigma^2$ AG

*[4 marks]*

**d i** $E(M) = \frac{2}{5}E(X_1) + \frac{3}{5}E(X_2)$ M1

$= \frac{2}{5}\mu + \frac{3}{5}\mu$ A1

$= \mu$ AG

**ii** $\text{Var}(M) = \frac{4}{25}\text{Var}(X_1) + \frac{9}{25}\text{Var}(X_2)$ M1

$= \frac{13}{25}\sigma^2$ A1

$> \frac{1}{2}\sigma^2$ therefore $\bar{X}$ is a more efficient estimator. R1

*[5 marks]*

**e i** $L = P(Y = a)P(Y = b)$ M1

$= p(1-p)^{a-1} \times p(1-p)^{b-1}$ A1

**ii** $L = p^2(1-p)^{a+b-2}$

$\frac{dL}{dp} = 2p(1-p)^{a+b-2} - (a+b-2)p^2(1-p)^{a+b-3}$ M1A1

At a max, $\frac{dL}{dp} = 0$ M1

$p(1-p)^{a+b-3}(2(1-p) - (a+b-2)p) = 0$ M1

Since $p \neq 0$ and $p \neq 1$ at the maximum value of $L$ R1

$2 - 2p = ap + bp - 2p$

$2 = ap + bp$

$p = \frac{2}{a+b}$ A1

*[8 marks]*

**f i** $S^2 = \frac{4^2 + 8^2}{2} - 6^2 = 4$ M1

Unbiased estimate of $\sigma^2 = 2S^2 = 8$ A1

**ii** $p = \frac{2}{4+8} = \frac{1}{6}$ A1

*[3 marks]*

*Total [25 marks]*

**2 a** $|e^{\frac{2\pi i}{3}} - 1| = |\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} - 1|$ M1

$= \left|-\frac{1}{2} + \frac{\sqrt{3}}{2}i - 1\right|$ A1

$= \sqrt{\left(-\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{3}$ A1

*[3 marks]*

| | | | |
|---|---|---|---|
| **b** | Bill: $e^{\frac{2\pi i}{3}}$ | A1 | |
| | Charlotte: $e^{\frac{4\pi i}{3}}$ | A1 | |
| | | | *[2 marks]* |
| **c** | Using part **a**: $\sqrt{3}$ units in $\sqrt{3}$ seconds. | A1A1 | |
| | | | *[2 marks]* |
| **d** | The direction from $Z_A$ to $Z_B$ is $Z_B - Z_A$ | R1 | |
| | The distance travelled per unit time is one, so this is $\dfrac{Z_B - Z_A}{\lvert Z_B - Z_A\rvert}$ | R1 | |
| | | | *[2 marks]* |
| **e** | $Z_B = e^{\frac{2\pi i}{3}} Z_A$ | A1 | |
| | | | *[1 mark]* |
| **f** | $\dfrac{dZ_A}{dt} = \dfrac{dr}{dt} e^{i\theta} + ire^{i\theta}\dfrac{d\theta}{dt}$ | M1A1 | |
| | | | *[2 marks]* |
| **g** | $\dfrac{dr}{dt} e^{i\theta} + ire^{i\theta}\dfrac{d\theta}{dt} = \dfrac{e^{\frac{2\pi i}{3}} Z_A - Z_A}{\left\lvert e^{\frac{2\pi i}{3}} Z_A - Z_A\right\rvert} = \dfrac{Z_A\left(e^{\frac{2\pi i}{3}} - 1\right)}{\lvert Z_A\rvert \left\lvert e^{\frac{2\pi i}{3}} - 1\right\rvert}$ | M1A1 | |
| | $= \dfrac{re^{i\theta}\left(e^{\frac{2\pi i}{3}} - 1\right)}{r\left\lvert e^{\frac{2\pi i}{3}} - 1\right\rvert}$ | M1A1 | |
| | $= \dfrac{e^{i\theta}}{\sqrt{3}}\left(-\dfrac{3}{2} + \dfrac{i\sqrt{3}}{2}\right)$ | A1 | |
| | $= e^{i\theta}\left(-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right)$ | | |
| | Dividing through by $e^{i\theta}$: | | |
| | $\dfrac{dr}{dt} + ir\dfrac{d\theta}{dt} = -\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i$ | | |
| | Comparing real and imaginary parts: | | |
| | $\dfrac{dr}{dt} = -\dfrac{\sqrt{3}}{2}$ | A1 | |
| | $r\dfrac{d\theta}{dt} = \dfrac{1}{2}$ | A1 | |
| | | | *[7 marks]* |
| **h** | $r = -\dfrac{\sqrt{3}}{2}t + c$ | M1 | |
| | When $t = 0$, $r = 1$ so $c = 1$ | M1 | |
| | $r = 1 - \dfrac{\sqrt{3}}{2}t$ | A1 | |
| | $\dfrac{d\theta}{dt} = \dfrac{1}{2\left(1 - \frac{\sqrt{3}}{2}t\right)} = \dfrac{1}{2 - \sqrt{3}t}$ | M1 | |
| | $\theta = \dfrac{1}{\sqrt{3}}\ln(2 - \sqrt{3}t) + c$ | A1 | |
| | When $t = 0$, $\theta = 0$ so $c = \dfrac{1}{\sqrt{3}}\ln 2$ | M1 | |
| | $\theta = -\dfrac{1}{\sqrt{3}}\ln\left(\dfrac{2}{2 - \sqrt{3}t}\right)$ | A1 | |
| | | | *[7 marks]* |
| **i** | Meet when $r = 0$ | M1 | |
| | This happens when $1 - \dfrac{\sqrt{3}}{2}t = 0$ | | |
| | So $t = \dfrac{2}{\sqrt{3}}$ | A1 | |
| | Since $v = 1$ the distance travelled is $\dfrac{2}{\sqrt{3}}$ units. | A1 | |
| | As $t \to \dfrac{2}{\sqrt{3}}$, $\theta \to \infty$ so the snails make an infinite number of rotations | A1 | |
| | | | *[4 marks]* |
| | | | *Total [30 marks]* |

