1. [5 *points*]

Find the area under the graph of $y = \arcsin x$ between $x = \frac{1}{2}$ and $x = \frac{\sqrt{3}}{2}$.

We can do $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \arcsin x \; dx$, which can be done using integration by parts. We can also interchange the x- and y-axis and get that:

$$Area = \frac{\pi}{3} \times \frac{\sqrt{3}}{2} - \frac{\pi}{6} \times \frac{1}{2} - \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin y \; dy = \frac{2\pi\sqrt{3} - \pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{2\pi\sqrt{3} - \pi + 6 - 6\sqrt{3}}{12}$$

2. [5 *points*]

Given that:

$$\int_0^{\frac{\pi}{12}} \tan^2(kx)\, dx = \frac{4-\pi}{12}$$

find the value of k, with $0 < k < 6$.

We need to replace $\tan^2(kx)$ with $\sec^2(kx) - 1$ to get:

$$\int_0^{\frac{\pi}{12}} \tan^2(kx)\, dx = \int_0^{\frac{\pi}{12}} \sec^2(kx) - 1\, dx = \left[\frac{1}{k}\tan(kx) - x\right]_0^{\frac{\pi}{12}} = \frac{1}{k}\tan\left(\frac{k\pi}{12}\right) - \frac{\pi}{12}$$

This gives:

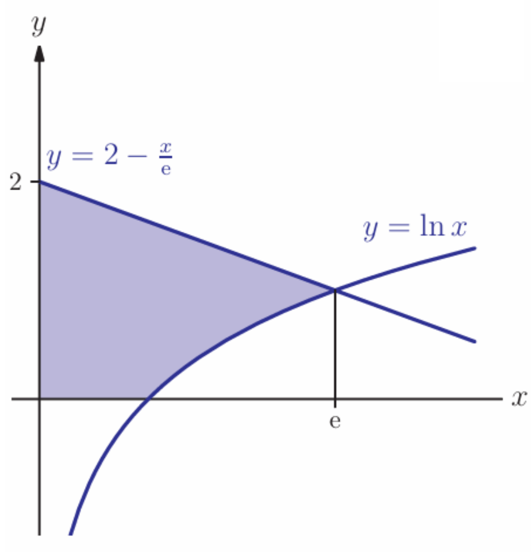
$$\frac{1}{k}\tan\left(\frac{k\pi}{12}\right) - \frac{\pi}{12} = \frac{4-\pi}{12}$$

$$\frac{1}{k}\tan\left(\frac{k\pi}{12}\right) = \frac{1}{3}$$

Since we're looking for **the** value of k, we can see that $k = 3$ works, so this is our solution.

3. [5 *points*]

The diagram shows the curves $y = 2 - \frac{x}{e}$ and $y = \ln x$. The two graphs intersect at $(e, 1)$. The shaded region is rotated 360° around the x-axis. Find the exact value of the volume of revolution.



We want:

$$Volume = \pi \int_0^e \left(2 - \frac{x}{e}\right)^2 dx - \pi \int_1^e (\ln x)^2 \, dx$$

We will do the second integral separately using integration by parts (twice):

$$\int (\ln x)^2 \, dx = x(\ln x)^2 - \int 2 \ln x \, dx = x(\ln x)^2 - 2x \ln x + 2x + c$$

We then have:

$$\pi \int_0^e \left(2 - \frac{x}{e}\right)^2 dx - \pi \int_1^e (\ln x)^2 \, dx =$$

$$= \pi \left[-\frac{e}{3}\left(2 - \frac{x}{e}\right)^3 \right]_0^e - \pi \left[x(\ln x)^2 - 2x \ln x + 2x \right]_1^e =$$

$$= \pi \left(-\frac{e}{3} + \frac{8e}{3} \right) - \pi \left(e - 2 \right) = \frac{\pi(4e + 6)}{3}$$

4. [5 *points*]

By using the substitution $u = e^x$, find the exact value of $\int_0^{\frac{1}{2}\ln 3} \frac{1}{e^x + e^{-x}}\, dx$.

Using the subsistution give we have $dx = \frac{1}{e^x} dx = \frac{1}{u} dx$, and when $x = 0, u = 1$ and when $x = \frac{1}{2}\ln 3, u = \sqrt{3}$, so the integral becomes:

$$\int_0^{\frac{1}{2}\ln 3} \frac{1}{e^x + e^{-x}}\, dx = \int_1^{\sqrt{3}} \frac{1}{u + \frac{1}{u}} \times \frac{1}{u}\, du = \int_1^{\sqrt{3}} \frac{1}{u^2 + 1}\, du = \Big[\arctan u\Big]_1^{\sqrt{3}} = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$