# Practice Set C Paper 1: Mark scheme

| | |
|---|---|
| **1** $a + 4d = 7$, $a + 9d = 81$ | M1A1 |
| Solving: | |
| $a = -52.2$, $d = 14.8$ | A1 |
| $S_{20} = \frac{20}{2}(-104.4 + 19 \times 14.8)$ | (M1) |
| $= 1768$ | A1 |
| | *Total [5 marks]* |
| **2 a** Stratified sampling | A1 |
| | *[1 mark]* |
| **b** Correct regression line attempted | M1 |
| $y = -1.33x + 6.39$ | A1 |
| | *[2 marks]* |
| **c** For every extra hour spent on social media, 1.33 hours less spent on homework. | A1 |
| No social media gives around 6.39 hours for homework. | A1 |
| | *[2 marks]* |
| | *Total [5 marks]* |
| **3 a** $\frac{4}{3}\pi(3^3) \times 1.45$ | (M1) |
| $= 164$ g | A1 |
| | *[2 marks]* |
| **b** Each volume [mass] is $\frac{1}{8}$ the previous one | A1 |
| Sum to infinity $= \frac{164}{1 - \frac{1}{8}} = 187$ g | M1A1 |
| Hence the mass is always smaller than 200 g | A1 |
| | *[4 marks]* |
| | *Total [6 marks]* |
| **4 a** $\frac{1}{4} + k + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 1$ | M1 |
| $k = \frac{5}{16}$ | A1 |
| | *[2 marks]* |
| **b** $E(G) = \left(0 \times \frac{1}{4}\right) + \left(1 \times \frac{5}{16}\right) + \left(2 \times \frac{1}{4}\right) + \left(3 \times \frac{1}{8}\right) + \left(4 \times \frac{1}{16}\right) = \frac{23}{16}$ | M1A1 |
| Their $E(G)$ multiplied by 38 | M1 |
| $= 54.6$ | A1 |
| | *[4 marks]* |
| | *Total [6 marks]* |
| **5** Write $z = x + iy$ | (M1) |
| Then $3x + 3iy - 4x + 4iy = 18 + 21i$ | A1 |
| Compare real and imaginary parts | M1 |
| $z = -18 + 3i$ | A1 |
| $\left\|\frac{z}{3}\right\| = \sqrt{6^2 + 1^2}$ | M1 |
| $= \sqrt{37}$ | A1 |
| | *Total [6 marks]* |
| **6 a** Translation 2 left | A1 |
| | *[1 mark]* |
| **b** $y = 0$ | A1 |
| $x = -2$ | A1 |
| | *[2 marks]* |
| **c** $x = \frac{1}{y + 2}$ | (M1) |
| $x(y + 2) = 1$ | (M1) |
| **Note:** The first method mark for switching $x$ and $y$ can be awarded before or after the second method mark. | |
| $y = \frac{1}{x} - 2 = f^{-1}(x)$ | A1 |
| | *[3 marks]* |

**d** $y = -2$ A1
$x = 0$ A1
*[2 marks]*
*Total [8 marks]*

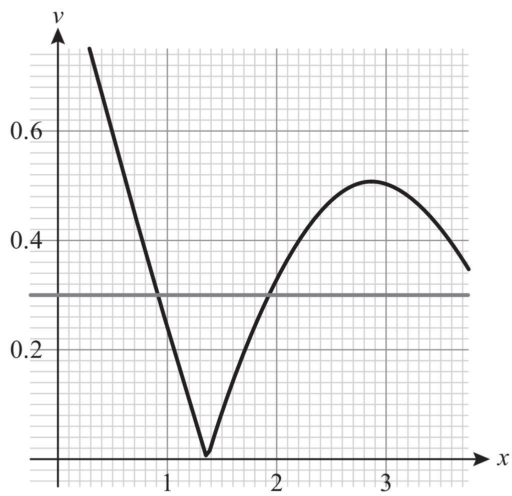
**7 a** A A1
Gradient is zero and changing from positive to negative A1
*[2 marks]*

**b** B, D A1
and E A1
Second derivative is zero and changes sign A1
*[3 marks]*
*Total [5 marks]*

**8** $|\mathbf{a}|\,|\mathbf{b}|\cos\theta = 17$ M1
$|\mathbf{a}|\,|\mathbf{b}|\sin\theta = \sqrt{4+1+25}$ $[= \sqrt{30}]$ M1
$\tan\theta = \frac{\sqrt{30}}{17}$ M1A1
$\theta = 17.9°$ A1
*Total [5 marks]*

**9 a** Integrate $|v|$ (M1)
With limits 0 and 5 (M1)
Distance = 1.8 m A1
*[3 marks]*

**b** Sketch $\left|\frac{dv}{dt}\right|$ [or $\frac{dv}{dt}$] (M1)




Intersect with $y = 0.3$ [or with both 0.3 and –0.3] (M1)
$t = 0.902$ and 1.93 seconds A1
*[3 marks]*
*Total [6 marks]*

**10 a** The underlying population distributions of lifetimes need to be normal. A1
*[1 mark]*

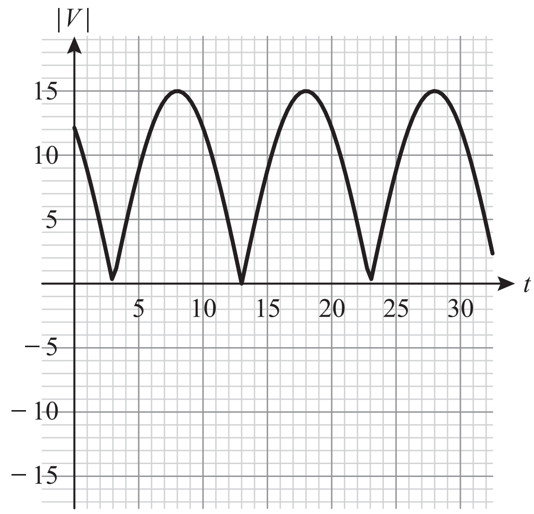
**b** $H_0$: The two population means are equal
$H_1$: The two population means are different A1
Using two-sample $t$-test (pooled) M1
$p = 0.0263$ A1
$<0.05$ M1
Sufficient evidence that the population mean lifetimes are different. A1
*[5 marks]*
*Total [6 marks]*

**11 a** $M = \frac{1}{2}((8+5)\mathbf{i} + (-3+1)\mathbf{j})$ M1
$= 6.5\mathbf{i} - \mathbf{j}$ A1
*[2 marks]*

**b** $\overrightarrow{AB} = -3\mathbf{i} + 4\mathbf{j}$ A1
Attempt to find vector perpendicular to $\overrightarrow{AB}$: $4\mathbf{i} - 3\mathbf{j}$ M1
$\mathbf{r} = 6.5\mathbf{i} - \mathbf{j} + \lambda(4\mathbf{i} - 3\mathbf{j})$ A1
*[3 marks]*

| | | | |
|---|---|---|---|
| | **c** | $\lvert 4\mathbf{i} - 3\mathbf{j}\rvert = \sqrt{4^2 - (-3)^2} = 5$ | (M1) |
| | | $\mathbf{v} = \frac{20}{5}(4\mathbf{i} - 3\mathbf{j}) = 16\mathbf{i} - 12\mathbf{j}$ | A1 |
| | | $6.5\mathbf{i} - \mathbf{j} = a\mathbf{i} + b\mathbf{j} + 0.2(16\mathbf{i} - 12\mathbf{j})$ | (M1) |
| | | $\mathbf{r} = 3.3\mathbf{i} + 1.4\mathbf{j} + t(16\mathbf{i} - 12\mathbf{j})$ | A1 |
| | | | *[4 marks]* |
| | | | *Total [9 marks]* |
| **12** | **a** | Use $\bar{X} \approx$ normal | M1 |
| | | mean = 7.6 | A1 |
| | | variance $= \frac{3.7}{40}$ | A1 |
| | | $P(\bar{X} > 8) = 0.0942$ | A1 |
| | | | *[4 marks]* |
| | **b** | We are not told that the population distribution is normal. | A1 |
| | | | *[1 mark]* |
| | | | *Total [5 marks]* |
| **13** | **a** | Substitute: $3.6 \times 10^{-6} = \frac{k}{0.002^2}$ | M1 |
| | | $k = 1.44 \times 10^{-11}$ | A1 |
| | | | *[2 marks]* |
| | **b** | Use $\frac{dr}{dt} = 0.07$ | A1 |
| | | Use $\frac{dF}{dr} = -\frac{2k}{r^3}$ | M1 |
| | | $\frac{dF}{dt} = -\frac{2k}{r^3}\frac{dr}{dt}$ | A1 |
| | | $= 2.52 \times 10^{-4}$ (N s$^{-1}$) | A1 |
| | | | *[4 marks]* |
| | | | *Total [6 marks]* |
| **14** | **a** | $\begin{pmatrix} 0.5 & 0.1 & 0 \\ 0.45 & 0.5 & 0.1 \\ 0.05 & 0.4 & 0.9 \end{pmatrix}$ | M1A1 |
| | | | *[2 marks]* |
| | **b** | $\begin{pmatrix} 0.5 & 0.1 & 0 \\ 0.45 & 0.5 & 0.1 \\ 0.05 & 0.4 & 0.9 \end{pmatrix}\begin{pmatrix} G \\ S \\ T \end{pmatrix} = \begin{pmatrix} G \\ S \\ T \end{pmatrix}$ | (M1) |
| | | $0.5G + 0.1S = G$ | |
| | | $0.45G + 0.5S + 0.1T = S$ | A1 |
| | | $0.05G + 0.4S + 0.9T = T$ | |
| | | $G + S + T = 1$ | A1 |
| | | | *[3 marks]* |
| | **c** | Reduce to system of three equations in three unknowns | M1 |
| | | $G = \frac{2}{53}, S = \frac{10}{53}, T = \frac{41}{53}$ | A1 |
| | | | *[2 marks]* |
| | | | *Total [7 marks]* |
| **15** | | Separate variables and attempt to integrate: | |
| | | $\int dy = \int \frac{4x}{3x^2 + 1}\,dx$ | M1 |
| | | Use substitution $u = 3x^2 + 1$ | (M1) |
| | | Obtain $k\ln(3x^2 + 1)$ | A1 |
| | | $y = \frac{2}{3}\ln(3x^2 + 1) + c$ | A1 |
| | | Use $x = 0, y = 1$ | M1 |
| | | Obtain $c = 1$ | A1 |
| | | | *Total [6 marks]* |
| **16** | **a** | $a = 15$ | A1 |
| | | $b = \frac{2\pi}{\text{period}}$ | M1 |
| | | $= 0.314$ | A1 |
| | | $c = 3$ | A1 |
| | | | *[4 marks]* |

**b** A1




*[1 mark]*
*Total [5 marks]*

**17 a** Two equations, with the first one correct M1

$\frac{dx}{dt} = y, \frac{dy}{dt} = -0.49x$ A1

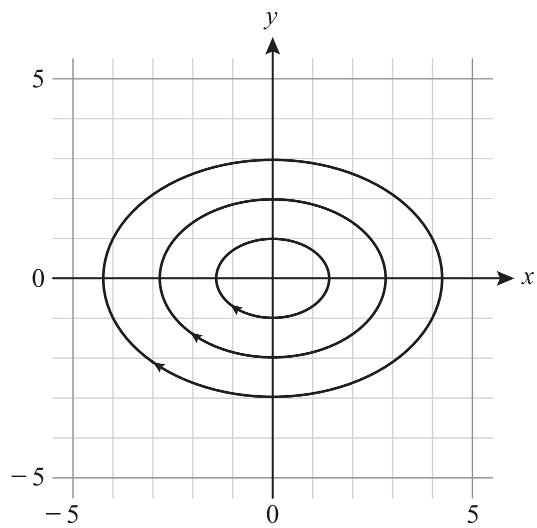
*[2 marks]*

**b** $\det\begin{pmatrix} -\lambda & 1 \\ -0.49 & -\lambda \end{pmatrix}$ OR $\lambda^2 + 0.49 = 0$ M1

$\lambda = \pm 0.7\mathrm{i}$ A1

Circle in the phase plane A1

Arrow indicating clockwise direction A1




*[4 marks]*

**c** The number of fish varies periodically / oscillates M1

Around 400 A1

*[2 marks]*
*Total [8 marks]*

**18 a** $H_0$: $\mu = 8.7$, $H_1$ : $\mu > 8.7$ A1

*[1 mark]*

**b** $P(X \geq a|\mu = 8.7) < 0.1$ M1

$a = 14$ A1

$P(X < 14|\mu = 9.6)$ M1A1

**Note:** Award M1 for attempt to find probability with their $a$ and $\mu = 9.6$

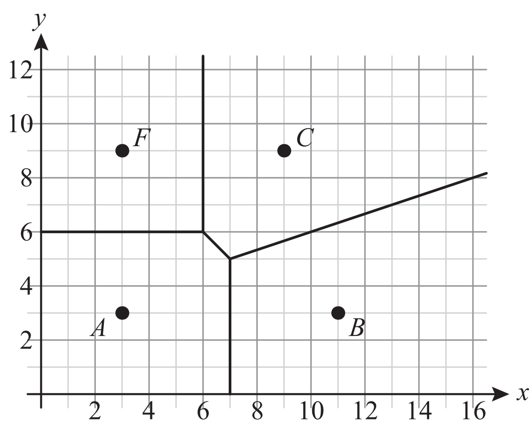
$= 0.892$ A1

*[5 marks]*
*Total [6 marks]*

# Practice Set C Paper 2: Mark scheme

| | | | | |
|---|---|---|---|---|
| **1** | **a** | | Paper 1: mean = 78.9, SD = 17.4 | A1 |
| | | | Paper 2: mean = 74.0, SD = 15.1 | A1 |
| | | | Paper 1 has higher marks on average. | A1 |
| | | | Paper 2 has more consistent marks. | A1 |
| | | | | *[4 marks]* |
| | **b** | | $r = 0.868$ | A1 |
| | | | $> 0.532$ | M1 |
| | | | There is evidence of positive correlation between the two sets of marks. | A1 |
| | | | | *[3 marks]* |
| | **c** | **i** | Find regression line $y$ on $x$ | M1 |
| | | | $y = 0.755x + 14.4$ | A1 |
| | | | $0.755 \times 95 + 14.4 \approx 86$ marks | A1 |
| | | **ii** | Cannot be used | A1 |
| | | | Mark is outside the range of available data (extrapolation) | R1 |
| | | | | *[5 marks]* |
| | **d** | **i** | Boundary for 7: inverse normal of 0.88 | M1 |
| | | | Boundary = 81 | A1 |
| | | | 5 students | A1 |
| | | **ii** | Use $B(12, 0.12)$ | (M1) |
| | | | $P(> 5) = 1 - P(\leq 5)$ | (M1) |
| | | | $= 0.00144$ | A1 |
| | | | | *[6 marks]* |
| | **e** | | Scaled mark $= \frac{80}{110} \times$ original mark | (M1) |
| | | | Mean = 57.4 | A1 |
| | | | SD = 12.7 | A1 |
| | | | | *[3 marks]* |
| | | | | *Total [21 marks]* |
| **2** | **a** | | Attempt to differentiate the fraction: obtain $\frac{1}{(x-a)^2}$ | M1 |
| | | | Obtain $1 - \frac{1}{(x-a)^2}$ | A1 |
| | | | | *[2 marks]* |
| | **b** | | Use their $f'(x) > 0$ or set $f'(x) = 0$ | (M1) |
| | | | $(3 - a)^2 = 1$ | M1 |
| | | | $a = 2$ only | A1 |
| | | | | *[3 marks]* |
| | **c** | | $f'(x) = 1 - \frac{1}{2^2} = \frac{3}{4}$ | (M1) |
| | | | Gradient of normal is $-\frac{4}{3}$ | A1 |
| | | | $y - 4.5 = -\frac{4}{3}(x - 4)$ | A1 |
| | | | Intersect with the curve: $4.5 - \frac{4}{3}(x - 4) = x + \frac{1}{x-2}$ | M1 |
| | | | Q(2.21, 6.88) | A1 |
| | | | | *[5 marks]* |
| | **d** | | $\int_{2.21}^{4} x + \frac{1}{x-a}\,dx$ | M1 |
| | | | $= 7.81$ | A1 |
| | | | | *[2 marks]* |
| | **e** | | Integrate $\left(x + \frac{1}{x-a}\right)^2$ | M1 |
| | | | Limits 2.21 and 4 | M1 |
| | | | Volume $= \pi \int_{2.21}^{4} \left(x + \frac{1}{x-2}\right)^2 dx = 109$ | A1 |
| | | | | *[3 marks]* |
| | **f** | | $\int_{4}^{5} x + \frac{1}{x-a}\,dx = \left[\frac{1}{2}x^2 + \ln(x-a)\right]_4^5$ | M1 |
| | | | $= 4.5 + \ln(5-a) - \ln(4-a)$ | A1 |
| | | | Solve this = 5 | (M1) |
| | | | $a = 2.46$ | A1 |
| | | | | *[4 marks]* |
| | | | | *Total [19 marks]* |

| | | | |
|---|---|---|---|
| **3** | **a** | $\sqrt{2^2 + 6^2}$ | (M1) |
| | | $= 6.32\,\text{km}$ | A1 |
| | | | *[2 marks]* |
| | **b** | $\tan^{-1}\begin{pmatrix}2\\6\end{pmatrix}$ OR $\tan^{-1}\begin{pmatrix}6\\2\end{pmatrix}$ | M1 |
| | | 360 – 18.4 OR 270 + 71.6 | (M1) |
| | | $= 342°$ | A1 |
| | | | *[3 marks]* |
| | **c** | Bisector of AB: $x = 7$ | A1 |
| | | Midpoint of BC: (10, 6) | A1 |
| | | Gradient of BC $= -3$ | (M1) |
| | | Bisector of BC: $y - 6 = \frac{1}{3}(x - 10)$ | A1 |
| | | $x = 7$, solve for $y$ | M1 |
| | | $y = 6 + \frac{1}{3}(7 - 10)$ | A1 |
| | | $x = 7, y = 5$ | AG |
| | | | *[6 marks]* |
| | **d** | B: $\frac{5 + 8}{2} \times 9$ | M1 |
| | | $= 58.5\,\text{km}^2$ | A1 |
| | | C: $(16 \times 12) - A - B$ | M1 |
| | | $= 74\,\text{km}^2$ | A1 |
| | | | *[4 marks]* |
| | **e** | Lines $x = 6$ and $y = 6$ added | M1 |
| | | Correct parts made solid | A1 |
| | | Rest of the diagram correct | A1 |




| | | | |
|---|---|---|---|
| | | | *[3 marks]* |
| | **f** | Calculate distance from A or C to (6, 6) and (7, 5) | M1 |
| | | $\sqrt{18} < \sqrt{20}$ | A1 |
| | | The location of the restaurant is at (7, 5) | A1 |
| | | | *[3 marks]* |
| | | | *Total [21 marks]* |
| **4** | **a** | $\begin{pmatrix}5 & a\\-1 & 7\end{pmatrix}\begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}2a - 5\\15\end{pmatrix}$ | M1 |
| | | $(2a - 5, 15)$ | A1 |
| | | | *[2 marks]* |
| | **b** | $\begin{pmatrix}5 & a\\-1 & 7\end{pmatrix}^{-1} = \frac{1}{a + 35}\begin{pmatrix}7 & -a\\1 & 5\end{pmatrix}$ | M1A1 |
| | | | *[2 marks]* |
| | **c** | $\frac{1}{a + 35}\begin{pmatrix}7 & -a\\1 & 5\end{pmatrix}\begin{pmatrix}a - 20\\11\end{pmatrix}$ | M1 |
| | | $= \frac{1}{a + 35}\begin{pmatrix}-140 - 4a\\a + 35\end{pmatrix}$ | A1 |
| | | P(–4, 1) | A1 |
| | | | *[3 marks]* |
| | **d** | $\begin{vmatrix}5 - \lambda & -3\\-1 & 7 - \lambda\end{vmatrix} = 0$ | M1 |
| | | $\lambda = 4, 8$ | A1 |
| | | Eigenvectors $\begin{pmatrix}3\\1\end{pmatrix}, \begin{pmatrix}1\\-1\end{pmatrix}$ | (M1)A1 |
| | | | *[4 marks]* |

**e** $y = \frac{1}{3}x$ and $y = -x$ A1A1

*[2 marks]*

**f** Area of $S = 3k$ A1

det $\mathbf{M} = 32$ A1

$32 \times 3k = 720$ M1

$k = 7.5$ A1

*[4 marks]*

*Total [17 marks]*

**5 a** A–B–D A1

240 (GBP) A1

*[2 marks]*

**b** At least two correct rows M1

All correct (Give A1 if rows and columns are swapped.) A1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 1 | 0 |

*[2 marks]*

**c** Cube the adjacency matrix (M1)

3 ways A1

*[2 marks]*

**d** Consider A, $A^2$ and $A^3$ M1

Looking for all three zero entries (M1)

A or D A1

*[3 marks]*

**e** A1 for each correct entry.

| | **B** | **C** | **D** | |
|---|---|---|---|---|
| **A** | 128 | 225 | 240 | A1 |
| **B** | – | 180 | 112 | A1 |
| **C** | – | – | 96 | A1 |

*[3 marks]*

**f** **i** Show the circuit with length 561 (e.g. ABDCA) A1

**ii** Minimum spanning tree for B, C, D: C–B–D (208) M1

Add edges AB and AC: 208 + 128 + 225 = 561 A1

**iii** The upper and lower bounds are equal, so this is the solution to the travelling salesman problem. R1

*[4 marks]*

*Total [16 marks]*

**6 a** **i** As $R \to 0$, $V \to \infty$ A1

**ii** As $R \to \infty$, $V \to 0$ A1

*[2 marks]*

**b** $\frac{A}{r^{12}} - \frac{B}{r^6} = 0$

$Br^6 = A$ (M1)

$r = \left(\frac{A}{B}\right)^{\frac{1}{6}}$ A1

*[2 marks]*

**c** $\frac{dV}{dr} = -12Ar^{-13} + 6Br^{-7}$ A1A1

$-12Ar_0^{-13} + 6Br_0^{-7} = 0$ M1

$-2Ar_0^{-6} + B = 0$

$r_0^{-6} = \frac{B}{2A}$

$r_0 = \left(\frac{2A}{B}\right)^{\frac{1}{6}}$ A1

*[4 marks]*

**d** **i** $\frac{d^2V}{dr^2} = 156Ar^{-14} - 42Br^{-8}$ M1A1

**ii** Substitute in their value of $r_0$ M1

$\frac{d^2V}{dr^2} = 2r^{-8}(78Ar^{-6} - 21B)$

$= 2\left(\frac{2A}{B}\right)^{-\frac{4}{3}}\left(78A\left(\frac{2A}{B}\right)^{-1} - 21B\right)$

$= 36B\left(\frac{2A}{B}\right)^{-\frac{4}{3}}$ A1

$> 0$ since $A, B > 0$, so the point is a local minimum. R1

*[5 marks]*

**e** Substitute their equilibrium separation into $V$ M1

$V_{\text{min}} = \frac{1}{\left(\frac{2A}{B}\right)}\left(\frac{A}{\left(\frac{2A}{B}\right)} - B\right)$ A1

$= \frac{B}{2A}\left(\frac{B}{2} - B\right)$ A1

$= -\frac{B^2}{4A}$ AG

*[3 marks]*

*Total [16 marks]*

# Practice Set C Paper 3: Mark scheme

**1 a i** $\begin{pmatrix} 6500 \\ -4400 \\ 0 \end{pmatrix} + (t-9)\begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$ (M1)A1

**ii** e.g. This model assumes that the helicopter is a point mass.
Or it assumes the world is flat R1
**Note:** Accept any reasonable assumption

*[3 marks]*

**b i** When $t = 9$, $B = \begin{pmatrix} 5400 \\ -3500 \\ 10 \end{pmatrix}$ A1

**ii** The velocity vector is $\begin{pmatrix} 600 \\ -500 \\ 0 \end{pmatrix}$ (M1)

The speed is $\sqrt{600^2 + (-500)^2 + 0^2} = 781 \text{ km hr}^{-1}$ A1

**iii** e.g. comparing $x$ coordinates: M1

$600t = 6500$ so $t = 10\frac{5}{6}$ (10 : 50) A1

This is not consistent with the $z$-coordinate (10 vs 5.5) so the two objects do not collide R1

**Note:** $t = 12\frac{1}{3}$ will be found if $z$-coordinates are compared first.
$t = 10.8$ will be found if $y$-coordinates are compared first. These should also get full credit if part of a coherent argument.

**iv** The distance between A and B is (M1)(A1)
$d_{AB} = \sqrt{(600t - 6500)^2 + (1000 - 500t + 4400)^2 + (10 - 3(t-9))^2}$
This can be minimized graphically.
Using the GDC the minimal distance is 13.6 km (3 s.f.) therefore there is no need to provide an alert. R1

*[9 marks]*

**c i** $C = \begin{pmatrix} -100 \\ -200 \\ 0 \end{pmatrix} + (100t + 5000t^2)\begin{pmatrix} 1 \\ 2 \\ 0.1 \end{pmatrix}$ A1

Which is of the form of a straight line. R1

**ii** For a movement of 0.1 up, the horizontal movement is $\sqrt{1^2 + 2^2} = \sqrt{5}$ A1

Therefore the angle of elevation is $\tan^{-1}\left(\frac{0.1}{\sqrt{5}}\right) = 2.56$ (M1)A1

**iii** The velocity is given by

$$\begin{pmatrix} 100 + 10\,000t \\ 200 + 20\,000t \\ 10 + 1000t \end{pmatrix}$$

The acceleration is given by (A1)

$$\begin{pmatrix} 10\,000 \\ 20\,000 \\ 1000 \end{pmatrix}$$

The magnitude of the acceleration is $\sqrt{10\,000^2 + 20\,000^2 + 1000^2}$ M1
$\approx 22\,383 \text{ km}^{-2}$ A1

*[8 marks]*

**d i** $\mathbf{q} = \begin{pmatrix} 100 \sin 3\pi t \\ 100 \cos 3\pi t \\ t \end{pmatrix} - (t)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 100 \sin 3\pi t \\ 100 \cos 3\pi t \\ 0 \end{pmatrix}$ A1

This is the horizontal component of the vector $d$ A1

**ii** $\boldsymbol{v}_d = \begin{pmatrix} 300\pi \cos 3\pi t \\ -300\pi \sin 2\pi t \\ 1 \end{pmatrix}$ A1

$$\boldsymbol{v}_d \cdot \mathbf{q} = \begin{pmatrix} 300\pi \cos 3\pi t \\ -300\pi \sin 2\pi t \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 100 \sin 3\pi t \\ 100 \cos 3\pi t \\ 0 \end{pmatrix}$$

$= 30\,000\pi \sin 3\pi t \cos 3\pi t - 30\,000\pi \sin 3\pi t \cos 3\pi t + 0 = 0$ A1

Therefore the velocity is always perpendicular to the horizontal component of the displacement, so it is a spiral. R1

*[5 marks]*

*Total [25 marks]*

| | | | | |
|---|---|---|---|---|
| **2** | **a** | **i** | Using the quotient rule | (M1) |
| | | | $u = \alpha S$ | |
| | | | $w = \beta + S$ | |
| | | | $\frac{du}{dS} = \alpha$ | |
| | | | $\frac{dw}{dS} = 1$ | |
| | | | $\frac{dv}{dS} = \frac{\alpha(\beta + S) - \alpha S}{(\beta + S)^2} = \frac{\alpha\beta}{(\beta + S)^2}$ | A1 |
| | | | Since both the numerator and the denominator are positive, this is a positive quantity so $v$ is increasing as S increases. | R1 |
| | | **ii** | Dividing the top and the bottom of the fraction by $S$: | (M1) |
| | | | $v = \frac{\alpha}{\frac{\beta}{S} + 1}$ | |
| | | | Therefore as $S \to \infty$, $v \to \alpha$ | A1 |
| | | | Therefore α is the maximum value of $v$ (when there is an excess of the reactant) | R1 |
| | | **iii** | When $S = \beta$ then $v = \frac{\alpha\beta}{\beta + \beta} = \frac{\alpha}{2}$ | A1 |
| | | | So $\beta$ is the concentration of reactant which is required to get to half of the maximum reaction rate. | R1 |
| | | | | *[8 marks]* |
| | **b** | | $\frac{1}{v} = \frac{\beta + S}{\alpha S} = \frac{\beta}{\alpha}\frac{1}{S} + \frac{1}{\alpha}$ | M1 |
| | | | This is a straight line with gradient $\frac{\beta}{\alpha}$ | A1 |
| | | | and intercept $\frac{1}{\alpha}$ | A1 |
| | | | | *[3 marks]* |
| | **c** | **i** | (table below) | (A1) |

| Observation | 1/*S* | 1/*v* |
|---|---|---|
| *A* | 1 | 0.0556 |
| *B* | 0.2 | 0.0227 |
| *C* | 0.1 | 0.0161 |
| *D* | 0.05 | 0.0128 |
| *E* | 0.0333 | 0.0123 |

| | | | | |
|---|---|---|---|---|
| | | | Line of best fit is $\frac{1}{v} = 0.0442\frac{1}{S} + 0.0117$ | A1 |
| | | **ii** | So $\frac{\beta}{\alpha} = 0.0442$ and $\frac{1}{\alpha} = 0.0117$ | (M1) |
| | | | Therefore $\beta \approx 3.79$ | A1 |
| | | | $\alpha \approx 85.7$ | A1 |
| | | | | *[5 marks]* |
| | **d** | **i** | $r = 0.997$ | A1 |
| | | **ii** | $H_0: \rho = 0$ | A1 |
| | | | $H_1: \rho \neq 0$ | A1 |
| | | **iii** | $p = 1.56 \times 10^{-4}$ | A1 |
| | | | This is less than 0.05 so there is significant evidence that there is a non-zero underlying correlation. | R1 |
| | | | | *[5 marks]* |
| | **e** | | If $v_A = 19.8$ then $\frac{1}{v} = 0.0388\frac{1}{S} + 0.0122$ | A1 |
| | | | Then $\beta = 3.19$ | A1 |
| | | | So the percentage error in the quoted value is $\frac{3.79 - 3.19}{3.19}$ | M1 |
| | | | = 19% error. | A1 |
| | | | So the error is amplified by linearization. | R1 |
| | | | | *[5 marks]* |

**f** The predicted values of $y$ are: (M1)

| Observation | $\hat{y}$ |
|---|---|
| $A$ | 0.055 919 |
| $B$ | 0.020 524 |
| $C$ | 0.016 099 |
| $D$ | 0.013 887 |
| $E$ | 0.013 149 |

Therefore $MS_E = 2.26 \times 10^{-6}$ (A1)

$SS_X = (1 - 0.277)^2 + (0.2 - 0.277)^2 + ... = 0.671$

Therefore the confidence interval for the intercept is

$0.008\,997 < c < 0.014\,352$ (A1)

Since $\alpha = \frac{1}{c}$

$69.7 < \alpha < 111$ A1

*[4 marks]*

*Total [30 marks]*